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LRS Bianchi Type-V Anisotropic Universe without Big Smash Driven by Law of Variation of Hubble's Parameter

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Abstract

Homogeneous and anisotropic LRS Bianchi type-V universe has been studied with ω<-1 without Big Smash driven by Hubble's law of variation of parameter. It is investigated that if cosmic dark energy behaves like a fluid with equation of state $p = \omega\rho$, (p and ρ being pressure and energy density respectively) as well as generalized Chaplygin gas simultaneously, Big Rip or Big Smash problem does not arise even for equation of state parameter $\omega < -1$. Also the scale factor for the future universe is found to be regular for all time.

Keywords: Dark Energy, Big Smash, LRS Bianchi type-V Universe.

Introduction

Recent observations of type Ia Supernova[1-4], Cosmic Microwave Background Radiations (CMBR) and galaxy clustering strongly indicate that our universe is spatially flat and accelerating. There exists an unknown exotic physical entity called as Dark Energy (DE) with negative pressure which is responsible for the same. The existence of dark energy was confirm by several high precision observations, specially the Wilkinson Microwave Anisotropy Prob (WMAP) satellite experiments[5-6] and large scale structure[7-11]. It is conclude that the dark energy constitutes 75% of total energy of universe, the dark matter about 23% and 4% of the total energy is occupied by the usual baryon matter. The dark energy is usually described by the equation of state (EOS) parameter $\omega = \frac{p}{\rho}$, the ratio of spatially homogeneous dark energy pressure p to its energy density ρ. The simplest explanation for dark energy is cosmological constant for which $\omega = -1$, also termed as ΛCDM cosmological model. Caldwell[12] noted that observational data do not rule-out the possibility that $\omega < -1$, such models are termed as phantom dark energy models. However the dark energy models, where $-1 < \omega < -\frac{1}{3}$ are not excluded by observational data[13] called quintessence dark energy models. There are some other dark energy models which can cross the phantom divide $\omega = -1$ both sides called quintom. Phantom dark energy can lead to a singularity in which the scale factor and density become infinite at a finite time called as Big Rip or Cosmic Dooms Day[14,15]. Cosmologist started making efforts to avoid this problem using $\omega < -1$ [16-17].

Sahni and Shtanov[18] have obtained cosmological model for the future universe without Big Rip problem with $\omega < -1$ in contest of Brain world scenario. Ghate and Patil[19] have investigated higher dimensional dissipative future universe without Big Rip.

Chaplygin gas is also considered as a good source of dark energy for having negative pressure with equation of state

$$P = -\frac{A}{\rho}, \tag{1}$$

where p and ρ are respectively pressure and energy density in co-moving frame of reference such that $\rho > 0$ and A is positive constant.

Bertolami *et al.*[20] have noted that Generalized Chaplygin Gas (GCG) is better for latest supernova data. The equation of state is modified in case of GCG as

$$P = -\frac{A}{\rho^{\frac{1}{\alpha}}}, \tag{2}$$

where $1 \leq \alpha < \infty$.

In particular for $\alpha = 1$, equation (2) becomes equation (1).

In this paper, LRS Bianchi type-V space-time is considered. In Section 2, the model and field equations have been presented by assuming dark energy as generalized Chaplygin gas. In Section 3, the law of variation of Hubble's parameter have been used to find the solution. In the last section 4, concluding remarks have been expressed.

Metric and Field Equations

We consider the LRS Bianchi type-V universe given by ,

$$ds^2 = dt^2 - a^2(t)dx^2 - b^2(t)e^{-2mx}(dy^2 + dz^2), \quad (3)$$

where $a(t)$, $b(t)$ are scale factors and are functions of t only, m is a constant and t is a cosmic time.

The energy momentum tensor is given by ,

$$T_j^i = (\rho + p)u^i u_j - pg_j^i, \quad (4)$$

where p is isotropic pressure, ρ is energy density and g_j^i is metric tensor.

With the help of above equations, matter tensor is given by

$$T_j^i = (\rho, -p, -p, -p). \quad (5)$$

The Einstein's field equations are,

$$R_j^i - \frac{1}{2}g_j^i R = -T_j^i, \quad (6)$$

where R_j^i is Ricci tensor , R is Ricci scalar, T_j^i is energy momentum tensor for bulk viscous cosmology.

From equations (1), (2), (3), Einstein's field equation (4) gives

$$\frac{\dot{b}^2}{b^2} + \frac{2\ddot{b}}{b} - \frac{m^2}{a^2} = -p, \quad (7)$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} - \frac{m^2}{a^2} = -p, \quad (8)$$

$$\frac{\dot{b}^2}{b^2} + \frac{2\dot{a}\dot{b}}{ab} - \frac{3m^2}{a^2} = \rho, \quad (9)$$

$$\frac{\dot{a}}{a} - \frac{\dot{b}}{b} = 0 \quad (10)$$

Here (.) dot represents the differentiation with respect to t. The energy conservation equation is given by

$$T^{ij}_{;j} = 0, \quad (11)$$

where $T^{ij}_{;j} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^j}[T^{ij}\sqrt{-g}] + T^{ik}\Gamma^i_{jk}$,

which simplifies to

$$\dot{\rho} + (p + \rho)\left(\frac{\dot{a}}{a} + \frac{2\dot{b}}{b}\right) = 0, \quad (12)$$

where $\dot{\rho}$ is differentiation of ρ with respect to t.

Define average scale factor as ,

$$B = (ab^2)^{\frac{1}{3}} \quad (13)$$

so that we have ,

$$\frac{3\dot{B}}{B} = \left(\frac{\dot{a}}{a} + \frac{2\dot{b}}{b}\right). \quad (14)$$

Using equation (14), equation (12) can be written as

$$\dot{\rho} = -\frac{3\dot{B}}{B}(p + \rho). \quad (15)$$

Using equation (2), equation (15) can be written as

$$\dot{\rho} = -\frac{3\dot{B}}{B}\left(\frac{\rho^{\frac{1+\alpha}{\alpha}} - A}{\rho^{\frac{1}{\alpha}}}\right), \quad (16)$$

where A is a constant.

Integrating equation (16) we get

$$\rho^{\frac{1+\alpha}{\alpha}} = A + [\rho_0^{\frac{1+\alpha}{\alpha}} - A]\left(\frac{B_0}{B}\right)^{\frac{3(1+\alpha)}{\alpha}}. \quad (17)$$

In present model, the equation state parameter for dark energy is the ratio of pressure p and energy density ρ given by

$$\omega = \frac{p}{\rho}. \quad (18)$$

we assumed that the dark energy behaves like Generalized Chaplygin Gas obeying equation of state as in equation (2) . Using equations (2) and (18) we get

$$\omega(t) = -\frac{A}{\rho^{\frac{1+\alpha}{\alpha}}}. \quad (19)$$

At present time $t = t_0$ and $\omega(t_0) = \omega_0$, equation (19) can be put as

$$A = -\omega_0 \rho_0^{\frac{1+\alpha}{\alpha}}. \quad (20)$$

Using equation (20) in equation (17) we get

$$\rho = \rho_0\left[-\omega_0 + (1+\omega_0)\left(\frac{B_0}{B}\right)^{\frac{3(1+\alpha)}{\alpha}}\right]^{\frac{\alpha}{(1+\alpha)}}, \text{ with } \omega < -1. \quad (21)$$

In homogeneous model of universe, a scalar field $\phi(t)$ with potential $v(\phi)$ has energy density

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + v(\phi) \quad (22)$$

and pressure

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - v(\phi)\,. \tag{23}$$

Using equations (20), (22) and (23) we get

$$\dot{\phi}^2(t) = \frac{\rho^{\frac{(1+\alpha)}{\alpha}} + \omega_0\,\rho_0^{\frac{(1+\alpha)}{\alpha}}}{\rho^{\frac{1}{\alpha}}}\,. \tag{24}$$

Putting value of ρ in equation (24) we get

$$\dot{\phi}^2(t) = \frac{\rho_0(1+\omega_0)\left(\frac{B_0}{B}\right)^{\frac{3(1+\alpha)}{\alpha}}}{\left[-\omega_0 + (1+\omega_0)\left(\frac{B_0}{B}\right)^{\frac{3(1+\alpha)}{\alpha}}\right]^{\frac{1}{(1+\alpha)}}}\,. \tag{25}$$

From equation (25), it is clear that if $\omega_0 > -1$, $\dot{\phi}^2(t) > 0$ giving positive kinetic energy and if $\omega_0 < -1$, $\dot{\phi}^2(t) < 0$ giving negative kinetic energy. It is important to note that similar results are obtained by Hoyle and Narlikar for C-field with negative kinetic energy for steady state theory of the Universe[21, 22].

Law of variation of Hubble's Parameter

The Hubble's parameter H and deceleration parameter q plays important roll to show the physical relevancy of the model in cosmology. The first represents the present time scale of expansion while second gives that the present stage is speeding up instead of slowing down.

We define, the generalized mean Hubble's parameter H as

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{26}$$

where $H_1 = \frac{\dot{a}}{a}$, $H_2 = H_3 = \frac{\dot{b}}{b}$ are directional Hubble's factors in the directions of x, y and z-axes respectively.

Using equations (13) and (26), we get

$$H = \frac{\dot{B}}{B} = \frac{1}{3}\left(\frac{\dot{a}}{a} + \frac{2\dot{b}}{b}\right), \tag{27}$$

where an over dot $\left(\dot{}\right)$ denotes derivative with respect to cosmic time t.

Since the line element is completely characterized by Hubble's parameter H, therefore, let us consider that mean Hubble's parameter is related to average scale factor B by following relation

As stated by Berman[23] for FRW model by,

$$H = DB^{-n}, \tag{28}$$

where D and n are positive constants.

The deceleration parameter is given by

$$q = -\frac{B\ddot{B}}{\dot{B}^2}\,. \tag{29}$$

From equations (27) and (28), we get

$$\dot{B} = DB^{-n+1}, \tag{30}$$

$$\ddot{B} = -D^2\,(n-1)\,B^{-2n+1}\,. \tag{31}$$

From (29), (30), (31), we get

$$q = (n-1)\,. \tag{32}$$

Equation (32), shows that deceleration parameter q is constant. If $n > 1$, q is positive gives standard model and if $0 \le n < 1$, $-1 \le q < 0$ which shows inflation. However the current observations of SNIa and CMBR favor accelerating model, i.e. $q < 0$.

From equations (27) and (28), we get

$$\frac{\dot{B}}{B} = DB^{-n}\,. \tag{33}$$

On integrating equation (33), we get

$$B = (nDt + c_0)^{\frac{1}{n}}, \tag{34}$$

where c_0 is a constant of integration.

From equation (34), it is clear that $B(t) \to \infty$ as $t \to \infty$ which shows universe is accelerating and support observational evidences of Ia Supernova [1, 3] and WMAP[24, 25]. Also this model is free from finite time future singularity.

In this model Hubble's distance is given by

$$H^{-1} = (nt + D_0)\,, \tag{35}$$

where $D_0 = \frac{c_0}{D}$ is constant.

From equation (35) we have, if $t \to 0$ then $H^{-1} \to D_0$ and if $t \to \infty$ then $H^{-1} \to \infty$. Thus in present case galaxies will not disappear when $t \to \infty$.

The horizon distance is obtained as

$$d_H = B(t)\int_0^t \frac{dt}{B(t)}. \tag{36}$$

From equations (34) and (36), we get

$$d_H = \frac{1}{D(n-1)}\left[(nDt+c_0) - D^{'}(nDt+c_0)^{\frac{1}{n}}\right], \tag{37}$$

which gives

$$d_H > B(t), \text{ for } t > \frac{(1+D')^{\frac{n}{(n-1)}} - c_0}{nD}.$$

That is horizon grows more rapidly than scale factor. It is like flat universe dominated by dark energy.
From equations (34) and (21) energy density is given by

$$\rho = \rho_0\left[-\omega_0 + (1+\omega_0)\left(\frac{B_0}{(nDt+c_0)^{\frac{1}{n}}}\right)^{\frac{3(1+\alpha)}{\alpha}}\right]^{\frac{\alpha}{(1+\alpha)}}$$

(38)

Conclusion

LRS Bianchi type-V universe is studied by using the law of variation of Hubble's parameter stated by Berman[23]. It is found that when the cosmic dark energy behaves like a fluid as well as generalized Chaplygin gas simultaneously then Big Smash problem does not arise unlike other phantom models. Singh *et al.*[26] have established ω_0 for the model in the range $-2.4 < \omega_0 < -1.74$ up to 95% confidence level based on I Supernova data. For this model in particular $\alpha = 3$, $\rho_\infty = \rho,\ t \to \infty$ is found in the range $1.51\,\rho_0 < \rho_\infty < 1.92\,\rho_0$, which is analogous with the value obtained by Yadav[27] for Bianchi type-I universe where it is close to the value obtained for FRW model by Srivastava[28] ($1.15\,\rho_0 < \rho_\infty < 2.24\,\rho_0$). It is interesting to note that Big Smash problem does not arise in the present model.

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