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General Higher Order Intermodal Antibunching in two-mode Bose Einstein Condensates

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Abstract

We solve analytically the fully quantum mechanical Hamiltonian of a two-mode Bose Einstein Condensates (BECs) system using Sen-Mandal approach which give more precise solution than that obtained using short-time approximation. These solutions are used to obtain the general higher order intermodal antibunching in the two mode BECs. We find the time dependent antibunching parameter in the inter-mode and the degree of antibunching parameter increases with order. The degree of nonclassicality can be manipulated with the magnitude of chemical potential difference between the modes and the interaction constants.

Keywords: Nonclassical effects, BEC, Sen-Mandal approach, Short-time technique, Higher order antibunching, Degree of nonclassicality.

Introduction

The nonclassical states of a quantum system are essential requirements for applications of quantum states. For example nonclassical states are required for continuous variable quantum cryptography, teleportation of coherent states, quantum computation and communication, useful in building a single-particle (single photon) sources[1-4]. It is reported that such nonclassical states are present in two-mode BECs system and they has many practical applications. Such as, two weekly coupled BECs confined in double well trap can produce Josephson charged qubits, quantum states can transfer using two-component BECs coupled by optical fibre and many applications of two-mode BECs in quantum information processing are reported[5-8]. Recently, the multiparticle entanglement and spin-squeezed state on a two layer atom chip are experimentally realized with two-component BECs[8]. These facts had motivated some of the present authors to systematically investigate the nonclassical properties of two-mode BECs systems of two categories: i. atom-atom BEC, where total no of bosons present in the system is conserved, and ii. atom-molecule BEC, where total no of bosons present in the system is not conserved[9,10]. Those recent works reported the signature of a group of experimentally realizable nonclassical criteria which have practical relevance, such as intermodal entanglement of lower-order and higher-order, intermodal antibunching of lower-order, antibunching of lower-order and higher-order in pure mode, squeezing in a two-mode BECs system[9,10]. Till now, no one have investigated the higher order intermodal antibunching in two-mode BECs system. Most of the works on two-mode BECs system reported the nonclassical properties in lower-order. Recently, a number of experimental and theoretical observations of higher-order nonclassicalities are reported in quantum optical, BECs system and it also reported that the week nonclassicalities is easily detectable using higher-order nonclassical criteria[9-12]. Keeping this in mind we will investigate the possibilities of higher-order intermodal antibunching in two-mode BECs system.

We organise this paperas follows. In sec. II we introduce the model Hamiltonian of a two-mode BECs system and solve this Hamiltonian using Sen-Mandal approach[13]. In Sec. III we investigate the higher-order antibunching in the coupled mode. Finally we conclude in Sec. IV.

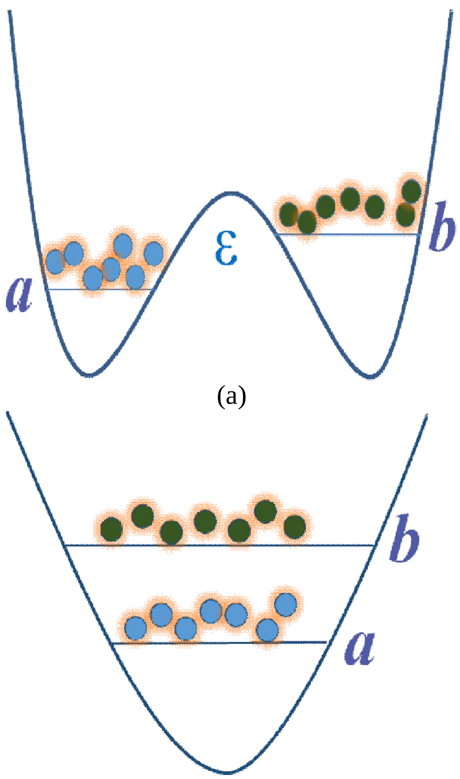
Model Hamiltonian

The completely quantum mechanical description of repulsive two-mode BECs system is denoted by the Hamiltonian as[9]:

$$H = \frac{\kappa}{4}(a^{\dagger 2}a^2 + b^{\dagger 2}b^2) - \frac{\Delta\mu}{2}(a^\dagger a - b^\dagger b) - \frac{\varepsilon}{2}(a^\dagger b + b^\dagger a) \quad (1)$$

Throughout our present study, we consider ħ=1. The single-particle annihilation (creation) operators $a\,(a^\dagger)$ and $b\,(b^\dagger)$ correspond to the modes a and b respectively. The parameter $\varepsilon\, denotes$ the single atom tunnelling amplitude,$\Delta\mu$ is the chemical potential difference between the modes and κ denotes the coupling constant for intra-mode interactions. This Hamiltonian describes both, the double-well BEC (external Josephson effect) and the single-well two level BEC (internal Josephson effect) as shown in Figures-1(a) and 1(b) respectively. In case of double-well BECs system the intermodal coupling is considered very small, whereas in the single-well two-mode BECs system there is no such restriction. To study the antibunching properties of the two-mode BECs system, we need to find the temporal variation of the operators for the two modes by solving the Hamiltonian (1). The Heisenberg equations of motion corresponding to the Hamiltonian (1) are:

$$\dot{a}(t) = -i\left(\frac{\kappa}{2}a^{\dagger}(t)a^{2}(t) - \frac{\Delta\mu}{2}a(t) - \frac{\varepsilon}{2}b(t)\right),$$
$$\dot{b}(t) = -i\left(\frac{\kappa}{2}b^{\dagger}(t)b^{2}(t) + \frac{\Delta\mu}{2}b(t) - \frac{\varepsilon}{2}a(t)\right). \quad (2)$$



(a)
(b)

Figure-1
Schematic diagram of two-mode BEC system;
(a) double-well BEC (b) single-well two-mode BEC

These are the coupled nonlinear differential equations of field operators and are not exactly solvable in closed analytic form. The well-known method to solve the above two equations is short-time approximation. We solve these coupled equations using Sen-Mandal approach[13].

The solutions obtained from Sen-Mandal approach is more exact than the solutions obtained from short-time approximations[14]. The methodology used here is available in our previous papers and details of the present solutions are available in our recent publication[9,13,15-18]. The solutions of Equation 2 are:

$$a(t) = f_1 a(0) + f_2 b(0) + f_3 a^{\dagger}(0)a^{2}(0) + f_4 a(0) f_5 a^{\dagger}(0)a^{2}(0) f_6 a^{\dagger 2}(0)a^{3}(0) f_7 a^{2}(0)b^{\dagger}(0) + f_8 a^{\dagger}(0)a(0)b(0) f_9 b^{\dagger}(0)b^{2}(0),$$

$$b(t) = g_1 b(0) + g_2 a(0) + g_3 b^{\dagger}(0)b^{2}(0) + g_4 b(0) + g_5 b^{\dagger}(0)b^{2}(0) + g_6 b^{\dagger 2}(0)b^{3}(0) + g_7 a^{\dagger}(0)b^{2}(0) + g_8 b^{\dagger}(0)b(0)a(0) + g_9 a^{\dagger}(0)a^{2}(0). \quad (3)$$

The parameters $f_i (i = 1,2, \ldots ,9)$ and $g_i (i = 1,2, \ldots ,9)$ are:

$$f_1(t) = g_1^*(t) = e^{\frac{i\Delta\mu t}{2}},\ f_2(t) = -g_2^*(t) = \frac{\varepsilon}{2\Delta\mu}G(t)f_1(t),$$
$$f_3(t) = -g_3^*(t) = -\frac{i\kappa t}{2}f_1(t),\ f_4(t) = g_4^*(t) = \left[\frac{i\varepsilon^2 t}{4\Delta\mu} - \frac{\varepsilon^2}{4\Delta\mu^2}G(t)\right]f_1(t),$$
$$f_5(t) = g_5^*(t) = -\frac{\kappa^2 t^2}{8}f_1(t),\ f_6(t) = g_6^*(t) = -\frac{\kappa^2 t^2}{8}f_1(t),$$
$$f_7(t) = g_7^*(t) = \left[-\frac{i\kappa\varepsilon t}{4\Delta\mu} - \frac{\kappa\varepsilon}{4\Delta\mu^2}G^*(t)\right]f_1(t),$$
$$f_8(t) = g_8^*(t) = \left[-\frac{i\kappa\varepsilon t}{2\Delta\mu} + \frac{\kappa\varepsilon}{2\Delta\mu^2}G(t)\right]f_1(t),$$
$$f_9(t) = g_9^*(t) = \left[\frac{i\kappa\varepsilon t}{4\Delta\mu}e^{-i\Delta\mu t} - \frac{\kappa\varepsilon}{4\Delta\mu^2}G(t)\right]f_1(t). \quad (4)$$

where: $G(t) = (1 - e^{-i\Delta\mu t})$. Deriving the above solutions we consider up to the second order in κ,ε with the restriction $\kappa t < 1$ *and / or* $\varepsilon t < 1$ to obey perturbation theory. We use these solutions to study the higher-order intermodal antibunching in the two-mode BECs system.

Higher Order Intermodal Antibunching

To investigate the higher order intermodal antibunching, we consider that both the two modes are initially coherent. So, the initial composite coherent state of the system will be:

$$|\psi(0)\rangle = |\alpha\rangle \otimes |\beta\rangle. \quad (5)$$

Where $|\alpha\rangle$and $|\beta\rangle$ are Eigen states of a and b respectively.

The Eigen value equations of the field operator $a(0)$ operating on the composite coherent state $|\psi(0)\rangle$ is:

$$a(0)|\psi(0)\rangle = \alpha|\alpha\rangle \otimes |\beta\rangle. \quad (6)$$

Where the complex number α is the eigen value for the field operator operator $a(0)$ operating on composite state $|\psi(0)\rangle$. Similarly the eigenvalue for the field operator $b(0)$ operating on composite state $|\psi(0)\rangle$ is β.

C. T. Lee introduced the concept of higher-order antibunching in photon statistics[19]. In order to investigate the n^{th}order antibunching in two-mode BEC, we use the following criterion[20].

$$\langle a^{\dagger}(t)^{n}a(t)^{n}b^{\dagger}(t)^{n}b(t)^{n}\rangle - \langle a^{\dagger}(t)a(t)\rangle^{n}\langle b^{\dagger}(t)b(t)\rangle^{n} < 0 \quad (7)$$

where: n is the positive integer and $n > 1$ gives the higher order. Now using (3), (4), (5) and (7) we obtain the analytical expression for higher order intermodal antibunching which is

$$\langle a^{\dagger}(t)^n a(t)^n b^{\dagger}(t)^n b(t)^n\rangle - \langle a^{\dagger}(t)a(t)\rangle^n \langle b^{\dagger}(t)b(t)\rangle^n = \left[nf_1^{*}f_9\left(\alpha^{*}|\alpha|^{2(n-1)}|\beta|^{2n}\beta + \alpha^{*}|\alpha|^{2n}|\beta|^{2(n-1)}\beta\right) + c.c.\right] \quad (8)$$

In order to get the flavour of the equation (8), we plot the right hand side of the equation (8) with respect to the dimensional time κt. The negative region of Figure-2(a) illustrate the inter-modal antibunched state of two-mode BECs system and the inter-modal state oscillate between classical (bunching) and nonclassical (antibunching) regions.

It is clear from Figure-2(b) that the signature of the nonclassicality depends on the particular value of $\Delta\mu/\kappa$ irrespective the order. So, the signature of antibunching can be manipulated with the chemical potential difference between the modes. The depth of antibunching increases with $\Delta\mu/\kappa$ and/or ε/κ which is shown in Figures-2(b) and 2(c) respectively. So, the degree of antibunching can be manipulated with the chemical potential difference and/or single-atom tunnelling amplitude between the modes.

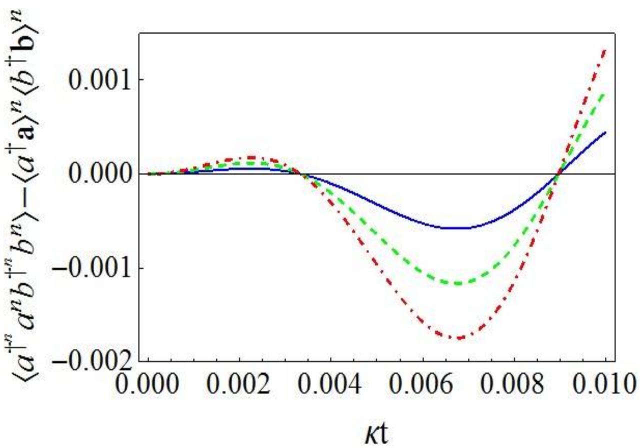
All three plots of Figure-2 show that the amount of nonclassicalities increases with the order. So, for weak nonclassical effects which is difficult to detect using normal order (n=1), can easily detectable by means of higher order criteria. The signature of nonclassicalities is independent of order number as all the orders represent the same physical system.

Conclusion

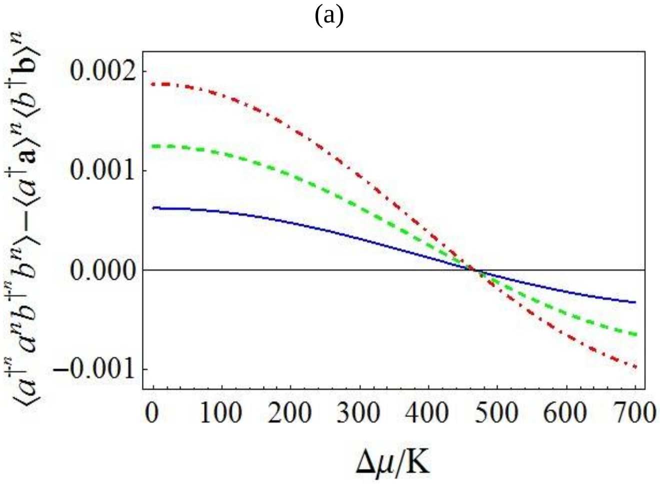
Heisenberg's equations of motion involving operators for two mode BECs system are solved analytically using perturbative Sen-Mandal technique. These solutions are utilized to investigate the higher order antibunching for two mode BECs system.

It is reported that the depth of antibunching increases with the order which enable us to detect weak nonclassicalities. It is interesting that the amount of nonclassicality can be manipulated with the magnitude of the interaction constants and chemical potential difference between the modes. As the two-mode

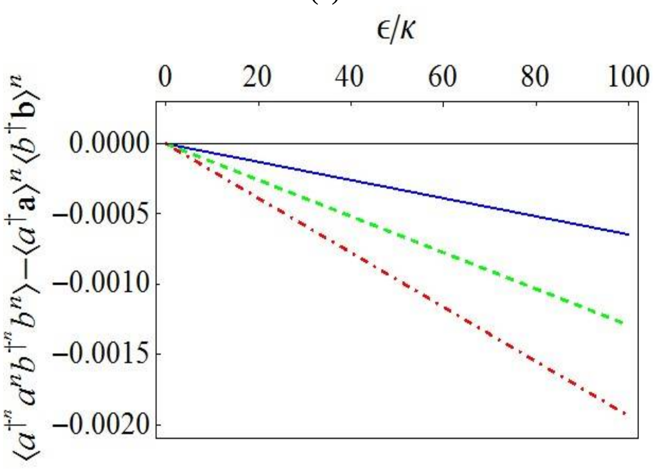
BECs system is experimentally realizable, our result reported here can be verified experimentally and can apply in quantum information processing. Hamiltonian similar to the present work is also appear in optical systems[15-19]. Thus, the methodology adopted here may also be used in quantum optical systems to study the higher order photon antibunching.



(a)



(b)



(c)

Figure-2

Plot of right hand side of Equation 8 with, (a) rescaled interaction time κt for $\alpha = \beta = 1, \kappa = 10\ Hz, \varepsilon = 500\ HZ$and $\Delta\mu = 7000\ Hz$, (b) $\Delta\mu/\kappa$ for $\alpha = \beta = 1, \kappa = 10\ Hz, \varepsilon = 500\ HZ$ and $\kappa t = 0.0005$, (c) ε/κ for $\alpha = \beta = 1, \kappa = 10\ Hz,\ \Delta\mu = 7000\ HZ$ and $\kappa t = 0.005$. The smooth blue line, dashed green line and the dot-dashed red line are for n = 1; n = 2; n = 3 respectively

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