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Existence of Solutions for Fractional Differential Equation with Nonlocal Boundary Condition

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Abstract

By using standard Riemann-Liouville differentiation and Leray- Shauder theory, existence of non negative solutions for fractional differential equation with global boundary condition $D_{0+}^{b}v(d_{01}) + s(d_{01})\ g(d_{01}, v(d_{01})) = 0,\ 0 < d_{01} < 0.99$, $v(0)=0,\ v(0.99)=\sum_{h=1}^{\infty} b_h v(\eta_h)$ *is considered, here b∈ (1,2] is a real number, the standard Riemann-Liouville differentiation is* D_{0+}^{b}, *and* $\eta_h \in (0, 0.99),\ b_h \in [0,\infty)$ *with* $\sum_{h=1}^{\infty} b_h \eta_h^{b-1} < 1,\ s(d_{01}) \in C([0,1],[0,\infty)),\ g(t,v) \in C([0,1]\times[0,\infty),[0,\infty))$.

Keywords: Fixed point theorem, Leray-Shauder theory, standard Riemann-Liouville differentiation, fractional calculus theory, Fractional differential equations and Nonlocal boundary condition.

Introduction

The ardent improvement of an exposition of the abstract principles of a science of art Fractional Calculus has been induced by Fractional Differential Equations. The application of such constructions in the field of science such as Dynamics, Statics, Bio-Chemistry, Chemistry and Engineering[1-6]. Many things in the world are always changing in fraction of time which is one factor in deciding the nature of the particular thing.

Solving a Differential Equation of integral order is a well known process for all of us. There was an ambiguity while solving Differential Equation whose order is not an integral that is a fraction. To avoid such an ambiguity, there was an invention of how to solve a Fractional Differential Equation.

Many articles and books on Fractional Calculus are consecrated to the solution of Linear Initial Fractional Differential Equations in terms of some specific functions[6-8]. At present many papers have been revealed for proving existence of non negative solutions for the initial Fractional Differential Equations with global boundary conditions using non linear analysis[9-17].

Not long ago, the existence of positive solutions of nonlinear fractional differential equation has been revealed by Bai and L $\ddot{u}$ [15].

$$D_{0+}^{b}v(d_{01}) + g(d_{01}, v(d_{01})) = 0,\ 0 < d_{01} < 0.99, \tag{1.1}$$

$$v(0) = v(0.99) = 0,$$

here $b \in (1,2]$ is a non imaginary number, the standard Riemann-Liouville differentiation is D_{0+}^{b} and $g : [0,1] \text{ x } [0,\infty) \to [0,\infty)$ is continuous.

Here existence of non negative solutions for fractional differential equation with global boundary condition will be proved by taking the following FDE

$$D_{0+}^{b}v(d_{01}) + s(d_{01}\ t)\ g(d_{01}, v(d_{01})) = 0,\ 0 < t < 0.99, \tag{1.2}$$

$$v(0) = 0, \qquad v(0.99) = \sum_{h=1}^{\infty} b_h v(\eta_{h)},)$$

here $b \in (1,2]$ is a non imaginary number, the standard Riemann-Liouville differentiation is D_{0+}^{b}, and $\eta_h \in (0, 0.99),\ b_h \in [0,\infty)$ *with*

$\sum_{h=1}^{\infty} b_h \eta_h^{b-1} < 1,\ s(d_{01}) \in C([0,1],\ [0,\infty)),$

$g(d_{01}, v) \in C([0,1] \text{ x } [0,\infty),\ [0,\infty)).$

The following conditions will be assumed in the whole paper for proving the main result:

$(J1)\ \eta_h \in (0,1), b_h \in [0,\infty)$ are constants with? $\sum_{h=1}^{\infty} b_h \eta_h^{\alpha-1} < 0.99,$

(J2)$s(d_{01}) \in C([0,1],[0,\infty)),\ s(d_{01}) \not\equiv 0\ on\ [a,b] \subset (0,1),$

(J3) $g(d_{01}, v) \in C([0,1], \text{ x } [0,\infty),[0,\infty))$.

Remark 1. 1. Existence of non negative solutions for problem (1.2) have not been derived. We have to derive it for (1.2).

The Preliminary Lemmas

The necessary conditions of definitions from fractional calculus theory are as follows: Definition 2. 1. A function *f*: $(0,\infty)\rightarrow$ R whose fractional integral of order b>0 is written as follows $I_{o+}^{b} f(d_{01}) = \frac{1}{\Gamma(b)}\int_0^{d_{01}} (d_{01}-s)^{b-1} f(s)\, ds,$ (2.1)

where the side in right is point wise defined on $(0,\infty)$.

Definition 2.2. A function *f*: $(0,\infty)\rightarrow$R whose fractional derivative of order $b>0$ is written as follows

$$D_{0+}^{b} f(d_{01}) = \frac{1}{\Gamma(n_1-b)}\left(\frac{d}{dt}\right)^{n_1}\int_0^t \frac{f(s)}{(t-s)^{b-n_1+1}}\, ds, \quad (2.2)$$

here $n_1 = [n_0] + 1$, such that the side in right is point wise defined on $(0,\infty)$.

Definition 2. 3. Let ϕ : Q $\rightarrow$ $(0,\infty)$ be a positive continuous concave functional on a cone Q of a real Banach space E, given that ϕ is continuous and

$$\phi(tx_1+(1-t)y_1) \geq t\phi(x_1)+(1-t)\,\phi(y_1), \quad (2.3)$$

For all x_1 , $y_1 \in$ Q and $0 \leq t \leq 1$

Example 2. 4. By fixing $\mu > -1$,

$$D_{0+}^{b} t^{\mu} = \frac{\Gamma(\mu+1)}{\Gamma(\mu-b+1)} t^{\mu-b}, \quad (2.4)$$

Specifically giving $D_{0+}^{b} t^{b-r} = 0$, r=1,2,… ,J, where J is the smallest integer greater than or equal to b .

Using the above basic definitions and Lemma, we can obtain the following statements.

Lemma 2. 5 Assume v$\in N(0,1)\cap M(0,1)$ *such that* $b>0$, then the fractional differential equation

$$D_{0+}^{b} v(t) = 0$$

has v(d_{01}) =$N_1 d_{01}^{\,b-1} + N_2 d_{01}^{\,b-2} + \ldots + N_J d_{01}^{\,b-J}$, $N_k \in$ R,k=1,2,…,J, for the smallest integer J not less than or equal to b , which is a unique solution. (2.5)

Lemma 2. 6. Let v$\in$ N(0,1) $\cap$M(0,1) be assumed with a fractional derivative of order b>0. Then,

$$I_{0+}^{b} D_{0+}^{b} v(d_{01}) = v(d_{01}) + N_1 d_{01}^{\,b-1} + N_2 d_{01}^{\,b-2} + \ldots + N_J d_{01}^{\,b-J}, \quad (2.6)$$

for some $N_k \in$ R, k=1, 2, …, J.

Lemma 2. 7. Assume f$\in$ N[0,1] and $b \in (1,2]$, the solution[15] of

$D_{0+}^{b} v(d_{01}) + f(d_{01}) = 0,\ 0 < d_{01} < 1,$ which is unique (2.7)
v(0) = v (1) = 0
is

$$v(d_{01}) = \int_0^1 H(d_{01}, m_1)\, f(m_1)\, dm_1, \quad (2.8)$$

Where

$$H(d_{01},m_1) = \begin{cases} \frac{[d_{01}(1-m_1)]^{b-1} - (d_{01}-m_1)^{b-1}}{\Gamma(b)}, & 0 \leq m_1 \leq d_{01} \leq 1, \\ \frac{[d_{01}(1-m_1)]^{b-1}}{\Gamma(b)}, & 0 \leq d_{01} \leq m_1 \leq 1, \end{cases} \quad (2.9)$$

Lemma 2.8 Suppose (J1) Holds. Given f$\in$ N where 0≤N≤ 1 and 1<$b \leq$2, the unique solution of

$D_{0+}^{b} v(d_{01}) + f(d_{01}) = 0,\ d_{01} \in (0,1),$

$$v(0) = 0, \quad v(1) = \sum_{h=1}^{\infty} b_h v(\eta_h) \quad (2.10)$$

$$v(d_{01}) = \int_0^1 H(d_{01}, m_1)\, f(m_1)\, dm_1 + B(m_1)\, d_{01}^{\,b-1}, \quad (2.11)$$

$$Where \quad H(d_{01},m_1) = \begin{cases} \frac{[d_{01}(1-m_1)]^{b-1} - (d_{01}-m_1)^{b-1}}{\Gamma(b)}, & 0 \leq m_1 \leq d_{01} \leq 1, \\ \frac{[d_{01}(1-x)]^{b-1}}{\Gamma(b)}, & 0 \leq d_{01} \leq m_1 \leq 1, \end{cases} \quad (2.12)$$

$$B(f) = \frac{\sum_{h=1}^{\infty} b_h \int_0^1 H(\eta_h, m_1)\, f(m_1)\, dm_1}{1-\sum_{h=1}^{\infty} b_h \eta_h^{b-1}}$$

Proof: Using Lemmas 1.2 and 1.3, the following will be proved

$$v(d_{01}) = \int_0^1 H(d_{01}, m_1)\, f(m_1)\, dm_1 = N_1 d_{01}^{\,b-1} + N_2 d_{01}^{\,b-2} \quad (2.13)$$

Because

$$\sum_{h=1}^{\infty} b_h \int_0^1 H(\eta_h, m_1)\, dm_1 = \frac{\sum_{h=1}^{\infty} b_h \eta_h^{b-1}(1-\eta_i)}{b\Gamma(b)},\ b_h \eta_h^{b-1}(1-\eta_h) < b_h \eta_h^{b-1} \quad (2.14)$$

applying (J1), $\sum_{h=1}^{\infty} b_h \eta_h^{b-1}(1-\eta_h)$ converges. andso $\sum_{h=1}^{\infty} b_h \int_0^1 H(\eta_h, x)\, dm_1$ is converging. f(d_{01}) is a function which is continuous on [0,1], so $\sum_{h=1}^{\infty} b_h \int_0^1 H\,(\eta_h, m_1)\, f(m_1)\, dm_1$ also converges. By v(0) = 0, v(0.99) = $\sum_{h=1}^{\infty} b_h v\,(\eta_h)$,there are $N_2 = 0$, $N_1 = \left[\sum_{h=1}^{\infty} b_h \int_0^1 H\,(\eta_h, m_1)\, f\,(m_1)\, dm_1\right] / (1- \sum_{h=1}^{\infty} b_h\, \eta_h^{b-1})$. hence

$$v(d_{01}) = \int_0^1 H\,(d_{01}, m_1)\, f(m_1)\, dm_1 + B(f)\, d_{01}^{\,b-1}, \quad (2.15)$$

Hence we have the proof

$$B(f) = \frac{\sum_{h=1}^{\infty} b_h \int_0^1 H\,(\eta_h, m_1)\, f\,(m_1)\, dm_1}{1-\sum_{h=1}^{\infty} b_h \eta_h^{b-1}}$$

Lemma 2. 9. The function $H(d_{01}, m_1)$ defined by 2.9 satisfies the following conditions: i. $H\,(d_{01}, m_1) > 0$, for $d_{01}, m_1 \in (0,1)$, ii. there exists a positive function $\gamma \in N(0,1)$ such that $\min_{(0.25)\le m_1 \le (0.75)} H(d_{01}, m_1) \ge \gamma\,(m_1)$, $\max_{0\le d_{01}\le 1} H(d_{01}, m_1) = \gamma\,(m_1)\, H(m_1, m_1)$, $0 < m_1 < 1$.

Lemma 2. 10. Assuming M is a Banach space[18], let $Q \subseteq M$ be a cone and Ψ_1, Ψ_2 two sets which are open and bounded belongs to M with $0 \in \Psi_1 \subset \Psi_2$ in case of $A : Q \cap (\overline{\Psi}_2 \setminus \Psi_1) \to Q$ is a operator which is completely continuous in order to have either

$(i)\ \|Az_1\| \le \|z_1\|,\ z_1 \in Q \cap \partial\Psi_1\ and\ \ \|Az_1\| \ge \|z_1\|,\ z_1 \in Q \cap \partial\,\Psi_2\ \ or$

$(ii)\ \|Az_1\| \ge \|z_1\|,\ z_1 \in Q \cap \partial\Psi_1\ \ and\ \ \|Az_1\| \le \|z_1\|,\ z_1 \in Q \cap \partial\,\Psi_2\ \ or$

holds.

Then, there is a fixed point for A in $Q \cap (\overline{\Psi_2} \setminus \Psi_1)$

Lemma 2.11. Having a cone Q in non imaginary Banch space M[19], $Q_c = \{\, z_1 \in Q \,|\, \|z_1\| \le c \,\}$, θ a concave functional which is continuous with non negativity on Q for which $\theta\,(z_1) \le \|z_1\|,\ for\ all\ z_1 \in \overline{Q}_c$, and $Q(\theta, p, q) = \{z \in Q |\ \ p \le \theta(z_1), \|z_1\| \le q\}$. suppose for a completely continuous function

$A : \overline{Q}_C \to \overline{Q}_C$, there exist constants $0 < p_1 < p < q \le q_1 c$ such that

$(K1)\ \{z_1 \in Q\,(\theta, p, q)\,|\,\theta\,(z_1) > p\} \ne \emptyset\ and\ \theta\,(Az_1) > p,\ z_1 \in Q(\theta, p, q),$

$(K2)\ \|Az_1\| \le p_1,\ for\ z_1 \le p_1,$

$(K3)\ \theta(Az_1) > p\ \ for\ z_1 \in Q(\theta, p, q_1)\ with\ \|Az_1\| > q.$

Therefore, A has at least three fixed points z_{11}, z_{22}, z_{33} with

$$\|z_{11}\| < p_1, \quad p < \theta(z_{22}), \qquad p_1 < \|z_{33}\|, \qquad \theta(z_{33}) < p.$$

Note 2.12. If there holds $q = q_1$, then $(K1)\ \{z_1 \in Q\,(\theta, p, q)\,|\,\theta\,(z_1) > p\} \ne \emptyset\ and\ \theta\,(Az_1) > p,\ z_1 \in Q(\theta, p, q),$ implies condition $(K3)\ \theta(Az_1) > p\ \ for\ z_1 \in Q(\theta, p, q_1)\ with\ \|Az_1\| > q.$

The Main Results

Assume M = N [0,1] has furnished with the command $v \le \upsilon$ if $v\,(d_{01}) \le \upsilon\,(d_{01})$ such that $d_{01} \in [0,1]$, and the norm with maximum, $\|v\| = \max_{0\le d_{01}\le 1} |v(d_{01})|$. Name the cone $Q \subset M\ by\ Q = \{v \in M\ |v\,(d_{01}) \ge 0\}$.

Define the concave functional θ with positive continuous on the cone Q has explained by

$$\theta(v) = \min_{(0.25)\le t \le (0.75)} v(d_{01}).$$

Lemma 3. 1. Permit J: Q $\to$ M as a [operator] process [15] explained by $Jv(d_{01}) := \int_0^1 H\,(d_{01}, m_1)\ g\,(m_1, v(m_1))$ ds, then J: Q $\to$ Q is completely continuous function so that the process leads to the main result.

Lemma 3. 2. Define A: Q $\to$ M be the operator defined by

$$Av(d_{01}) := \int_0^1 H(d_{01}, m_1)\, s(m_1)\, g\,(m_1, v(m_1))\, dm_1 + B\,(s(.)\, g\,(\cdot, v(.)))\, d_{01}^{\,b-1}, \quad (3.1)$$

then the operator A: Q $\to$ Q is completely continuous.

Proof: We can prove this using J : Q $\to$ M as a [operator] process[15] explained by $Jv(d_{01}) := \int_0^1 H\,(d_{01}, m_1)\ g\,(m_1, v(m_1))$ ds, then J: Q $\to$ Q is completely continuous Denote

$$E = \left(\int_0^1 H(m_1, m_1)\, s\,(m_1)\, dm_1 + \frac{\sum_{h=1}^{\infty} b_h \int_0^1 H(\eta_h, m_1)\, s\,(m_1)\, dm_1}{1-\sum_{h=1}^{\infty} b_h \eta_h^{b-1}}\right)^{-1} \quad (3.2)$$

$$F = \left(\int_{0.25}^{0.75} \gamma(s) H(m_1, m_1) s(m_1) dm_1\right)$$

Theorem 3.3. consider (J1)-(J3) hold, and there be two non negative constants $r_2 > r_1 > 0$ such that $g\,(d_{01}, v) \le E\, r_2$, for every d_{01} that is $0 \le d_{01} \le 1$, $v \in [0, r_2]$,

$(d_{01}, v) \ge E\, r_1$, for every d_{01} lies between 0 and 1, $v \in [0, r_1]$, here E, F is explained in (3.2), therefore there exists one or more solutions for problem (1.2) that is v such that $r_1 \le |v| \le r_2$.

Proof: Since by lemmas 2.8 and 3.2 A:Q $\to$Q is entirely continuous, hence there is a solution v = v(d_{01}) for the problem (1.2) if and only if the operator equation v = Av has been solved by v. In case of applying the following in order to have either

$$(i)\ \|Az_1\| \le \|z_1\|,\ z_1 \in Q \cap \partial\Psi_1\ and\ \ \|Az_1\| \ge \|z_1\|,\ z_1 \in Q \cap \partial\Psi_2\ or;$$

$$(ii)\ \|Az_1\| \ge \|z_1\|,\ z_1 \in Q \cap \partial\Psi_1\ \ and\ \ \|Az_1\| \le \|z_1\|,\ z_1 \in Q \cap \partial\ \Psi_2\ \ or$$

holds. We need the two steps as follows:

Step 1: Put $\Psi_2 = \{v \in Q \,|\, \|v\| \le r_2\}$. Here v$\in\partial\,\Omega_2$, where 0 $\le$ v(t) $\le$ r_2 for every d_{01}, $0 \le d_{01} \le 1$.

Hence we have

$$\|Av\| \le \int_0^1 H(m_1,m_1)s(m_1)\ g\ (m_1,v(m_1))\,dm_1 + \frac{\sum_{h=1}^{\infty} b_i \int_0^1 H(\eta_h,m_1)\ s\ (m_1)\ g\ (m_1,v(m_1))dm_1}{1-\sum_{h=1}^{\infty} b_h\ \eta_h^{b-1}}$$

$$\le Er_2\left[\int_0^1 H(m_1,m_1)\ s(m_1)dm_1 + \frac{\sum_{h=1}^{\infty} b_h \int_0^1 H(\eta_h,m_1)\ s\ (m_1)\ dm_1}{1-\sum_{h=1}^{\infty} b_h \eta_h^{b-1}}\right] \quad (3.2)$$

$$= r_2 = \|\ v\ \|.$$

Therefore,

$$\|\ Av \,\| \le \|\ v \,\|,\ \ v \in Q \bigcap \partial\Psi_2.$$

Step 2: Let $\Psi_1 = \{v \in Q \,|\, \|v\| \le r_1\}$. For v $\in\partial\ \Psi_1$, and 0 $\le$ v(d_{01}) $\le$ r_1 for every d_{01} $\in$ [0,1]. Since by (2) in assumption, for d_{01} $\in$ [0.25, 0.75], we have

$$\|Av\|(d_{01}) = \int_0^1 H(d_{01},m_1)s(m_1)\ g\ (m_1,v(m_1))\,dm_1 + \frac{t^{b-1}\sum_{h=1}^{\infty} b_h \int_0^1 G(\eta_h,m_1)\ s\ (m_1)\ g\ (m_1,v(m_1))dm_1}{1-\sum_{h=1}^{\infty} b_h\ \eta_h^{b-1}}$$

$$\ge \int_0^1 \gamma\ (m_1)\ H(m_1,m_1)\ s(m_1) g(m_1,v(m_1))dm_1 \qquad (3.4)$$

$$= r_1 = \|\ v\ \|.$$

therefore, $\|\ Av \,\| \ge \|\ v \,\|,\ \ v \in Q \bigcap \partial\Psi_1$ (3.5)

since, there exists a positive function γ $\in$ N(0,1) such that

$\min_{(0.25)\le m_1 \le (0.75)}$ H(d_{01}, m_1) $\ge$ γ (m_1), $\max_{0\le d_{01}\le 1}$ H(d_{01}, m_1) = γ (m_1) H(m_1, m_1), 0 < m_1 < 1.

therefore there exists one or more solutions for problem (1.2) that is v such that $r_1 \le |v| \le r_2$.

Hence proved.

Example 3.4. Consider the problem

$$D_{0+}^{3/2} v(d_{01}) + v^2 + \frac{\sin d_{01}}{4} + \frac{2}{10} = 0, \quad 0 < d_{01} < 1, \qquad (3.6)$$

$$v(0) = 0, \quad v(1) = \sum_{h=1}^{\infty} b_h v(\eta_h),$$

Where : $\sum_{h=1}^{\infty} b_h \eta_h^{1/2} = \frac{2}{10}$

Calculating E and F by using the above basics, we have the values $E \ge 1,4, F \approx 13.665$. Selecting r_1=(1/69), r_2 = (10/11),

$$g(d_{01},v) = v^2 + \frac{\sin d_{01}}{4} + \frac{2}{10} \le 1.2199 \le Er_2 \ \ (d_{01},v) \in [0,1]x[0,\frac{10}{11}], \qquad (3.7)$$

$$g(d_{01},v) = v^2 + \frac{\sin d_{01}}{4} + \frac{2}{10} \ge \frac{2}{10} \ge Fr_1, \quad (d_{01},v) \in [0,1]x[0,\frac{1}{69}]$$

Hence, there exists one or more non negative solutions v for problem (3.6) using statement of (3.3) such that (1/69) $\le \|v\| \le (10/11)$.

Theorem 3.5. Let (J1)-(J3) hold, and there be constants $0 < p_1 < p < q_1$ in such a way that the consideration holds as given below:

(A1) g (t, v) < Ea for (t, v) $\in [0,1]x[0,a]$

(A2) g (t, v) $\ge Fb\ for\ (t,v) \in [1/4,3/4]x[b,c]$

(A3) g(t, v) $\le Ec\ for\ (t,v) \in [0,1]$ x $[0,$x$]$; where E, F is defined in (*).

Then, there must be three or more non negative solutions arbitrarily named v_1, v_2, v_3 [20] for the problem (1.2)

$$\|v_1\| < p_1, \quad \min_{0.25\le d_{01}\le 0.75} |v_2| < \|v_2\| \le q_1\ ,\ \ p_1 < \|v_3\| \le q_1,$$

$$\min_{0.25\le d_{01}\le 0.75} |v_3| < p \qquad (3.8)$$

Proof: Since the function H(d_{01}, m_1) satisfies the following conditions: i. H (d_{01}, m_1) > 0, for d_{01}, m_1 $\in$ (0,1), ii. there exists a positive function γ $\in$ N(0,1) such that

$\min_{(0.25)\le m_1 \le (0.75)}$ H(d_{01}, m_1) $\ge$ γ (m_1), $\max_{0\le d_{01}\le 1}$ H(d_{01}, m_1) = γ (m_1) H(m_1, m_1), 0 < m_1 < 1.

If

$v \in \overline{Q}_c,\ then\ \|v\| \le q_1$. Consideration $(A3)$ $implies$ $g(d_{01}, v(d_{01})) \le Eq_1\ \ for\ \ 0 \le d_{01} \le 1$.

Similarly,

$$\|Av\| = \max_{0 \le d_{01} \le 1} \left| \int_0^1 H(d_{01}, m_1) s(m_1) g(m_1, v(m_1)) dm_1 + \frac{d_{01}^{\,b-1} \sum_{h=1}^{\infty} b_h \int_0^1 H(\eta_h, m_1) s(m_1) g(m_1, v(m_1)) dm_1}{1 - \sum_{h=1}^{\infty} b_h \eta_h^{\,b-1}} \right|$$

$$\le \int_0^1 H(m_1, m_1) s(m_1) g(m_1, v(m_1)) dm_1 + \frac{d_{01}^{\,b-1} \sum_{h=1}^{\infty} b_h \int_0^1 G(\eta_h, m_1) s(m_1) g(m_1, v(m_1)) dm_1}{1 - \sum_{h=1}^{\infty} b_h \eta_h^{\,b-1}} |$$

$$\le \left[\int_0^1 H(m_1, m_1) s(m_1) dm_1 + \frac{\sum_{h=1}^{\infty} b_h \int_0^1 H(\eta_i, m_1) s(m_1) dm_1}{1 - \sum_{h=1}^{\infty} b_h \eta_h^{\,b-1}} \right] \| v \|$$

$$\le \|v\| \tag{3.9}$$

That is, A: $\overline{Q}c \to \overline{Q}c$. Similarly for v $\in \overline{Q}_{p_1}$, the consideration (A1) implies $g\left(d_{01}, v\left(d_{01}\right)\right) < Mp_1$, $0 \le d_{01} \le 1$. From that condition (K2) is satisfied in Lemma 2.11.

Assume $v\left(d_{01}\right) = \left(p + q_1\right)/2, \quad 0 \le d_{01} \le 1$ to satisfy condition (K1) of Lemma 2.11. It is easy to see that $v\left(d_{01}\right) = \left(p + q_1\right)/2,$

$\in Q(\theta, p, q_1), \theta(v) = (\theta(p + q_1))/2 > p_1,$

$and\ consquently, \{v \in Q(\theta, p, q) \mid \theta(v) > p\} \ne \emptyset\ Hence, if\ v \in Q(\theta, p, q_1),$

$then\ \ p \le v(d_{01}) \le q_1\ for\ 0.25 \le d_{01} \le 0.75.$

From condition (A2), we have $f\left(d_{01}, v\left(d_{01}\right)\right) Fp$

for $0.25 \le d_{01} \le 0.75$.

(ie) $\theta(Av) = \min_{0.25 \le d_{01} \le 0.75} \left| Av(d_{01}) \right|$

$$\ge \int_0^1 \gamma(m_1) H(m_1, m_1) s(m_1) g(m_1, v)) dm_1$$

$$> Fp \int_{0.25}^{0.75} \gamma(m_1) H(m_1, m_1) s(m_1) dm_1$$

$$= p = \| v \|$$

$$\theta(Av) > p, for\ every\ \ v \in Q(\theta, p, q_1). \tag{3.10}$$

Which proves first condition of Lemma 2.11.

Hence, there exists three non negative solutions namely $v_1, v_2\ and\ v_3$ or more for the boundary value problem (1.2).From that we have

$$\|v_1\| < p_1, \quad \min_{0.25 \le d_{01} \le 0.75} |v_2| < \|v_2\| \le q_1\ , \quad p_1 < \|v_3\| \le q_1,$$

$$\min_{0.25 \le d_{01} \le 0.75} |v_3| < p \tag{3.11}$$

Hence the proof.

Example 3.6. Consider the problem (3.12) given below

$$D_{0+}^{3/2} v(d_{01}) + g(d_{01}, v) = 0, 0 < d_{01} < 1$$

$$v(0) = 0, v(1) = \sum_{h=1}^{\infty} b_h v(\eta_h) \tag{3.12}$$

Where $\sum_{h=1}^{\infty} b_h \eta_h^{\frac{1}{2}} = \frac{1}{5},$

$$\text{g(t, v)=} \begin{cases} \frac{(t)}{39} + 13v^2, v \le 1, \\ 14 + (\frac{1}{39}) + v, v > 1 \end{cases} \tag{3.13}$$

We have E $\ge 1.4, F \approx 13.665.$ Choosing $= (1/13),\ p = 1,\ \ q_1 = 35,\ there\,hold$

$$g(d_{01}, v) = \frac{d_{01}}{39} + 13v^2 \le 0.098 \le Ep,\ (d_{01}, v) \in [0,1] x \left[0, \frac{1}{13}\right]$$

$$g(d_{01}, v) = 12 + \frac{d_{01}}{40} + v \ge 14.025 \ge Fb \approx 13.7, (d_{01}, v) \in \left[\frac{1}{4}, \frac{3}{4}\right] \text{x}[1,36] \tag{3.14}$$

$$g(d_{01}, v) \le 12 + \frac{d_{01}}{39} + v \le 48.136 \le Ec \approx 50.3\ (d_{01}, v) \in [0,1]\text{x}[0,36]$$

Hence the problem has three non negative solutions $v_1, v_2\ and\ v_3$ by considering the followings:

$$\max_{0 \le d_{01} \le 1} | v_1(d_{01}) | < \frac{1}{13}, \quad 1 < \min_{(1/4) \le d_{01} \le (3/4)} | v_2(d_{01}) | \quad \max_{0 \le d_{01} \le 1} | v_3(d_{01}) | \le 36, \tag{3.15}$$

$$\frac{1}{13} < \max_{0 \le d_{01} \le 1} | vs(d_{01}) | \le 36, \quad \min_{(1/4) \le d_{01} \le (3/4)} | v_3(d_{01}) | < 1$$

Hence the proof.

Conclusion

In this paper, non negative solutions for Fractional Differential Equations with global boundary conditions have been derived and various examples were discussed by applying the main result.

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