Face Recognition Using Non-negative Matrix Factorization (NMF) An Analysis of Order of Decomposition on Recognition Rate

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Abstract

Non-negative Matrix Factorization (NMF) is a well established dimension reduction technique. NMF reduces the dimension of the non-negative data matrix by applying the non-negativity constraint. One of the applications of NMF is recognition problem (e.g. face recognition). To solve recognition problem using NMF, the order of decomposition play an important role on the recognition rate. This paper presents the details analysis of the order of decomposition on the recognition rate. A standard face dataset A and T is used for analysis.

Keywords: Non-negative matrix factorization (NMF), face recognition, dimension reduction.

Introduction

Face recognition is one of the most heavily researched area in the field of computer vision and pattern recognition for last few decades. Many algorithms are presented to address the problem and the performance of current algorithms decreases on the face images which are captured under different lighting conditions, view angles and expressions.

Different approaches are adopted to solve the face recognition problem. Template based approaches compared to set of given templates. Statistical techniques like Support Vector Machine (SVM), Principle Components Analysis (PCA), Linear Discriminate Analysis (LDA) and Independent Component Analysis (ICA) can be used to construct the templates. The feature based approaches used local facial features and their geometric relationships to recognize the face image.

Some approaches used less information of face feature to recognize in which they process the facial features information separately without considering the relationship between the features and with the whole face. M. Nixon used human eyes as a feature, R. Brunelli and T. Poggio utilized a combination of features to recognize the faces. Ashraf et al used geodesic distance transform to solved the problem.

In model-based approaches developed a human face model for recognition. Generally, these models are morphable and the morphable nature of the model allows them to recognize the human face with the varying face pose. These approaches can be 2D or 3D. The 3D face models are more complicated as compared to 2D because it attempt to capture the three dimensional features of the human faces. Elastic Bunch Graph Matching is the example of the 3D Morphable models.

The subspace approaches PCA, LDA, ICA, Sub-Holistic are the most popular among the methods used for face recognition. PCA, LDA and ICA are the example of subspace approaches. The first most popular subspace technique is Eigenfaces method (PCA). The conventional Eigen faces method has number of limitations; first it has a poor discriminatory power although it provides a very good representation of the face images. The second limitation is that the basis images of PCA do not provide any natural visual meaning. In addition, the global feature extraction nature of the approach fails to handle the occlusions problem in the images.

A powerful subspace technique, named Non-negative Matrix Factorization (NMF), is introduced. NMF applied non-negatively constraints on the face images which allow them to handle the occlusion problems in the images.

This paper provides the details analysis of the dimension reduction role of NMF technique in the face recognition problem.

Non- Negative Matrix Factorization (NMF)

Non-negative matrix factorization (NMF) is used to determine the linear approximate representation of a non-negative data matrix. For a data matrix X where each column matrix ‘m’ represents the dimension of the data and ‘n’ is the number of samples. The NMF provides the linear approximation of data matrix X is given as

\[ X_{m \times n} \approx W_{m \times d} H_{d \times n} \]

Where, W is a basis matrix with columns representing basis vector and H is a coefficient matrix with columns as coefficient vector. The constant d is the rank of the approximation and to achieved the dimension reduction its value is chosen as \((m + n)\) if \(d < n \times m\).
The basis matrix $W$ represents the order of decomposition. The $WH$ is a lower-rank approximation to $A_{train}$. For a given matrix $X$, the optimal value of $W$ and $H$ can be obtained by iteratively minimizing the error function,

$$ f(W,H) = \frac{1}{2} ||X - WH||^2 $$

The product of $WH$ is called NMF of $X$. Alternate least square optimization method is used to minimize (2) due to its fast convergence rate and consistency.

**Face Recognition Using Non-Negative Matrix Factorization (NMF)**

A standard database for face recognition AT&T is used. The algorithm of the system is implemented in the Matlab the details are given below.

Initially, the pre-processing is performed on the images of given data set which involve the resizing of the original images. After preprocessing on images of database build the training set for the NMF algorithm. The training set $A$ is formed by linearizing the 2-D images and then arrange the images in such a way that the each column representing the single image.

$$ A_{train} = \begin{bmatrix} F_1^1 & F_2^1 & \cdots & F_n^1 \\ F_1^2 & F_2^2 & \cdots & F_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ F_1^m & F_2^m & \cdots & F_n^m \end{bmatrix} $$

The dimension of the training set $A$ is $m \times n$, where $m$ is the dimension of each image and $n$ is the total number of images in the training set $A$.

The NMF algorithm is applied to the training set $A_{train}$. $A_{train} = W_{train}H_{train}$

Where: $W$ is the basis matrix with the dimension $m \times r$ and $H$ is the coefficient matrix with the dimension of $r \times n$. The constant $r$ represents the order of decomposition. The $WH$ is a lower-rank approximation to $A_{train}$. The basis matrix $W_{train}$ of the training set $A_{train}$ is used to approximate the test image $T_{test}$ as $T_p = W_{train}H_p$

The above equation provides the coefficient matrix $H_{test}$ to the test image $B$. The most closest column matrix $F_q$ representing a face, in the training set $A_{train}$ to $F_q$ is obtained by minimizing equation (2) over the entire training set $A_{train}$ as $F_q = argmin \| H_p - H_{test} \|^2$

**Results and Discussion**

The algorithm used the standard database AT and T for evaluation. The database contains 40 classes and each class has 10 images. The algorithm used 5 images for training and testing. The recognition criteria of the proposed algorithm are that for each test images. It should have the top 5 matches to the same class of the test image for 100 percent recognition.

The algorithm is tested on the number of training classes 10, 20 and 40 classes. Per class recognition is also calculated for class.

<table>
<thead>
<tr>
<th>No. of Classes</th>
<th>NMF</th>
<th>PCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>98%</td>
<td>94.6%</td>
</tr>
<tr>
<td>20</td>
<td>92%</td>
<td>88.3%</td>
</tr>
<tr>
<td>40</td>
<td>81%</td>
<td>76.8%</td>
</tr>
</tbody>
</table>

This section provides the comparison of proposed algorithm with PCA as shown in table-1.
Figure-1
Training and testing Images of dataset

Figure-2
Recognition rate against decomposition order (between 1 to 20) for ten numbers of classes
**Figure-3**
Recognition rate of each class

**Figure-4**
Recognition rate against decomposition order (between 1 to 20) for 20 numbers of classes
Figure-5
Recognition rate of each class

Figure-6
Recognition rate against decomposition order (between 1 to 20) for 40 numbers of classes
Conclusion

This paper presents face recognition using NMF and also provides the details analysis of the role of order of decomposition on the recognition rate. It can be concluded from the obtain results that order of decomposition has an important role on the recognition rate.

References