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Bianchi Type-IX Dark Energy Model in Barber's Second Self-Creation Theory

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Abstract

A Bianchi type-IX dark energy cosmological model with variable equation of state (EoS) parameter is obtained in Barber's second Self-Creation theory of gravitation. The field equations have been solved by applying variation law for generalized Hubble's parameter given by Bermann. Some physical properties of the models are also discussed.

Keywords: Dark Energy, Bianchi type-IX universe, Self-Creation theory.

Introduction

The Recent cosmological observations has provided increasingly convincing evidence that the universe is currently experiencing a phase of accelerated expansion. In late ninetees, two teams studying distant type Ia-supernovae (SNe-Ia) independently presented evidence of expansion[1-4] and confirmed later by cross checks from the cosmic microwave background radiation (CMBR)[5-6] and large scale structure (LSS)[7-11]. To explain the cosmic positive acceleration, mysterious dark energy (DE) has been proposed. Several DE models are distinguished using variable EoS $p = \omega\rho$ (p is the field pressure and ρ is its energy density) during evolution of the universe. Many relativists[12-16] have obtained DE cosmological models in various theories of gravitation in different contexts. Recently, Ghate and Sontakke[17-19] have studied DE cosmological model in Lyra and Brans-Dicke theory of gravitation.

Einstein's theory of general relativity is considered as the only successful theory of gravitation in terms of geometry which has most beautiful structures of the theoretical physics. Attempts are made by the relativists for proposing several alternative theories of gravitation. The most popular amongst them are scalar-tensor theories of gravitation formulated by Brans-Dicke[20], Nordtvedt[21], Sen[22], Sen and Dunn[23], Wagonar[24], Saez-Ballester[25]. Barber[26] has proposed two self-creation cosmologies by modifying the Brans-Dicke theory and general relativity. Barber's first theory is not in disagreement with experiment but is actually inconsistent in general has been pointed out by Brans[27]. Amongst the modified theories of gravitation, the Barber's second self creation theory modifies general relativity to a variable G-theory which include continuous creation within observational limits. In this theory the scalar-field does not directly gravitate, but simply divides the matter tensor with the scalar acting as a reciprocal gravitational constant.It is postulated that this scalar field couples to the trace of the energy momentum tensor. FRW cosmological models have been discussed by Pimental[28] and Soleng[29] by using a power law relation between the expansion of the universe and the scalar field. Attempts are made to present Bianchi type cosmological models in Barber's self creation theory of gravitation by number of authors like Singh[30], Reddy[31], Reddy *et al.*[32], Reddy andVenkateswarlu[33], Shanti and Rao[34]. Ram and Singh[35] have discussed the spatially homogeneous and isotropic Robertson-Walker and Bianchi type-II models of the universe in Barber's self-creation theory in presence of perfect fluid by using gamma law equation of state. Pradhan and Pandey[36], Pradhan and Vishwakarma[37], Panigrahi and Sahu[38], Venkateshwarlu and Kumar[39], Singh and Kumar[40], Venkateswarlu*et al.*[41], Reddy and Naidu[42] and Katore*et al.*[43] are some of the authors who have studied Barber's second self creation theory in different contexts. Katore*et al.*[44] have studied accelerating and decelerating hypersurface-homogeneous cosmological models in Barbers's second self-creation theory. Recently Mahanta[45] studied a dark energy (DE) models with variable EoS parameter in self-creation theory of gravitation.

Bianchi type-IX cosmological models are very popular for relativistic studies. These models are also used to examine the role of certain anisotropic sources during the formation of large scale structures as we seen the universe today. Number of authors Chakraborty[46], Raj Bali and Dave[47], Raj Bali and Yadav[48], Pradhan[49], Tyagi*et al.*[50], Ghate and Sontakke[51-52] have studied Bianchi type-IX cosmological models in different context.

In this paper, we consider Bianchi type-IX space-time filled with DE in Self-Creation theory. This work is organized as follows: In Section 2, the model and field equations have been presented. The field equations have been solved by applying variation law for generalized Hubble's parameter given by Berman. The physical behavior of the model has been discussed. In the last concluding remarks have been expressed.

Metric and Field Equations

Bianchi type-IX metric is considered in the form,

$$ds^2 = -dt^2 + a^2dx^2 + b^2dy^2 + \left(b^2\sin^2 y + a^2\cos^2 y\right)dz^2 - 2a^2\cos y\,dx\,dz \quad (1)$$

Where a, b are scale factors and are functions of cosmic time t.
In Barber's second Self-Creation theory, the field equations are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -\frac{8\pi}{\varphi}T_{ij}\,, \quad (2)$$

where $$\phi^{k}_{;k} = \frac{8\pi}{3}\lambda T\,. \quad (3)$$

Here R_{ij} is the Ricci tensor, R is the Ricci scalar, T_{ij} is the energy tensor, T is thetrace of the energy momentum tensor, λ is a coupling constant to be determined from the experiment $(|\lambda| \le 0.1)$. A comma denotes partial differentiation while semicolon denotes covariant differentiation. and ϕ_i denotes ordinary derivatives with respect to x_i

The energy momentum tensor of the fluid is taken as

$$T_i^j = \left[T_0^0, T_1^1, T_2^2, T_3^3\right]\,. \quad (4)$$

One can parameterize energy momentum tensor as follows:

$$T_i^j = \left[-\rho, p_x, p_y, p_z\right],$$
$$T_i^j = \left[-1, \omega_x, \omega_y, \omega_z\right]\rho\,,$$
$$T_i^j = \left[-1, \omega, \omega+\delta, \omega+\delta\right]\rho\,. \quad (5)$$

Here ρ is the energy density of the fluid, p_x, p_y, p_z are the pressures and ω_x, ω_y and ω_z are the directional EoS parameters along *x, y* and *z* axes respectively, ω is the deviation free EoS parameter of the fluid.

Now, parameterizing the deviation from isotropy by setting $\omega_x = \omega$ and then introducing two skewness parameters δ and γ which aredeviations from ω on *y* and *z* axes respectively. Here δ and γ are not necessarily constants and can be functions of the cosmic time t.

In the co-moving coordinate system the field equations (2)–(4) for the metric (1) and with the help of energy momentum tensor (6) can be written as

$$2\frac{\dot a}{a}\frac{\dot b}{b} + \frac{\dot b^2}{b^2} + \frac{1}{b^2} - \frac{a^2}{4b^4} = \frac{8\pi}{\phi}\rho\,, \quad (6)$$

$$2\frac{\ddot b}{b} + \frac{\dot b^2}{b^2} + \frac{1}{b^2} - \frac{3a^2}{4b^4} = -\frac{8\pi}{\phi}\omega\rho\,, \quad (7)$$

$$\frac{\ddot a}{a} + \frac{\ddot b}{b} + \frac{\dot a}{a}\frac{\dot b}{b} + \frac{a^2}{4b^4} = -\frac{8\pi}{\phi}(\omega+\delta)\rho\,, \quad (8)$$

$$\frac{\ddot a}{a} + \frac{\ddot b}{b} + \frac{\dot a}{a}\frac{\dot b}{b} + \frac{a^2}{4b^4} = -\frac{8\pi}{\phi}(\omega+\gamma)\rho\,, \quad (9)$$

$$\ddot\phi + \dot\phi\left(\frac{\dot a}{a} + 2\frac{\dot b}{b}\right) = -\frac{8\pi}{3}\lambda T\,, \quad (10)$$

where the overdot $(\dot{})$ denotes the differentiation with respect to t.
From equations (8) and (9) we see that, the deviations from ω along y and z axes are same i.e. $\gamma = \delta$.

Solutions of Field Equations

The field equations (6)–(10) are four independent equations in sixunknowns $a, b, \omega, \rho, \delta, \phi$. Hence two additional conditions relating these unknowns may be used to obtain explicit solutions of the systems.

(i) Firstly, we assume that the expansion θ in the model is proportional to the shear σ. This condition leads to

$$a = b^m\,, \quad (11)$$

where*m* is proportionality constant.

The motive behind assuming condition is explained with reference to Thorne[53], the observations of the velocity red-shift relation for extragalactic sources suggest that Hubble's expansion of the universe is isotropic today within ≈ 30 percent[54,55]. To put more precisely, red-shift studies place the limit $\frac{\sigma}{H} \le 0.3$ on the ratio of shear σ to Hubble's constant *H* in the neighborhood of our galaxy today. Collin *et al.*[56] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant.

(ii) Secondly, the law of variation of Hubble's parameter that yields a constant value of deceleration parameter. Such types of relations have already been considered by Berman[57] for solving FRW models.
We consider the constant deceleration parameter model defined by

$$q = -\frac{R\ddot R}{\dot R^2} = \text{constant}, \quad (12)$$

where the scale factor *R* is given by

$$R = \left(ab^2\right)^{\frac{1}{3}}\,. \quad (13)$$

The solution of equations (12) and (13), gives

$$R = (\alpha t + \beta)^{\frac{1}{1+q}}, \tag{14}$$

where $\alpha(\neq 0)$, and β are constants of integration.
This condition implies that the condition of expansion is $1+q>0$.

Solving the field equations (7)-(10) with the help of equations (11), (13) and (14), we obtain the expansion for metric coefficients as follows:

$$a = (\alpha t + \beta)^{\frac{3m}{(1+q)(m+2)}}, \tag{15}$$

$$b = (\alpha t + \beta)^{\frac{3}{(1+q)(m+2)}}. \tag{16}$$

Through a proper choice of coordinates and constants, Bianchi type-IX DE cosmological model in Self-Creation theory of gravitation can be written as

$$ds^2 = -dt^2 + t^{\frac{6m}{(1+q)(m+2)}} dx^2 + t^{\frac{6}{(1+q)(m+2)}} dy^2 + \left(t^{\frac{6}{(1+q)(m+2)}} \sin^2 y + t^{\frac{6m}{(1+q)(m+2)}} \cos^2 y \right) dz^2 - 2t^{\frac{6m}{(1+q)(m+2)}} \cos y dx dz \tag{17}$$

Some Physical Properties of the Model

For the cosmological model (17), the physical quantities spatial volume V, Hubble parameter H, expansion scalar θ, mean anisotropy parameter A_m, shear scalar σ^2 and deceleration parameter q, are obtained as follows :

Spatial volume, $$V = ab^2 = t^{\frac{3}{1+q}}. \tag{18}$$

Hubble parameter, $$H = \frac{\alpha}{(1+q)t}. \tag{19}$$

Expansion scalar, $$\theta = \frac{3\alpha}{(1+q)t}. \tag{20}$$

Mean Anisotropy Parameter, $$A_m = \frac{2(m-1)^2}{(m+2)^2} = \text{constant} \tag{21}$$

Shear scalar, $$\sigma^2 = \frac{1}{2}\left[\frac{6\alpha^2(m-1)^2}{(1+q)^2(m+2)^2 t^2}\right]. \tag{22}$$

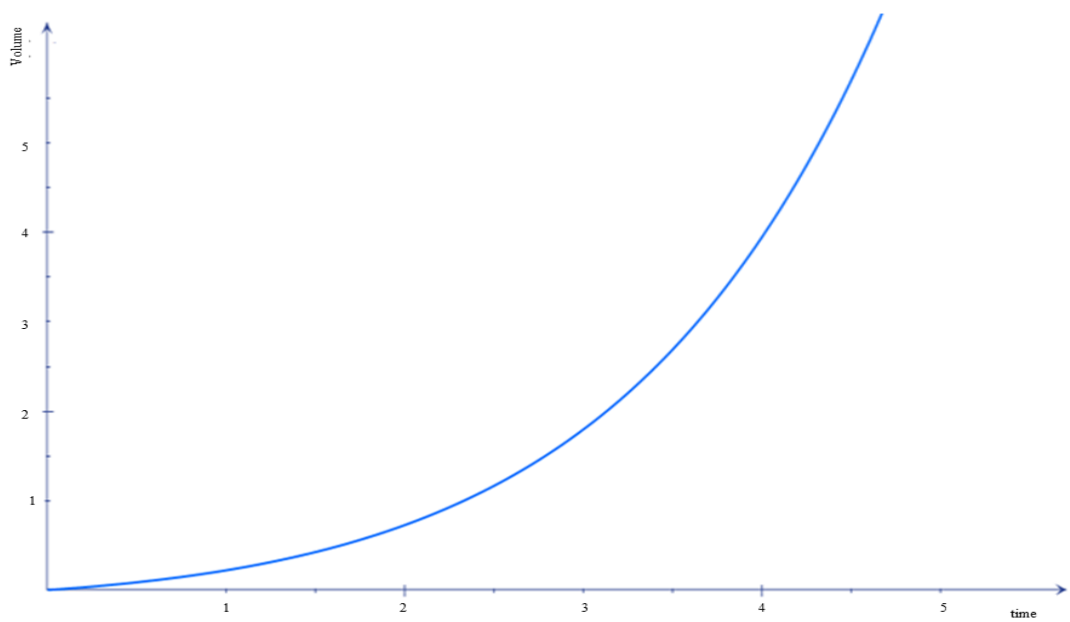

Figure-1
The plot of Volume verses time

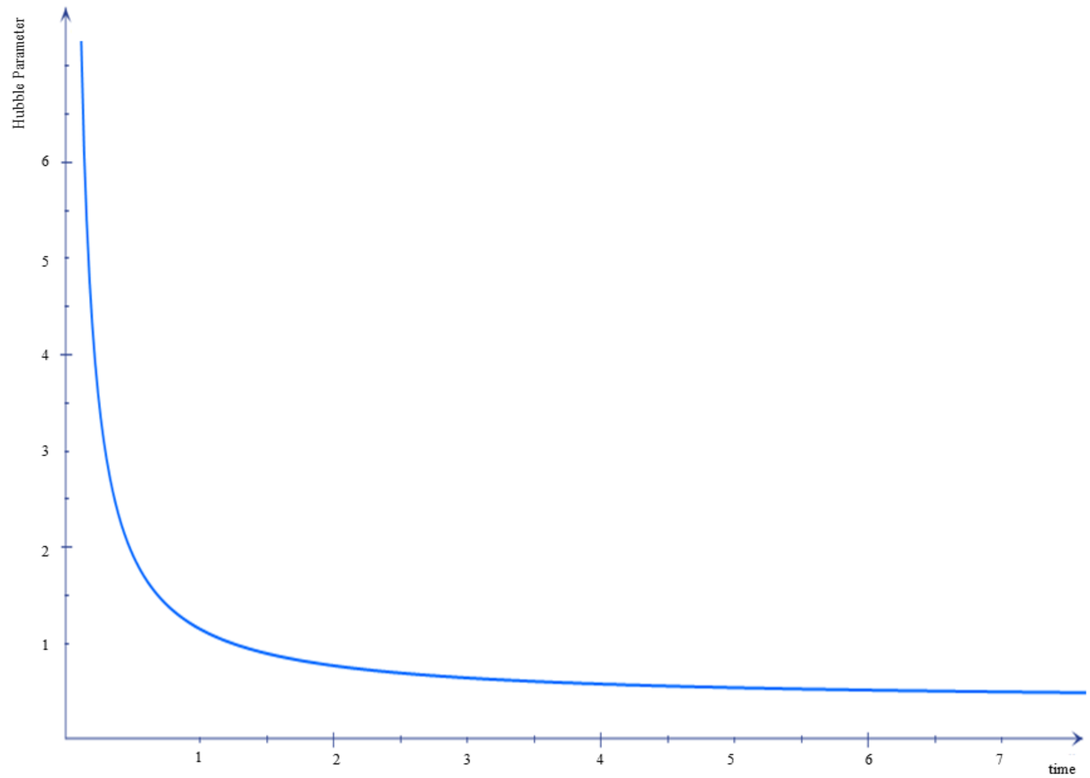

Figure-2
The plot of Hubble Parameter verses time

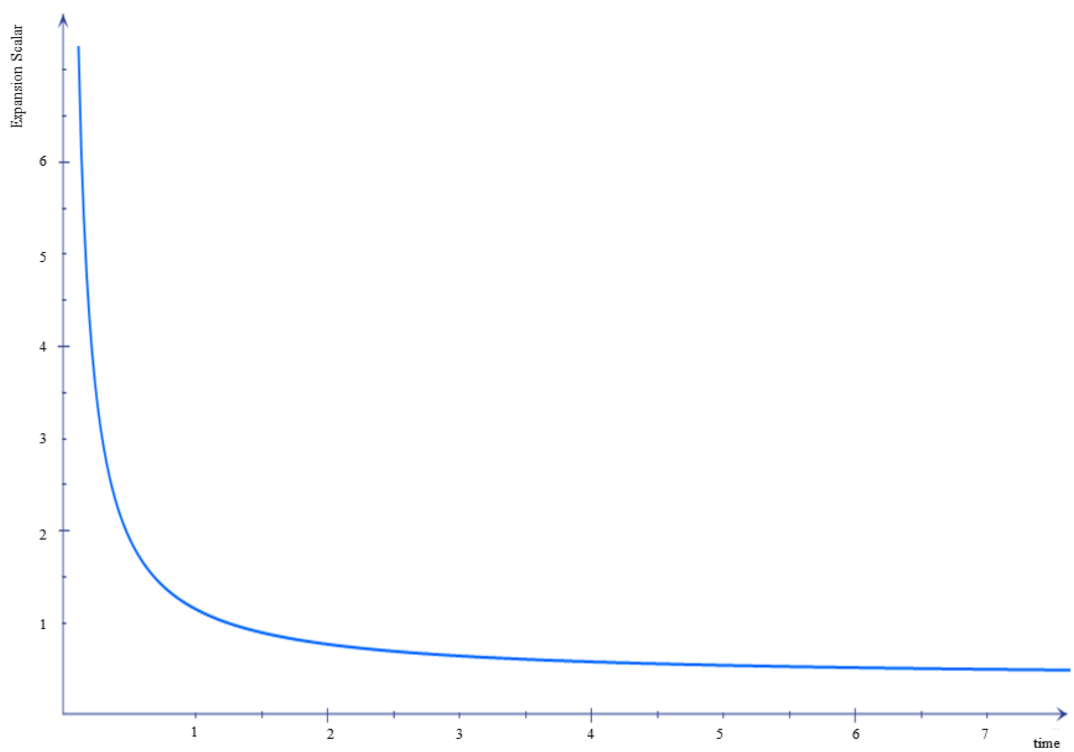

Figure-3
The plot of Expansion Scalar verses time

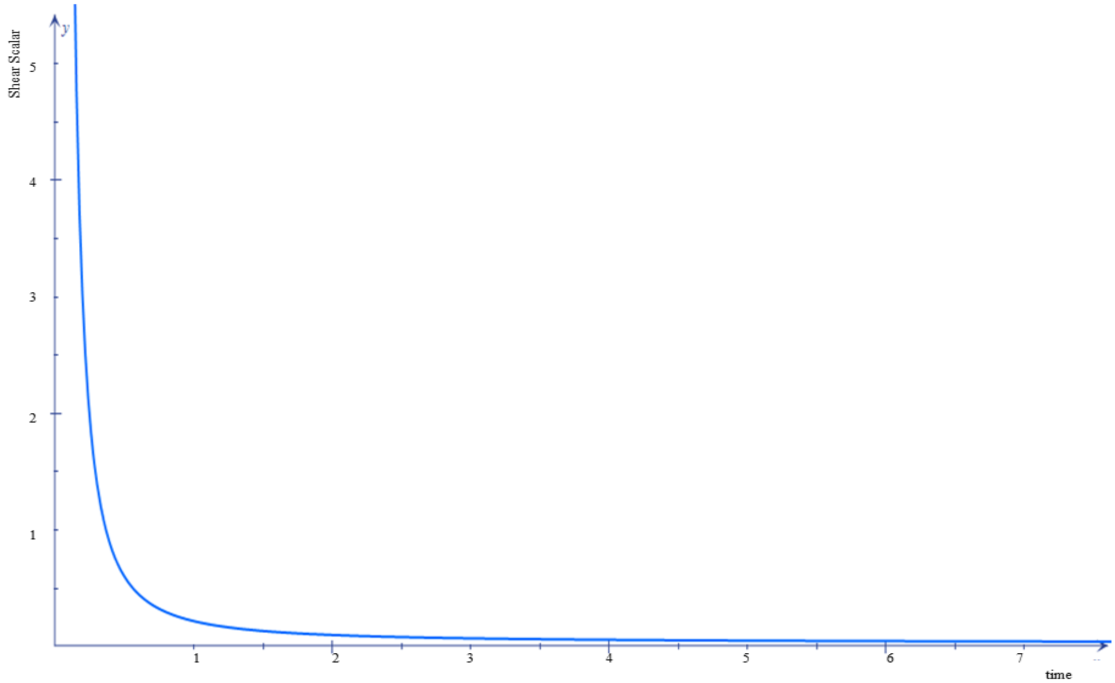

Figure-4
The plot of Shear Scalar verses time

Also, $$\frac{\sigma^2}{\theta^2}=\frac{(m-1)^2}{3(m+2)^2}\neq 0\ (m\neq 1)\ . \qquad (23)$$

The energy density,

$$\rho=\frac{\phi_0}{8\pi}\left[\frac{9(2m+1)}{(1+q)^2(m+2)^2}t^{\frac{(1+2q)}{(1+q)}}+t^{\frac{(m-4)}{(1+q)(m+2)}}-\frac{1}{4}t^{\frac{(7m-10)}{(1+q)(m+2)}}\right] \qquad (24)$$

EoS parameter,

$$\omega=-\frac{\frac{3(5-2m-2qm-4q)}{(1+q)^2(m+2)^2\,t^2}+t^{\frac{-6}{(1+q)(m+2)}}-\frac{3}{4}t^{\frac{6(m-2)}{(1+q)(m+2)}}}{\frac{9(2m+1)}{(1+q)^2(m+2)^2}t^{\frac{(1+2q)}{(1+q)}}+t^{\frac{(m-4)}{(1+q)(m+2)}}-\frac{1}{4}t^{\frac{(7m-10)}{(1+q)(m+2)}}}. \qquad (25)$$

Skewness parameter,

$$\delta=-\frac{\frac{3(2m^2+2m-qm^2-qm+2q-4)}{(1+q)^2(m+2)^2\,t^2}-t^{\frac{-6}{(1+q)(m+2)}}+t^{\frac{6(m-2)}{(1+q)(m+2)}}}{\frac{9(2m+1)}{(1+q)^2(m+2)^2\,t^2}+t^{\frac{-6}{(1+q)(m+2)}}-\frac{1}{4}t^{\frac{6(m-2)}{(1+q)(m+2)}}}\ . \qquad (26)$$

The cosmological model (17) has no initial singularity(*i.e.* at $t=0$). Physical quantities ρ,ω,δ diverge at $t=0$ while they tends to zero as $t\to\infty$. The scalar field $\phi\to\infty$ for large t. The spatial volume is zero at $t=0$ and increases as t increases which shows the accelerated expansion of the universe. Also, the scalar expansion θ, shear scalar σ^2, and the Hubble's parameter H tend to ∞ as $t\to 0$ and approach zero for large t. The mean anisotropy parameter is uniform throughout the evolution of the universe, since it does not depend on the cosmic time t. As $A_m\neq 0$ and σ^2/θ^2 is a constant, the model does not approach isotropy for large value of t.

Conclusion

Bianchi type-IX cosmological model has been obtained when the universe is filled with DE in Barber's second Self-Creation theory of gravitation by solving the field equations using a special law of variation of Hubble parameter proposed by Bermann[57]. The model obtained is expanding and free from initial singularity. It is observed that the model is non-singular, shearing and non-rotating and does not approach isotropy for large values of t. DEEoS parameters are time dependent. Since $A_m\neq 0$, indicating that the model is anisotropic throughout the evolution.We hope that the model will be useful for a better understanding the role of dark energy in cosmology.

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