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Kaluza-Klein Dust Filled Universe with Time Dependent Λ in Creation Field Cosmology

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Abstract

The solution of field equations in the creation field with variable cosmological constant have been obtained for Kaluza-Klein universe. Following Hoyle and Narlikar, we have assumed that universe is filled with dust distribution. To get deterministic solution, a relation between shear (σ) and expansion (θ) is assumed. The physical aspects of the model are also studied.

Keywords: Creation field cosmology, time dependent Λ, Kaluza-Klein Metric, Dust Distribution.

Introduction

The phenomenon of expanding universe, Primordial Nucleosynthesisand the observed isotropy of Comic Microwave Background Radiations (CMBR) are the three important observations in astronomy which were successfully explained the big-bang cosmology based on Einstein field equations called as big-bang model. This model is described by Friedman-Robertson-Walker line element and a matter density source which obeys equation of state $\rho = 3p$, where p and ρ are the fluid pressure and matter density respectively. In the late eighties, the astronomical observations revealed that the predictions of the big-bang model do not always exactly meet our expectations as was believed earlier[1]. So, alternative theories of gravitation were proposed by the researchers. The steady state theory by Bondi and Gold[2] is the most well-known theory in which the universe does not have any singular beginning or an end on the cosmic time scale. To maintain constancy of matter density, they envisaged a very slow but continuous creation of matter in constraint to explosive creation of standard model. The theory does not give any physical justification for continuous creation of matter and principle of conservation of matter was violated in this formalism. To overcome this difficulty, Hoyle and Narlikar[3] proposed C-field theory in which they adopted a field theoretical approach introducing a massless and chargeless scalar field in the Einstein-Hilbert action to account for the matter creation. In this theory, there is no big-bang type singularity as in steady state theory of Bondiand Gold. If a model explains successfully the creation of positive energy matter without violating conservation of energy then it is necessary to have some degrees of freedom which act as a negative energy mode. The matter creation is accomplished at the expense of negative energy. C-field theory solves horizon and flatness problem faced by big-bang model that was pointed out by Narlikar[4-5].Chatterjee and Banerjee[6] have investigated higher dimensional cosmology in C-field theory. Singh andChaubey[7] have studied Bianchi type I, III, V, VI_0 and Kantowski-Sachs universes in creation field cosmology. Recently, Katore[8] has investigated plane symmetric universe in C-field cosmology.

In the modern cosmological theories the dynamic cosmological term Λremains a focal point of interest as it solves the cosmological constant problem[9, 10] in a natural way. Since its introduction, its significance has been studied from time to time by many researchers[11-13]. A wide range of observations suggest that universe possesses a non-zero cosmological constant. During an early exponential phase, the vacuum energy is created as large cosmological constant, which is expected by Glashow-Salam-Weinberg and Grand Unified Theory as mentioned by Langacker[14]. Therefore, the present day observations of smallness of cosmological constant $(\Lambda \leq 10^{-56}\ cm^{-2})$ support to assume that the cosmological constant Λ is as time dependent. The relation $\Lambda \sim \frac{1}{t^2}$ seems to play a major role in cosmology[15]. Several attempts have made by many researchers[16-18] in the favor of time dependent $\Lambda \sim \frac{1}{t^2}$ in different contexts. Recently, Bali and Saraf[19-21] have obtained dust filled universe with varying Λ in C-field cosmology with different contexts.

The unification of gravity with other fundamental forces in nature is still a challenging problem today. Use of extra dimensional space time in the modern cosmological theories was made keeping in view the unification of the fundamental forces of nature and for developing Theory of Everything (ToE). Most recent efforts in this line demand the space time dimensions to be more than four. Also modern developments of superstring theory and Yang-Milles super gravity in its field theory limits need higher dimensional space time. Nordstrom[22] and Kaluza-Klein[23-24] suggested the idea of one extra space dimension in order to explain the unification of gravity with other fundamental forces. Numbers of authors have obtained cosmological models in the context of Kaluza-Klein theory viz. Marciano[25], Ponce[26], Chi[27], Fukui[28], Liu and Wesson[29], Coley[30], Tegmark[31] have studied Kaluza-Klein cosmological models with different matters. Excellent reviews of Kaluza-Klein theory and higher dimensional unified theories have been presented byOverduin and Wesson[32]. Li*et al0.*[33] have considered the inflation in Kaluza-Klein theory. Wesson[34] have investigated the cosmological constant problem in Kaluza-Klein cosmology. Baysal*et al.*[35] have investigated five dimensional cosmological model with variable G and Λ. Adhav*et al.*[36] have studied Kaluza-Klein universe in C-field cosmology.Purohit and Bhatt[37] have studied FLRW type Kaluza-Klein cosmological model with static extra dimension. Panigrahi*et al.*[38] have obtained five dimensional cosmological modelin the Kaluza-Klein theory. Recently, Shri RamandPriyanka[39] have investigated some Kaluza-Klein cosmological model in $f(R,T)$ gravity theory.

In this paper, we consider Kaluza-Klein space-time with varying cosmological constant in the presence of creation field. In section 2, the model and field equations have been presented. The field equations have been solved in section 3. The physical and geometrical aspects of the model have been discussed in section 4. In the last section 5 concluding remarks have been expressed.

The Metric and Field Equations

We consider the Kaluza-Klein metric in the form

$$ds^2 = dt^2 - A^2\left(dx^2 + dy^2 + dz^2\right) - B^2 d\psi^2, \qquad (1)$$

where A, B are scale factors which are functions of time t only and $\sqrt{-g} = A^3 B$.

The Einstein field equations by introduction of C- Field is modified by Hoyle and Narlikartheory[40-42] as

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G\left(\underset{(m)}{T_i^j} + \underset{(c)}{T_i^j}\right) - \Lambda g_i^j . \qquad (2)$$

The energy momentum tensor $\underset{(m)}{T_i^j}$ for perfect fluid and $\underset{(c)}{T_i^j}$ for creation field are given by

$$\underset{(m)}{T_i^j} = (\rho + p) v_i v^j - p g_i^j \qquad (3)$$

and

$$\underset{(c)}{T_i^j} = -f\left(c_i c^j - \frac{1}{2} g_i^j C^\alpha C_\alpha\right), \qquad (4)$$

where $f > 0$ is the coupling constant between matter and creation field and $C_i = \frac{dC}{dx^i}$.

The Einstein field equations (2) for the metric (1) for variable $\Lambda(t)$ leads to

$$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{B}}{AB} = 8\pi G\left(\rho - \frac{1}{2} f\dot{C}^2\right) + \Lambda \qquad (5)$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = 8\pi G\left(-p + \frac{1}{2} f\dot{C}^2\right) + \Lambda \qquad (6)$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} = 8\pi G\left(-p + \frac{1}{2} f\dot{C}^2\right) + \Lambda . \qquad (7)$$

Solution of Field Equations

The conservation equation

$$\left[8\pi G T_i^j + \Lambda g_i^j\right]_{;j} = 0 \qquad (8)$$

leads to

$$\frac{d}{dt}\dot{C}^2 + 2\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{C}^2 = \frac{2\dot{\rho}}{f} + \frac{2\rho}{f}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{\dot{\Lambda}}{4\pi G f}, \qquad (9)$$

where p being an isotropic pressure.

Following Hoyle and Narlikar Theory, we have taken $p = 0$, for dust universe.The source equation of C-field $C^j_{;i} = \frac{n}{f}$ leadsto $C = t$, for larger r.Thus $\dot{C} = 1$.

Using $p = 0$, $\dot{C} = 1$, equations (5), (6) and (7) lead to

$$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{B}}{AB} = 8\pi G\rho - 4\pi G f + \Lambda \qquad (10)$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = 4\pi G f + \Lambda \qquad (11)$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}}{A^2} = 4\pi G f + \Lambda . \qquad (12)$$

We assume that scalar expansion σ is proportional to θ which leads to

$$B = A^n, \qquad (13)$$

where n isanarbitrary constant and A, B are metric potentials. From equations (11) and (12), we get

$$\frac{\ddot{B}}{B}-\frac{\ddot{A}}{A}+2\frac{\dot{A}\dot{B}}{AB}-2\frac{\dot{A}^2}{A^2}=0. \qquad (14)$$

Using (13) in equations (14) therein we get

$$2\ddot{A}+2(n+2)\frac{\dot{A}^2}{A}=0. \qquad (15)$$

To get the deterministic value of A, we assume that

$\dot{A}=F(A)$ and $\ddot{A}=FF'$, where $F'=\frac{dF}{dA}$.

Equation (15) leads to

$$\frac{dF^2}{dA}+\frac{2(n+2)}{A}F^2=0. \qquad (16)$$

This after integration, we get

$$F^2=\frac{k_1^2}{A^{2(n+2)}}=\left(\frac{dA}{dt}\right)^2, \qquad (17)$$

where k_1^2 is the constant of integration.

Integrating equation (17), it transforms to

$$A=\left[(n+3)(k_1t+k_2)\right]^{\frac{1}{n+3}}, \qquad (18)$$

where k_2 is a constant of integration.

Equation (18) leads to

$$A=(at+b)^{\frac{1}{n+3}}, \qquad (19)$$

where $a=(n+3)k_1$, $b=(n+3)k_2$ and n is a positive integer.

Using equation (19) in equation (13), we get

$$B=(at+b)^{\frac{n}{n+3}}. \qquad (20)$$

From equation (11), we have

$$\Lambda=3\frac{\ddot{A}}{A}+3\frac{\dot{A}^2}{A^2}-4\pi Gf . \qquad (21)$$

Using equation (19) in equation (21), we get

$$\Lambda=-\frac{(n+1)(3a^2)}{(n+3)^2(at+b)^2}-k, \qquad (22)$$

where $4\pi Gf=k$.

Using equations (19), (20) and (22) in equation (10), we gat

$$8\pi G\rho=\frac{(n+1)(6a^2)}{(n+3)^2(at+b)^2}+2k. \qquad (23)$$

Using equations (19) and (20) in metric (1), the model is given by

$$ds^2=dt^2-(at+b)^{\frac{2}{n+3}}\left(dx^2+dy^2+dz^2\right)-(at+b)^{\frac{2n}{n+3}}d\psi^2. \qquad (24)$$

Using equations (19), (20), (22) and (23) in equation (9), we have

$$\frac{d}{dt}\dot{C}^2+\left[2\left(\frac{a}{at+b}\right)\right]\dot{C}^2=\left[2\left(\frac{a}{at+b}\right)\right]. \qquad (25)$$

This on integration gives

$$\dot{C}^2=1. \qquad (26)$$

Thus we have

$$\dot{C}=1, \qquad (27)$$

which leads to $C=t+b$, where b is a constant of integration. We find $\dot{C}=1$,which agrees with the value used in the source equation. Thus creation field C is proportional to time t.

Physical and Geometrical Aspects

The metric (1) for constrained mentioned above, leads to

$$ds^2=dt^2-(at+b)^{\frac{2}{n+3}}\left(dx^2+dy^2+dz^2\right)-(at+b)^{\frac{2n}{n+3}}. \qquad (28)$$

The homogeneous mass density (ρ), the cosmological constant (Λ), creation field (C), the spatial volume (R^4) and deceleration parameter (q) for the model (28) are given by

$$8\pi G\rho=\frac{(n+1)(6a^2)}{(n+3)^2(at+b)^2}+2k, \qquad (29)$$

$$\Lambda=-\frac{(n+1)(3a^2)}{(n+3)^2(at+b)^2}-k, \qquad (30)$$

$$C=t, \qquad (31)$$

$$R^4=(at+b), \qquad (32)$$

$$q=-\frac{\ddot{R}/R}{\dot{R}^2/R^2}=3, \qquad (33)$$

where $k=4\pi Gf$.

Conclusion

The spatial volume increases with time indicating that inflationary scenario is possible in Kaluza-Klein universe with variable Λ in presence of creation field. The deceleration parameter q which shows that the model is decelerating. The creation field C increases with time which matches the results as obtained by Hoyle and Narikar[40]. The matter density is throughout positive. The cosmological constant diverge initially at finite time $t = -\frac{b}{a}$ and decreases for increase in time. It remains constant for large values of t. (i.e. $t \to \infty$).

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