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Parameterization of Proton Structure Function using Fractal Inspired Model

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Abstract

In recent years, applicability of fractal geometry in the structure of nucleons is getting importance. This work reports a fractal inspired parameterization for the structure function of proton. In some earlier work by the author, a fractal model of proton structure was proposed with few fitted parameters. Later on this model was applied to few other aspects of nucleon structure. This paper reports a new parameterization applicable in a wide range of x and Q^2 of deep inelastic scattering in the light of recent data. The calculated chi square value proves the goodness of fit and therefore the model is expected to have wider range of applicability.

Keywords: Deep inelastic scattering, structure function, self-similar dimension.

Introduction

Measurement of nucleon structure functions has attracted the attention of theoretical and experimental high energy physicist since decades. Experimentally, the electron proton collider HERA (Hadron Electron Ring Accelerator) explores the proton structure at high centre of mass energy, $\sqrt{s} \approx 320 GeV$, where $s = 4E_e E_p$, E_e being the lepton beam energy and E_p is the proton beam energy[1,2,3]. Theoretically, the structure functions of proton are determined by QCD fits. In the method of parameterization, a certain functional form containing a set of parameters for the structure function is assumed. The model is then fitted to experimental data at different energy scales.

Fractal objects are characterised by scaling laws or power laws, reflecting a deep internal symmetry[4,5]. They are characterised by a self-similar dimension (D) and magnification factor (M) related by $D = \frac{\log M^D}{\log M}$. In some earlier work, parameterization of structure function of proton was reported on the basis of HERA data[6-8]. A statistical model for proton structure analysing fractal characteristics was proposed by S.N. Banerjee et al[9-11]. The fractal inspired model can be applied to deep inelastic neutrino-nucleon scattering, gluon distribution function inside the nucleon, ultra high energy neutrino nucleon scattering cross sections, longitudinal structure function of the nucleon and to other related areas[12-15]. In this paper the author reanalyses the fractal inspired model with recent data[16]. The motivation for this reanalysis is the applicability of the earlier model in a limited range of data and also the limitation of earlier model to describe neutrino-nucleon scattering and ultra high energy neutrino nucleon interaction cross section[12,14].

Methodology

Self-similar objects are described by a self-similar dimension and a magnification factor. Deep inside the proton structure, more gluon-gluon interactions are observed. So a scaling described by a power law, which is a characteristic of self-similar objects, is expected to exist in the proton structure. The un-integrated u-quark density exhibits a linear behavior as a function of x at fixed Q^2 and as a function of Q^2 at fixed x, where Q^2 is the four momentum transfer squared and x is the fraction of proton momentum carried away by a constituent parton in deep inelastic scattering[6]. Such linear behavior suggests that x and Q^2 or their suitable functions can be considered as the magnification factors for fractal proton. Considering $1/x$ and $\frac{Q_0^2}{Q_0^2+q^2}$ as the magnification factors, the following form for un-integrated parton density can be suggested,

$$F_2(x,Q^2) = \frac{(\exp D_0)Q_0^2 x^{-D_2+1}}{1-D_3-D_1\log\frac{1}{x}}\left(x^{D_1\log\left(1+\frac{Q^2}{Q_0^2}\right)}\left(1+\frac{Q^2}{Q_0^2}\right)^{-D_3+1}-1\right) \quad (1)$$

Now, the model needs to identify the four parameters best fitted to latest experimental data.

In equation (1), D_1 is the dimensional correlation relating the two magnification factors $1/x$ and $\frac{Q_0^2}{Q_0^2+q^2}$, whereas D_2 and D_3 are the fractal dimensions associated with $1/x$ and $\frac{Q_0^2}{Q_0^2+q^2}$ respectively, D_0 being the normalization constant.

Results and Discussion

In this work, the author has reported a parameterization of proton structure function $F_2(x,Q^2)$ with recent data reported by H1 and ZEUS collaboration[16]. Out of the four parameters (D_i where i=0-3), D_2 and D_3 should be positive and non zero since they are identified as the self-similar dimensions[4-5]. By

imposing a positivity constraint on the two parameters D_2 and D_3, a set of parameters as given in table-1 was obtained. $F_2(x,Q^2)$ is then calculated within the fractal approach using equation (1). Figure-1 shows the plot of estimated $F_2(x,Q^2)$ within the fractal inspired model and $F_2(x,Q^2)$ obtained from HERA as function of x for different values of Q^2, the four momentum transfer squared[16]. The chi square analysis is done to observe the reliability of the fit. The values of standard deviation and chi square are also recorded in table-1. The chi square value ensures the reliability of parameter estimation.

Table-1
The fitted parameters with χ^2 value for the fractal inspired model

D_0	D_1	D_2	D_3	Q_0^2	Standard deviation	χ^2/dof
1.03038	-0.07292	1.025473	0.76741	0.07939	0.0525	1.004

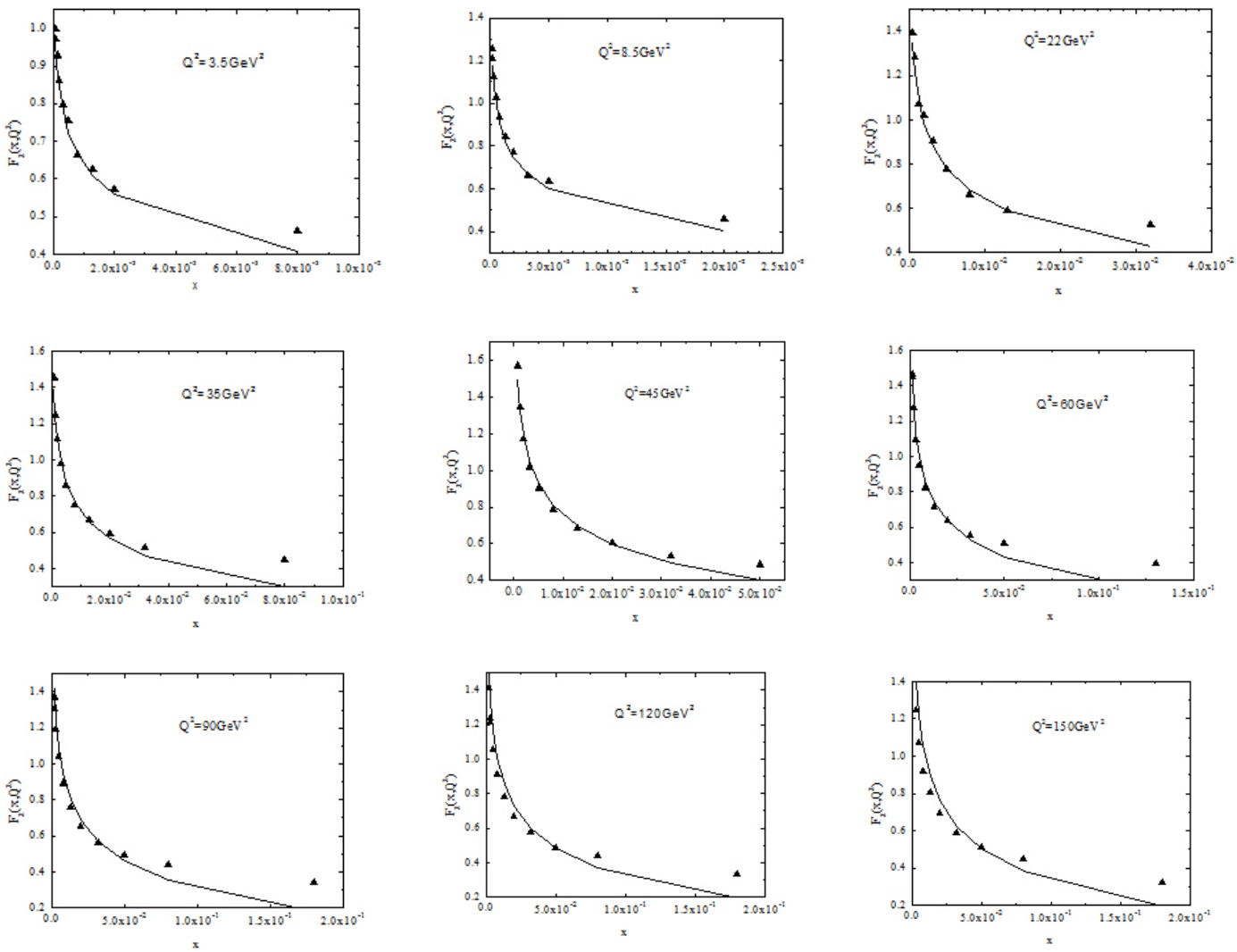

Figure-1
The plot of $F_2(x,Q^2)$ as a function of x in bins of Q^2. The lines represent estimated values from fractal inspired model of this work and the symbols represent the data presented by H1 and ZEUS collaboration[16].

Conclusion

In this parameterization of proton structure with recent data, the goodness of parameter estimation is reflected in the graphs and also in chi square analysis. Two of the parameters (D_2 and D_3) are identified as the self-similar dimensions of proton. The fractal characteristics of hadrons like proton as pursued in some references within a statistical quark model are found to be in good agreement with recent experimental data[9-11]. Our result compliments such idea in deep inelastic region. The fractal inspired model will be tested for a wider range of x and Q^2 available from experiments and also for other measures of proton structure which will provide a detailed analysis of the model.

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