Introduce and Compare Two Approaches for Monitoring a Two-Stage Process by Profile Quality Characteristic in the Second Stage

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Abstract
The quality of products is monitored according to the different control charts for quality characteristics of them. These quality characteristics are either variable or attribute can be a single variable, or a vector of variables or a profile relationship. However, because the quality of the product is the result of the performance of different procedures on the product, and usually these stages are not independent of each other, therefore, the assumption of independent process affecting error on the quality of the output. Up to now the effects of these situations on monitoring the multistage processes with single or multi variables were being examined. Though, multi-stage profile processes were less appealing for researchers. This paper introduces a model for a two-stage profile in addition to two different approaches that have been proposed for monitoring process. And the variation of the coefficients of the profile, as well as changes in the quality characteristics of the first stage, in a two-stage process, on the second phase control charts were reviewed.

Keywords: Average run length (ARL), exponentially weighted moving average (EWMA) control chart, multistage processes, $T^2$ multivariate control chart, profile monitoring.

Introduction
Nowadays, quality of many productions and service environment is monitored by statistical quality control based on statistical methods. Statistical process control, design of experiment and process capability are three major issues in statistical quality control. Control charts are powerful tools in statistical process control. Research activities undertaken in the field of control charts, have great emphasis on the proper use of control charts in the proper position and number of investigation has been done on the error resulting from improper use of them. Two of these studies are the major source of this article, which is trying to consider the both together. The first group believed that because many of the manufacturing processes are complex systems and this process is often not a single stage, hence, the output quality should be evaluated by monitoring several interdependent processes that take place. This type of control is called multistage processes monitoring. Multistage processes have cascade properties. This means that at each stage of the process, quality is dependent on two parameters. One is particular quality, which is the quality of operations in the current period. And the other is the overall quality, which is defined as the quality of pre-and current stages. The second group had tried to describe the quality of the product and the process performance by monitoring the relationship between a response variable and one or more independent variables. They have named this equation (relationship) as profile.

Zheng first carried out monitoring a multistage processes. The foundation of these efforts were based on the cascade property, then Hawkins provided similar charts regardless of the cascade property. This new control chart created new horizons in the analysis and improvement of a multistage processes, and then Wade and Woodall began to develop, expand, and emphasize the use of the charts. Several examples of multistage processes in the semiconductor industry by Skinner et al. and Jarkpaporn et al. have been raised, assuming that the data is not normalized. Loredo et al. Shu and Tsung and Yang and Yang conducted their research with premise of data correlation. Also using neural network by Niaki and Davoodi was studied.

In the first study in profile monitoring, Kang and Albine proposed two approaches including $T^2$ and EWMA-R control charts for monitoring simple linear profile in Phase I and II. Then, profile monitoring has been investigated by many authors. For example, in simple linear profile, Kim et al. coded the explanatory variables to achieve uncorrelated parameters. Then, they proposed using 3 EWMA control charts to monitor intercept, slope and variance of errors, separately. Afterward, Gupta et al. proposed a method in which the performance of the EWMA-3 method is justified by replacing Shewhart control charts. Zou et al. proposed a method based on generalized likelihood ratio (GLR) for monitoring simple linear profile in Phase II. The effects of non-normality residual on simple linear profile monitoring are investigated by Noorossana et al. and the effects of non-independent data on profiles monitoring are studied by Jensen et al. The major achievement in profile monitoring can be founded in Noorossana et al.

In the research that has been cited, few studies have been carried on these two topics; profile monitoring and controlling multistage processes. In this paper we will examine the
monitoring of a two-stage process in a manner that there is a simple linear profile in one of the stages. One of the researchers that has been done in this field can be Niakiet al^28 research that investigates the methods of monitoring the linear profiles in a two-stage process where at any stage, presents a profile instead of a quality characteristic.

In this paper we have considered a two-stage process. In the first stage, there is a quality characteristic as a random variable and in the second stage, there is a profile. Also, the quality characteristic of the first stage is one of the profile independent variables in the second stage. We decided to monitor the changes in the profile coefficients of the second stage and also the changes in first stage quality characteristic in the second stage of the process by the two approaches: $\overline{X} - R, T^2, \chi^2$ and EWMA $- R, T^2, \chi^2$ and then compare the results.

In the approach $\overline{X} - R, T^2, \chi^2$ for monitoring the quality characteristics of the first phase, diagram $\overline{X} - R$, for monitoring the profile parameters of the second phase, chart $T^2$, and for monitoring the amount of residuals, $\chi^2$ control chart are used. In the approach EWMA $- R, T^2, \chi^2$ for monitoring the quality characteristics of the first phase, graph EWMA $- R$, for monitoring the profile parameters of the second phase, chart $T^2$, and for monitoring the amount of residuals, chart $\chi^2$ are used. The paper is structured as follows:

In the second part, we analyze the problem and model assumptions and then in the third section, we introduce charts and statistics we use. In the fourth and fifth parts, respectively, method of data collection for monitoring profiles in the two-stage process and sensitivity analysis of the “Average Run Length” related to the changes of model parameters in the second phase are assessed. Finally, conclusions are presented.

Methodology

Defining the Problem and Model Assumptions: In many situations, the quality of a process or a product is characterized by the relationship between a response variable and one independent variable. Thus at each stage of sampling, a set of data is collected which can be shown by using a profile. But sometimes it is necessary that monitoring takes place at different stages of processes. This type of monitoring is named multistage processes monitoring. In fact, in this case the steps are not independent of each other. And based on the cascade property, former stages have their impact on the latter stages. Figure 1 shows the first stage of a two-stage process in which there is a quality characteristic $x_j$. According to equation 1 the quality characteristic $x_j$ has normal distribution.

\[
x_1 \sim N(\mu_i, \sigma_i^2)
\]  

Also as it is shown in Figure 2 in the second stage profile $y_2$ is available, as in equation 2. In the second phase profile, besides $x_2$ as an independent variable of the profile that gets constant values, the quality characteristic of the first stage ($x_1$) as another independent variable of the profile is considered to be constant like the assumptions.

\[
y_2 = \beta_0 + \gamma_i x_1 + \beta_2 x_2 + \varepsilon
\]  

In equation 2 $\beta_i$'s and $\gamma_i$ are coefficients of second stage profile so that the expected change in the $y_2$ per unit change in $x_1$ alone or $x_2$ in isolation with all other variables being constant shows that $x_2$ is the effective qualitative characteristics on the profile with constant values in the second stage and $x_1$ is the qualitative characteristics of the first step and $\varepsilon$ is the error term. Model assumptions are: i. The profiles are intended to be linear. ii. due to regression, the values of $x_2$ are constant (not random variables), iii. There is no autocorrelation within the profiles. iv. $\varepsilon$ has a normal distribution.

\[
\varepsilon \sim N(0, \sigma^2_e)
\]  

Control Charts and Statistics Used in the Phase II: According to the article, which is performed in the second phase of the control chart, the main objective of this phase is to explore changes in the process once possible. So, we review and explain each chart in both approaches $\overline{X} - R, T^2, \chi^2$ and EWMA $- R, T^2, \chi^2$.

$\overline{X} - R$ Control chart: $\overline{X} - R$ control chart can be one of the primary charts for monitoring a characteristic feature when the sample size in each sample is between 2 and 9. In this paper, we use this chart for monitoring the mean and distribution of the first stage quality characteristics $x_1$ of the $\overline{X} - R, T^2, \chi^2$ approach. Statistic of graphs $\overline{X}$ and $R$ are expressed in equations 4 and 5. $\overline{X}$ is the counter of the sample size $n_i$ of the quality characteristics of the first stage $(x_1)$. $i = 1, \ldots, n$. $j$ is the counter of the number of samples $(j = 1, \ldots, m)$. Also limits of the control chart $\overline{X}$ and $R$ are expressed in equations 6 and 7. $d_2, d_3$ are constants that are dependent on sample size.
Control chart: As mentioned in the introduction, one of the methods used for monitoring simple linear profiles is \( \text{EWMA} - R \) approach, usually in profile monitoring \( \text{EWMA} \) control chart for monitoring the mean residuals and \( R \) control chart for monitoring the diagram distribution are used. In this paper according to approach \( \text{EWMA} - R, T^2 \) the \( \text{EWMA} \) and \( R \) control charts are used to monitor the mean and distribution of the first stage quality characteristics \( x_1 \). The equations 8 and 9 show statistic and limits of \( \text{EWMA} \) control chart respectively.

It should be noted that \( \overline{x}_j \), the average \( j^{th} \) sample of quality characteristic \( x_1 \) for stage 1 and \( \hat{\lambda} \) has a constant value between zero and one, and \( L \) is the controlling factor that is calculated according to the type one error.

\[
\text{EWMA}_j = \lambda \overline{x}_j + (1-\lambda) \text{EWMA}_{j-1}
\]

\[
\text{EWMA}_o = \mu_{x_1} + L \sigma_{x_1} \sqrt{(2-\lambda)/n_i}
\]

The statistic and limits of \( R \) control chart are the same as equations 5 and 7.

\( T^2 \) Multivariate control chart: In quality control for monitoring a process that has more than one quality characteristic and quality characteristics are interdependent, \( T^2 \) multivariate control chart is used. That, when estimating the parameters of a simple linear regression according to equation 2, \( (\hat{\beta}_{i1}, \hat{\gamma}_{i1}, \hat{\beta}_{i2}) \) are dependent with the method of least squares error. So they can be monitored simultaneously by a \( T^2 \) multivariate control chart. Hence the statistic used in this diagram, is obtained from equation 10.

\[
T^2_j = (z_j - \mu)^T \sigma^{-1} (z_j - \mu)
\]

Where:

\[
z_j = (\hat{\beta}_{i1}, \hat{\gamma}_{i1}, \hat{\beta}_{i2})
\]

\[
\hat{\beta}_{i1} = r_{i1} r_{i2} r_{i2} r_{i3} \times S_{i2}/S_{i1}
\]

\[
\hat{\beta}_{i2} = r_{i2} r_{i2} r_{i2} r_{i3} \times S_{i2}/S_{i1}
\]

\[
\hat{\gamma}_{i1} = r_{i1} r_{i2} r_{i2} r_{i3} \times S_{i2}/S_{i1}
\]

\[
\mu = (\beta_{i1}, \gamma_{i1}, \beta_{i2})
\]

\[
\sigma = \begin{pmatrix}
\sigma_{0} & \sigma_{01} & \sigma_{02} \\
\sigma_{01} & \sigma_{11} & \sigma_{12} \\
\sigma_{02} & \sigma_{12} & \sigma_{22}
\end{pmatrix}
\]

Care must be taken that if the process is controlled, \( T^2_j \) has a chi-square distribution. The upper control limit for this control chart is in accordance with the following formula.

\[
UCL_{x^2} = \chi^2_{\alpha/\nu}
\]

In this paper, that we tend to monitor the three profile parameters by \( T^2 \) multivariate control chart the upper limit of the control chart has a chi-square distribution with \( \nu = 3 \) degrees of freedom.

\( \chi^2 \) Control chart: Usually \( \chi^2 \) control chart is used to monitor the distribution, so in this article we use it to monitor residuals. If the chart warns it is because it has only upper limit according to equation 15. It means at least of the residuals rises above the limit, and this means that the difference between the actual and predicted values of the profile has increased. In equation 16 the graph relation is expressed with regards to the residuals that have a normal distribution with zero mean and variance \( \sigma^2_e \).

\[
\chi^2 = \sum_{j=1}^{n} \sum_{k=1}^{n} \left( \frac{e_{jk}}{\sigma_{jk}} \right)^2
\]

\[
UCL_{\chi^2} = \chi^2_{\alpha, \nu, \sigma^2}
\]

In equation 15, \( k \) is the counter of the sample size of the effective qualitative characteristics of profile in the second phase \( (k = 1, \ldots, n_k) \).

How to Collect Data for Monitoring Profile in a Two-Stage Process: Since this model is different from the other models of multi-step processes, samples are obtained from relation (17).

\[
(y_{111}, x_{111}, x_{112}, y_{112}, x_{111}, x_{112}, \ldots, y_{11n}, x_{111}, x_{112})
\]

\[
(y_{221}, x_{221}, x_{222}, y_{222}, x_{221}, x_{222}, \ldots, y_{22n}, x_{221}, x_{222})
\]

\[
\cdots (y_{k11}, x_{k11}, x_{k12}, y_{k12}, x_{k11}, x_{k12}, \ldots, y_{kn1}, x_{kn1}, x_{kn2})
\]

\[
\cdots (y_{as1}, x_{as1}, x_{as2}, y_{as2}, x_{as1}, x_{as2}, \ldots, y_{asn}, x_{asn}, x_{an2})
\]

\[
\cdots (y_{asm1}, x_{asm1}, x_{asm2}, y_{asm2}, x_{asm1}, x_{asm2}, \ldots, y_{asmn}, x_{asmn}, x_{an2})
\]

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According to equation 17, \( x_{ji} \) is the \( i^{th} \) amount of stage one quality characteristic of the \( n_1 \) sample on the \( j^{th} \) sampling, and \( x_{ki} \) is the \( k^{th} \) amount of stage two quality characteristic of the \( n_2 \) sample, and \( y_{ijk} \) is the profile amount of the \( j^{th} \) sampling for the \( i^{th} \) amount of stage one quality characteristic of the \( n_1 \) sample and the \( k^{th} \) amount of stage two qualitative characteristic of the \( n_2 \) sample.

The \( \mathbf{X} \) vector and the \( \mathbf{Y} \) matrix are according to the equation 18.

\[
\mathbf{X} = \begin{bmatrix}
1 & x_{11} & x_{12} \\
1 & x_{11} & x_{22} \\
1 & x_{11} & x_{32} \\
\vdots & \vdots & \vdots \\
1 & x_{11} & x_{n_2} \\
1 & x_{12} & x_{22} \\
1 & x_{12} & x_{22} \\
\vdots & \vdots & \vdots \\
1 & x_{12} & x_{n_2} \\
\vdots & \vdots & \vdots \\
1 & x_{m_1} & x_{12} \\
\vdots & \vdots & \vdots \\
1 & x_{m_1} & x_{n_2} \\
1 & x_{m_1} & x_{m_2} \\
1 & x_{m_1} & x_{m_2} \\
\end{bmatrix},
\mathbf{Y} = \begin{bmatrix}
y_{111} \\
y_{112} \\
y_{113} \\
\vdots \\
y_{1n_2} \\
y_{211} \\
y_{212} \\
\vdots \\
y_{2n_2} \\
\vdots \\
y_{m_11} \\
y_{m_12} \\
y_{m_13} \\
\end{bmatrix}
\]

Then, using the least squares error, we have equation 19 for our model.

\[
\hat{\mathbf{b}} = (\mathbf{X}^\mathbf{X})^{-1} \mathbf{X} \mathbf{Y}
\]

Results and Discussion

Sensitivity Analysis of the Average Run Length to the Changes in Model Parameters: The structure of the sensitivity analysis: As it was the case, since the research has been done in phase II of the control chart, therefore at first according to the equation 1 and 2, we consider the model coefficients as given. Because the goal of the Phase II control chart is monitoring process. This is why we want to achieve: First, sensitivity of each of the proposed approaches for variation in the profile coefficients, and the changes in average of the first step quality characteristics. Second, among the proposed approaches, which one is the most sensitive to the variation in profile coefficients and changes in qualitative parameters of phase I. And finally, the best approach according to the model presented in equation 1 and 2.

Hence, in the first approach \( \bar{X} - R, T^2, \chi^2 \) for monitoring the phase one quality characteristics, a graph \( \bar{X} - R \), is used. Secondly profile parameters are monitored by \( T^2 \) control chart. And plot \( \chi^2 \) is the residuals monitoring chart. In the approach \( EWMA - R, T^2, \chi^2 \) for monitoring the quality characteristics of the first stage, a graph \( EWMA - R \), is used. Secondly, profile parameters are monitored by \( T^2 \) control chart. Also, a chart \( \chi^2 \) is used to monitor the residuals. Decision criterion is the average run length. Because one of the achievements of this paper is to compare the two approaches \( \bar{X} - R, T^2, \chi^2 \) and \( EWMA - R, T^2, \chi^2 \), both of these approaches must have the same values of \( ARL_0 \) and it should be considered at least equal to 200. Hence in this paper \( \alpha = 0.005 \) is equivalent to \( ARL_0 = 200 \).

According to the number of charts of each approach we achieved this objective. On the other hand, according to equation 1 the phase one quality characteristic has a standard normal distribution \( X_j \sim N(\mu_j, \sigma^2_j = 1) \) and in the simulation we assume \( n_1 = 5 \). The effective quality characteristics of the second stage on profile are constant numbers \( X_j = [2 \ 4 \ 6 \ 8] \), and \( n_2 = 4 \). These numbers are considered according to the second assumption of the model. The number of sampled loads is 100 \( (m = 100) \).

Because this research has been done on phase II of control chart, at first to control the diagram, the values of the coefficients are considered as given. The values \( \beta_0 = 1 \) and \( \beta_2 = 0.5 \) and \( \gamma_1 = 1 \) are considered for the state of control in a two-stage process as in equation 1 and 2. MATLAB software is used for simulation with 10,000 repetitions for each (ARL) output. At first we obtain control limits for the control charts with respect to the coefficients in the profiles in each proposed approach. Then with changes in each of these coefficients, it is possible that the charts detect the change and warn. From the time of change to
In the time that at least one of the charts identifies this change, it is called the run length. Then we repeat this activity as many as 10,000 times to obtain an average run length. Then we repeat the same operation for other profile changes for all parameters and qualitative characteristics of phase one.

In addition to the description above, in each step of the simulation, other outputs can be obtained. For example, the diagram $EWMA - R$ in figure 3 and the $T^2$ control chart in Figure 4 and the $\chi^2$ control chart in Figure 5 are given for the approach $EWMA - R, T^2, \chi^2$.

As you can see the $EWMA - R$ chart is drawn for the 570 samples. These two charts are used for monitoring the quality characteristics of the first stage. In figure 3 both diagrams show out-of-control state. At first figure 3a, the $EWMA$ control chart for monitoring the quality characteristics of the first stage, is focused on the first 100 samples in control mode and after simulation, as soon as the average quality characteristics of stage one changes, it immediately shows sensitivity to this shift. And by repeating the outside the control sequence, we can obtain a measure for average run length. Besides this, in figure 3b, $R$ control chart for monitoring the distribution of the quality characteristics of stage one has the same function.

Diagram shown in figure 4 is $T^2$ multivariate control chart which has been applied for the monitoring profile coefficients ($\beta_0, \gamma_1, \beta_2$) for both proposed approaches in this paper. As can be seen there has been a change in one of the profile coefficients in four hundred and sixtieth sample and the chart identifies this change. With performing more simulations and repetition, we can get the number of samples between the two out of control samples. And then their average is the average run length in $T^2$ control chart.

Diagram shown in figure 5 is $\chi^2$ control chart which has been applied for the monitoring residuals for both proposed approaches in this paper. As can be seen all of the residuals in 570 samples were drawn under control. It means between the actual values and the predicted values no significant difference exists. If this chart warns, at least one of the residuals shows out-of-control state, this means that it is out of range data and there is a difference between actual and predicted values.
Sensitivity analysis on the coefficients Profile: 1- Sensitivity Analysis on $\gamma$ coefficient: At this step, the changes on the coefficients $\beta_1 = 1$ and $\beta_2 = 0.5$ and $\gamma$ have been done from 1 to 3 to the amount of 0.05 for each parameter. Table 1 shows the simulated output for ARL calculation according to the rate of change $\gamma_1$ for both proposed approaches. As it is apparent, the best ARL is for $\gamma_1 = 1$. This is shown in figure 6. However, as stated the total ARL of both approaches has been fixed at 200. Column $\gamma_1$ shows the change in terms of $\gamma_1$. It is important to note that in this paper, the control mode has been considered for $\gamma_1 = 1$. SDRL column is the standard deviation run length. For this reason, in figure 6 we have the highest ARL for $\gamma_1 = 1$.

The ARL column states that per each $\gamma_1$, on average, how many samples have been plotted in control charts to show a warning. Calculated ARL for each of the approaches $X^2 - R$, $T^2$, $\chi^2$ and $EWMA - R$, $T^2$, $\chi^2$ is the total ARL. For example, in column ARL for approach $X^2 - R$, $T^2$, $\chi^2$ and for $\gamma_1 = 1.05$, amount 198.3621 is calculated. This number means that a positive change of 0.05 in $\gamma_1$ in the long run, shows at least one time out-of-the-control state in each 198 samples in at least one of the four graphs used in this approach.

As it is shown in figure 6, by an increase in $\gamma_1$ from value one, the ARL decreases. This is the same for both approaches. Both approaches $X^2 - R$, $T^2$, $\chi^2$ and $EWMA - R$, $T^2$, $\chi^2$ have the same reaction to changes in $\gamma_1$, because the changes applied to $\gamma_1$ have sensitivity affects on $T^2$ control chart, and the graph has the same performance in both approaches. Also for positive changes in $\gamma_1$ by more than 2, which means when $\gamma_1 \geq 3$, both approaches show the change in the first sample.

2-The Sensitivity Analysis on $\beta_0$ Coefficient: In the next step we perform changes on $\beta_0$ coefficient, which means $\beta_2 = 0.5$ and $\gamma_1 = 1$ and $\beta_0$ have changed from 1 to 3 to the amount of 0.05. In the next step we perform changes on $\beta_0$ coefficient, which means $\beta_2 = 0.5$ and $\gamma_1 = 1$ and $\beta_0$ have changed from 1 to 3 to the amount of 0.05. In table 2 as is defined the best ARL associated with the $\beta_0 = 1$ condition. This case is clear in figure 7. In this part, again, the total ARL for both approaches is the same, and it is fixed at least on 200.

Column $\beta_0$ shows the change in terms of $\beta_0$. It is important to note that $\beta_0 = 1$ is the control mode in this paper. For this reason, in figure 7 we have the highest ARL for $\beta_0 = 1$. ARL column shows each value of $\beta_0$ on average, how many samples within the control charts were plotted for each of the proposed approaches so that it is viewed as a warning. SDRL column is the standard deviation of the run length.

### Table 1

**Output of simulation to compute the ARL with change for $\gamma_1$ for both approaches**

<table>
<thead>
<tr>
<th>$X^2 - R$, $T^2$, $\chi^2$</th>
<th>$EWMA - R$, $T^2$, $\chi^2$</th>
<th>$\gamma_1$</th>
<th>$X^2 - R$, $T^2$, $\chi^2$</th>
<th>$EWMA - R$, $T^2$, $\chi^2$</th>
<th>$\gamma_1$</th>
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<td>$\beta_0 = 1 &amp; \beta_2 = 0.5$</td>
<td>$\beta_0 = 1 &amp; \beta_2 = 0.5$</td>
<td>$\beta_0 = 1 &amp; \beta_2 = 0.5$</td>
<td>$\beta_0 = 1 &amp; \beta_2 = 0.5$</td>
<td>$\beta_0 = 1 &amp; \beta_2 = 0.5$</td>
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<td>SDRL</td>
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As in figure 7 is determined, by increasing the value $\beta_0$ from one, the ARL is reduced. This is the same for both approaches. The two approaches $\bar{X} - R$, $T^2$, $\chi^2$ and $EWMA - R$, $T^2$, $\chi^2$ have the same functions to changes in $\beta_0$, because the changes applied to $\beta_0$ have sensitivity affects on $T^2$ control chart, and the graph has the same performance in both approaches. Also for positive changes in $\beta_0$ by more than 1.8, which means when $\beta_0 \geq 2.8$, both approaches show the change in the first sample.

![Figure-6](image)

**Figure-6**

Average run length curve for both proposed approaches for changes in $\gamma_1$

3-The sensitivity analysis on $\beta_2$ coefficient: In the next step we perform changes on $\beta_2$ coefficient, which means $\beta_1 = 1$ and $\gamma_1 = 1$ and $\beta_2$ have changed from 0.5 to 2.5 to the amount of 0.05. In table 3 as is defined the best ARL associated with the $\beta_2 = 0.5$ condition. This case is clear in figure 7. In this part, again, the total ARL for both approaches is the same, and it is fixed at least on 200. Column $\beta_2$ shows the change in terms of $\beta_2$. It is important to note that $\beta_2 = 0.5$ is the control mode in this paper. For this reason, in figure 8 we have the highest ARL for $\beta_2 = 0.5$. ARL column shows each value of $\beta_2$ on average, how many samples within the control charts were plotted for each of the proposed approaches so that it is viewed as a warning. SDRL column is the standard deviation of the run length. Because of the likeliness of simulated data, part of the curve is summarized.

**Table-2**

<table>
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<th>$\beta_2 = 0$</th>
<th>$\beta_2 = 0$</th>
<th>$\beta_2 = 0$</th>
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<th>$\beta_2 = 0$</th>
<th>$\beta_2 = 0$</th>
<th>$\beta_2 = 0$</th>
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<td>$X \bar{X} - R$, $T^2$, $\chi^2$</td>
<td>1.264955</td>
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<td>205.2427</td>
<td>207.3937</td>
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<td>212.3621</td>
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<td>1.05</td>
<td>2.05</td>
</tr>
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<td>$EWMA - R$, $T^2$, $\chi^2$</td>
<td>1.000369</td>
<td>1.005124</td>
<td>1.9</td>
<td>207.3937</td>
<td>200.6812</td>
<td>207.0338</td>
<td>205.7173</td>
<td>1.05</td>
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<td>0.768539</td>
<td>1.95</td>
<td>182.0202</td>
<td>183.0582</td>
<td>184.1808</td>
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<td>2</td>
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<tr>
<td>$\sigma_0 = 0.5$</td>
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<td>0.62132</td>
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<td>148.9851</td>
<td>150.1219</td>
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<td>154.3759</td>
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<td>1.3</td>
<td>1.5</td>
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<td>119.6498</td>
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<td>1.6</td>
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<td>0.395124</td>
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<td>0.147725</td>
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<td>0.105726</td>
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<td>2.5</td>
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<td>0.075302</td>
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<td>7.742323</td>
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<td>2.5</td>
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<td>0.062331</td>
<td>2.6</td>
<td>5.369095</td>
<td>5.9987</td>
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<td>2.75</td>
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<td>0.041198</td>
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<td>4.4556</td>
<td>3.899855</td>
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<td>2</td>
<td>3.5</td>
</tr>
<tr>
<td>$\sigma_0 = 0.5$</td>
<td>0.024489</td>
<td>0.031609</td>
<td>2.65</td>
<td>2.869813</td>
<td>3.4117</td>
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<tr>
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<td>2.180582</td>
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<td>3.75</td>
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<tr>
<td>$\sigma_0 = 0.5$</td>
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<td>0</td>
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<td>2.2119</td>
<td>1.669979</td>
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<td>1.8</td>
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</table>

![Figure-7](image)

**Figure-7**

Average run length curve for both proposed approaches for changes in $\beta_0$
Table-3
Output of simulation to compute the ARL with changes for $\beta_2$ in both approaches

| $\gamma$ = 1 & $\beta_0$ = 1 | $\bar{X} - R, T^2, \chi^2$ SDRL ARL $\bar{X} - R, T^2, \chi^2$ SDRL ARL $\bar{X} - R, T^2, \chi^2$ SDRL ARL $\bar{X} - R, T^2, \chi^2$ SDRL ARL $\bar{X} - R, T^2, \chi^2$ SDRL ARL |
|-----------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| $\beta_2$ | $\beta_2$ | $\beta_2$ | $\beta_2$ | $\beta_2$ | $\beta_2$ | $\beta_2$ | $\beta_2$ | $\beta_2$ |
| 0.5 0.2072517 0.2099821 0.2079379 0.2052427 | 0.7 1.141 0.407106 1.1375 0.394727 | 0.55 71.9864 71.72321 71.6243 69.60755 | 0.75 1.0093 0.097028 1.0082 0.090186 | 0.6 8.5529 8.077916 8.5114 7.963178 | 0.8 1.0004 0.019997 1.0003 0.017319 | 0.65 2.0541 1.520199 2.0586 1.483909 | 0.85 1 0 1 0 |

As in figure 8 is determined, ARL value decreases with the increase in $\beta_2$ amount. However, comparing figure 8 with figure 7 is observed for the reduction in ARL is faster with the changes in $\beta_2$ than with the changes in $\beta_0$. Both approaches are more sensitive to such changes in $\beta_2$ than in $\beta_0$. On the other hand, both approaches $\bar{X} - R, T^2, \chi^2$ and EWMA $- R, T^2, \chi^2$ have the same reaction to changes in $\beta_2$. The acquisition was obvious; since these changes applied to $\beta_2$ have sensitivity affects on $T^2$ control chart, and $T^2$ control chart works the same in both approaches. On the other hand, for each positive change in $\beta_2$ by more than 0.35, which means if $\beta_2 \geq 0.85$ both approaches show this change in the first sample.

The sensitivity analysis on the average quality characteristics of the stage one: In this step, the changes have applied on the average quality characteristics of the phase I, thus, the coefficients of the profile $\beta_0 = 1$ and $\gamma = 1$ and $\beta_2 = 0.5$ are in the controlled state. Table 4 shows the ARL values for changes in average qualitative characteristic of the first stage in both approaches. In this part, also, the total ARL for both approaches is the same, and is considered at least 200.Column $\mu_{s_1}$ shows changes based on the average qualitative characteristics of the first stage. It is important to note that in this article, $\mu_{s_1} = 0$ has been the control mode. For this reason, in figure 9 for $\mu_{s_1} = 0$ we have the highest ARL. ARL column states that per each value of $\mu_{s_1}$, on average, how many samples of the control charts were plotted for each approach so that an alert has been observed. SDRL column is the standard deviation of the run length.

As in figure 9 is shown, ARL value decreases with increasing $\mu_{s_1}$. Both approaches $\bar{X}$ $- R, T^2, \chi^2$ and EWMA $- R, T^2, \chi^2$ adjust differently to changes in $\mu_{s_1}$. The changes are not the same and as can be seen in the approach EWMA $- R, T^2, \chi^2$ for incremental changes in the average quality characteristics of the first stage $\mu_{s_1}$, better adjustment is seen in compare to approach $\bar{X}$ $- R, T^2, \chi^2$. The changes applied are on $\mu_{s_1}$ and the EWMA control chart is better than the $\bar{X}$ control chart for small changes in the mean.

Figure 10 is a general diagram to compare the average run length for changes in all parameters $\beta_0, \gamma, \beta_2$ and the average quality characteristic of stage one $\mu_{s_1}$.
Thus, R is the rate of change in any of the cases mentioned. If $R = 0$, i.e., the parameter is not changed, and if $R = 0.1$ for example for $\gamma_i$ it means that the parameter value is $\gamma_i = 1.1$.

According to the simulation and the two monitoring approaches presented: $\bar{X} - R, T^2, \chi^2$ and $EWMA - R, T^2, \chi^2$ it can be stated that, in general, the most sensitivity to changes is to changes in parameter $\beta_2$. Both approaches have the same performance to the changes. However, the worst-case detection is related to changes in the average quality characteristic of the first stage by the approach $\bar{X} - R, T^2, \chi^2$. In general it can be stated that the approach $EWMA - R, T^2, \chi^2$ for monitoring a two-stage process, as in figures 1 and 2 mentioned, is better than approach $\bar{X} - R, T^2, \chi^2$.

The results of this paper can have special importance in many industries where quality product is a function of more than one stage and profile monitoring is done in one of the stages. For example, products such as parts manufacturing, production of metals such as copper and loom, etc. that quality product is not formed at a particular stage and pre-processing steps which have impacts on the nature of the profile, is of the utmost importance. For example of the case in the textile industry. The first phase is the spinning part and the quality characteristics of it, is the thickness of the thread, which as one of the independent variables in the second phase profile has an effective role in resistance of the fabric. On the other hand $x_2$ can also be a place on coils where the fabric resistance is measured. This measurement happens in certain areas of Coils. This means that $x_2$ gets constant values.

<table>
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<tr>
<th>$\chi_i = 1$ &amp; $\beta_0 = 1$ &amp; $\beta_2 = 0.5$</th>
<th>$\chi_i = 0.5$ &amp; $\beta_0 = 1$ &amp; $\beta_2 = 0.5$</th>
<th>$\chi_i = 0.5$ &amp; $\beta_0 = 0.5$ &amp; $\beta_2 = 1$</th>
<th>$\chi_i = 0.5$ &amp; $\beta_0 = 0$ &amp; $\beta_2 = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{X} - R, T^2, \chi^2$</td>
<td>$EWMA - R, T^2, \chi^2$</td>
<td>$EWMA - R, T^2, \chi^2$</td>
<td>$EWMA - R, T^2, \chi^2$</td>
</tr>
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<td>$\mu_{1,1}$</td>
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<td>$ARL$</td>
<td>$SDRL$</td>
</tr>
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Table-4
The simulated output to compute ARL with changes of $\mu_{1,1}$ in quality characteristic of stage one for both approaches
Conclusion

This paper introduces and compares two approaches for monitoring a two-stage process with profile quality characteristics in the second stage. And given that the quality of products are monitored by a variety of control charts on their quality characteristics. These quality characteristics that are either variable or attribute can be, a single variable, a vector of variables, or a profile equation. However, because the quality of the performance of different procedures is the quality of the product and usually these steps are not independent of each other, therefore, the assumption of independent process affecting the quality of the output with error. So far, the impact of such conditions on multi-stage monitoring processes of univariate and multivariate have been studied. However, a profile has been less studied in multi-stage monitoring process.

This paper introduces a model for a two-step profile in addition to two different approaches that have been proposed for monitoring process.

And the variation of the coefficients of the profile, as well as changes in the qualitative characteristics of the first stage, in a two-stage process, on the second phase control charts were reviewed. And observed changes in the coefficients $\gamma_i, \beta_0$ have almost the same effect on a two-step monitoring process. While the rate of change in $\beta_2$ influences a two-stage process in a narrower range. This paper also proposes two approaches for monitoring such processes and the final analysis was carried out related to these comparisons.

This new topic of research activities can be considered in the following cases: i. EWMA chart to monitor the residuals of the profile and compare two approaches of this article. ii. Analysis of the interaction between Independent quality characteristics in the profile equation. iii. Using other multivariate charts such as MEWMA and MCUSUM instead of $T^2$ control chart and compare the outputs with each other. iv. Examining the performance of the simple linear profile coefficients in monitoring processes with more than two stages. v. Evaluation of the effect of the first stage on the profile slope. vi. Examining the performance of the simple linear profile coefficients in monitoring processes with more than two steps that have a profile in each stages.

References


