



# PID Controller for Robotic Manipulator Nonlinear Model and Compare with Sliding Mode Controller

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## Abstract

*In this paper, the nonlinear model of the robotic manipulator has been chosen as the model to be studied. Nowadays, complicated controllers are commonly discussed in many researches. In this work, it will be shown that a simple, practical PID controller operates much better than a robust and nonlinear sliding mode controller in the aforesaid system. Simulation results and the comparison of these two controllers prove this claim to be true. Finally, a robust analysis has been done by which the resistance of these controllers is assessed.*

**Keywords:** Robotic manipulator; sliding mode, PID controller.

## Introduction

Control of robotic trajectory is a very sophisticated problem because of its coupled and nonlinear structure for system dynamics. Also, when a robotic manipulator does the required operations at high speed, some things may cause a large system error<sup>1,2</sup>. These things are the effects of nonlinear properties, time variant coefficients and other unknown events, such as backlash, friction, etc. one of the most important works is related to finding an effective controller to attain accurate tracking of the desired motion. There are many algorithms for robot trajectory control, such as fuzzy control<sup>3</sup>, neural network<sup>4</sup>, sliding mode<sup>5</sup> and robust control<sup>6</sup>.

Sliding mode control acts as a robust strategy. It has applied in different areas, such as power converters, aerospace, robotics and industrial process<sup>7</sup>. In sliding mode control, all trajectories that exist in the state space are directed toward the sliding surface. Upon the reaching of system state to the sliding surface, it slides along it and the system will have no sensitivity to a group of disturbances and parameter variations. It must be mentioned that a traditional sliding mode control have some drawbacks in practical motion control. The first problem is related to obtain the system parameters. It will be difficult. In Neuro-Sliding<sup>8</sup>, two neural networks are used in parallel to calculate the equivalent control and also the correct control of the sliding mode. It can be seen from robust control<sup>6</sup> that a neural network is used for achieving the uncertainty related to a robust control system. The second drawback is the high frequency oscillation in the control input. This problem that exists always is called "chattering". Oscillations are generated by the required high speed switching. The "chattering" can excite no modeled high frequency plant dynamics; therefore, it can cause unpredictable instabilities. For these reasons,

chattering is not desirable in many real applications. A simple way to overcome the chattering is to consider a boundary layer<sup>9</sup> but it does not ensure that the state trajectories of system converge to the sliding surface. This situation may results the existence of the steady state error. Also, the system dynamics analysis in the boundary layer is very difficult. In order to eliminate the chattering<sup>10</sup> use the auto-tuning neuron as the direct adaptive neural controller. When the state trajectory of system goes into the boundary layer, this neuron replaces the sliding mode control.

In this paper, a simple and commonly used PID controller has been applied to control the dynamic robotic manipulator and obtained results are compared with those of the nonlinear sliding mode controller. It is obvious that applying a PID controller is practically quite simple and of low cost. In fact, this paper discusses that PID controllers can be effective in many complicated and nonlinear systems. Always, using controllers with different and complicated structures, whose disassembling would cause huge problems, is not a solution for controlling systems.

This paper is organized as follows. Section 2 describes the robot dynamics and some of its fundamental properties and the problem statement. In section 3, the design and analysis of the controller is presented. The simulation results of the two link manipulator are given in section 4 and finally the conclusion is in section 5.

## Dynamic of Robotic Manipulator

The dynamic equation of an n-link robotic manipulator is

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau \quad (1)$$

Where  $q$  joint position vector,  $\dot{q}$  joint velocity vector,  $\ddot{q}$  joint acceleration vector,  $M(q)$  inertia matrix,  $C(q, \dot{q})$  matrix of centripetal and Coriolis forces,  $G(q)$  the gravity vector,  $\tau$  the motor torque vector. This matrix is as follow<sup>1,11</sup>.

$$M(q) = \begin{bmatrix} \frac{1}{4}(m_1 + m_2)l_1^2 + \frac{1}{4}m_2l_2^2 + m_2l_1l_2 \cos q_2 & \frac{1}{4}m_2l_2^2 + m_2l_1l_2 \cos q_2 \\ \frac{1}{4}m_2l_2^2 + m_2l_1l_2 \cos q_2 & \frac{1}{4}m_2l_2^2 \end{bmatrix}$$

$$C(q) = \begin{bmatrix} -m_2l_1l_2\dot{q}_2 \sin q_2 & -\frac{1}{2}m_2l_1l_2\dot{q}_2 \sin q_2 \\ \frac{1}{2}m_2l_1l_2\dot{q}_1 \sin q_2 & 0 \end{bmatrix} \quad (2)$$

$$G(q) = \begin{bmatrix} (\frac{1}{2}m_1 + m_2)gl_1 \cos q_1 + \frac{1}{2}m_2l_2g \cos(q_1 + q_2) \\ \frac{1}{2}m_2l_2g \cos(q_1 + q_2) \end{bmatrix}$$

Where  $l_1$  and  $l_2$  are the lengths;  $m_1$  and  $m_2$  are the mass of the links, respectively. Figure-1 shows the geometric structure of a manipulator with two links.

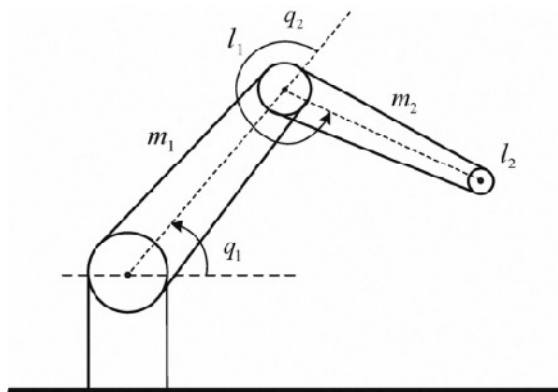


Figure-1  
Two-link robotic manipulator

The dynamics of a robotic manipulator has five properties as follows<sup>11</sup>: i. Symmetric and positive definite  $M^T=M$ . ii. The parameter  $M(q)$  is bounded, i.e.,  $\mu_1(q)I \leq M(q) \leq \mu_2(q)I$ , where  $\mu_1(q)$  and  $\mu_2(q)$  are scalars. For revolute links, they are constants.  $I$  is an identical matrix. iii. Matrix  $\dot{M} - 2C$  is skew symmetric, i.e., for any vector  $X$ ,  $X^T(\dot{M} - 2C)X = 0$ . iv.  $C(q, \dot{q})\dot{q}$  is quadratic  $\dot{q}$  to and bounded as  $\|C(q, \dot{q})\dot{q}\| \leq \mu_3(q)\|\dot{q}\|^2$ , where  $\mu_3(q)$  is a scalar constant for revolute links. v. The gravity vector  $G$  is bounded as

$\|G(q)\| \leq \mu_4(q)$ , where  $\mu_4(q)$  is a scalar constant for revolute links. It is independent of  $q$ .

## Design Controller

**Sliding Mode:** Sliding mode control is a tracking method. The purpose of this paper is track the reference signal by robotic manipulator. Therefore, error is defined as follows<sup>1</sup>:

$$\tilde{q} = q - q_d \quad (3)$$

Where  $\tilde{q}$  is tracking error and  $q_d$  is reference signal. The dynamic equations of the robot arm are of two orders; hence sliding surfaces are defined as follows.

$$S(x, t) = \left( \frac{d}{dt} + \lambda \right)^{n-1} \tilde{x} \quad (4)$$

Where  $n=2$  and

$$S = \tilde{q} + \lambda \tilde{q} \quad (5)$$

According to the sliding mode method  $\dot{S} = 0$  and control law is derived as follows.

$$\dot{S} = \tilde{q} + \lambda \tilde{q} = \ddot{q} - \ddot{q}_d + \lambda \dot{q} - \lambda \dot{q}_d \quad (6)$$

To derive the control law expression of the matrix  $M$  is multiplied on both sides.

$$M\dot{S} = M\ddot{q} - M\ddot{q}_d + M\lambda(\dot{q} - \dot{q}_d) \quad (7)$$

If the system parameters are constant values, then  $M\dot{S} = 0$ . therefore, using equation (1) control law is:

$$\tau_{eq} = C(q, \dot{q})\dot{q} + G(q) + M\ddot{q}_d - M\lambda(\dot{q} - \dot{q}_d) \quad (8)$$

Since the operating system parameters are not constant and change, Sliding mode method was chosen for this system. for change the parameters of the sliding mode controllers have good resistance, sign function is added to the control law. Hence

$$\dot{S} = -K\text{sign}(s) \Rightarrow \tau = \tau_{eq} - K\text{sign}(s) \quad (9)$$

For investigate the stability of the system is chosen Lyapunov function as below:

$$V = \frac{1}{2} s^2 \quad (10)$$

The derivative of  $V$  is

$$\dot{V} = \frac{1}{2}(s\dot{s}) = \frac{1}{2}s(-k\text{sign}(s)) = -\frac{1}{2}|s|k < 0, \text{ if } k > 0 \quad (11)$$

Therefore, system stability is guaranteed.

**PID controller:** To design a PID controller, it has been tried so that the tracking error would be minimized and system output could reach a desired level. Setting PID controller parameters has been done through the trial and error method. The controlling block of this system is shown in figure-2.

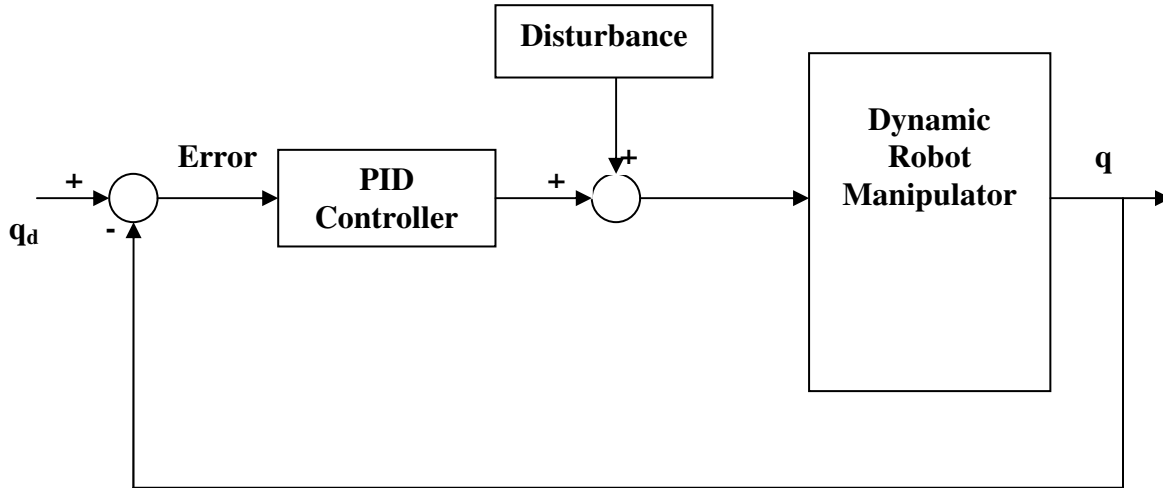


Figure-2  
 PID Controller Schematic

In this paper, two parallel PID controllers have been used to control the nonlinear dynamic of the robotic manipulator.

### Results and Discussion

Dynamic model of the robotic manipulator is shown in figure-1. Values of model parameters and initial conditions are as follows [1].

$$m_1 = 4kg, m_2 = 2kg, l_1 = 2m, l_2 = 1m, g = 9.8m/s^2, q_1(0) = 0.5, q_2(0) = 0.5, \dot{q}_1(0) = 0, \dot{q}_2(0) = 0.$$

The purpose of this paper is track the reference signal by controlling the robot arm. Reference signal is

$$q_d = [1.5 \sin(2\pi/3)t \quad 2 \sin(2\pi/3)t]^T \quad (12)$$

For simulation, a friction vector is considered, too. Hence, system model was considered as follows.

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F(\dot{q}) = \tau \quad (13)$$

Where

$$F(\dot{q}) = [20\dot{q}_1 + 0.8\text{sgn}(\dot{q}_1) \quad 4\dot{q}_2 + 0.16\text{sgn}(\dot{q}_2)]^T \quad (14)$$

To show the capability Controller is designed, an external disturbance  $D(t)$  is also considered. Despite disturbance, simulations have been performed. Disturbance equation is

$$D(t) = [560 \sin t \quad 85 \sin t]^T \quad (15)$$

In figure-3 position robot with reference signal and torque applied to robotic manipulator is demonstrated by applying the sliding mode controller. Chattering phenomenon in torque applied to the robotic manipulator clearly exists in this case.

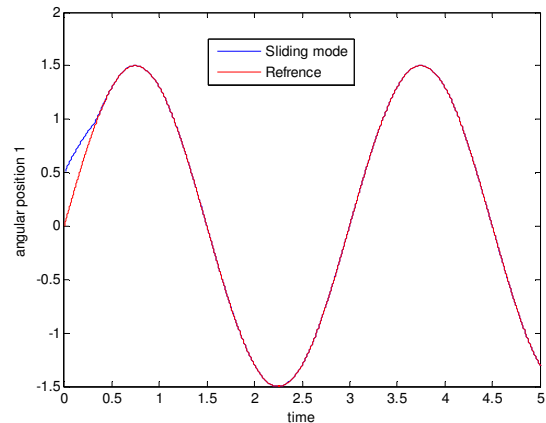


Figure-3a  
 Position first joint robot by sliding mode control

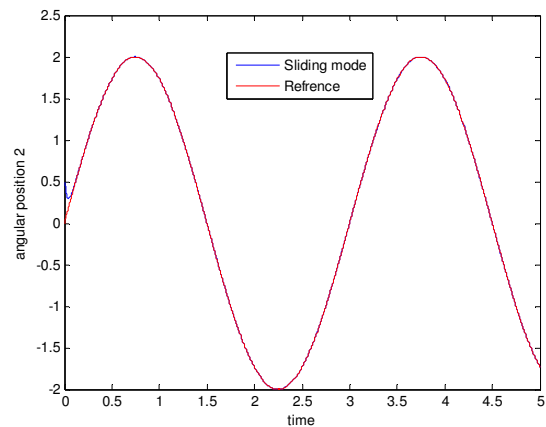
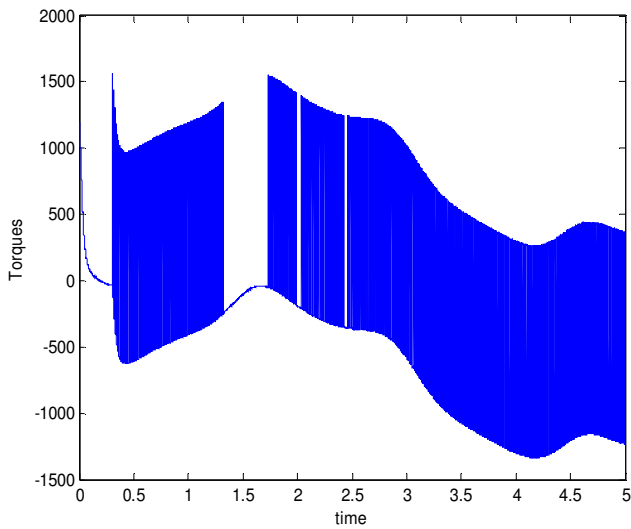
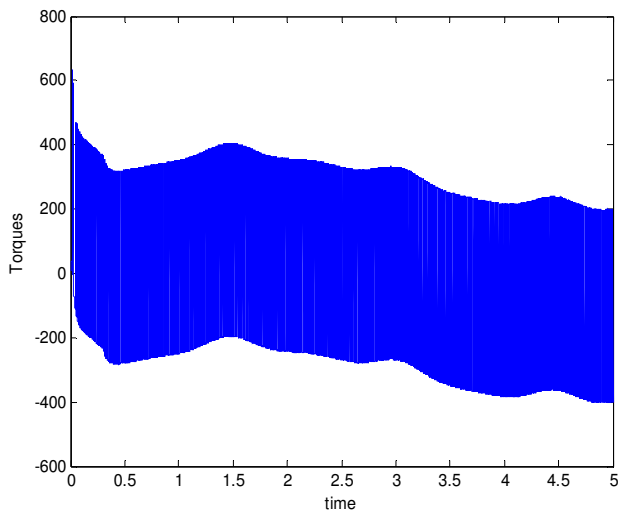


Figure-3b  
 Position second joint robot by sliding mode control



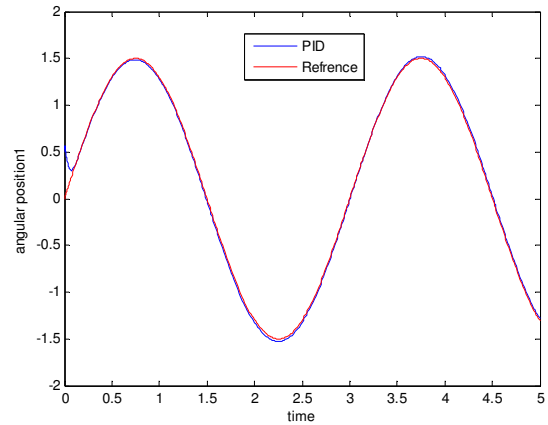
**Figure-3c**

**Torque first joint robot by sliding mode control**



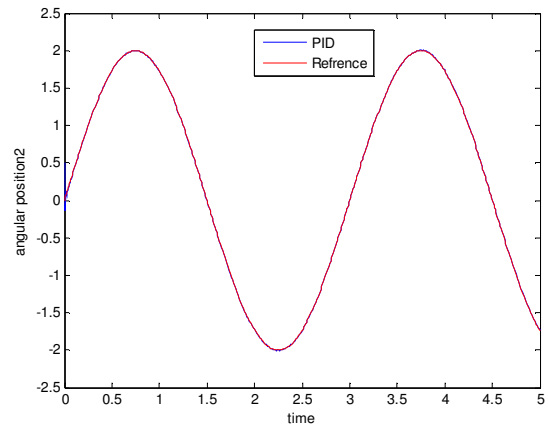
**Figure-3d**

**Torque second joint robot by sliding mode control**



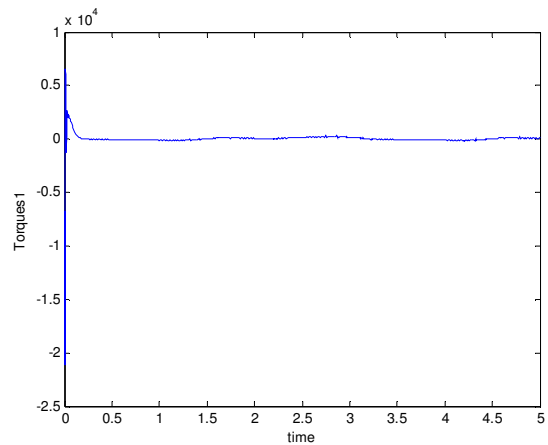
**Figure-4a**

**Position first joint robot by PID controller**



**Figure-4b**

**Position second joint robot by PID controller**



**Figure-4c**

**Torque joint first robot by PID controller**

In figures-4a and 4b, the position of the robotic manipulator is shown using a PID controller and the reference signal. It can be noticed that tracking has been done quite well and the PID controller has operated successfully. In 4c and 4d figures, the robotic manipulator applied torque is shown where no chattering of the sliding mode controller is observed. Figure-5 shows the robust analysis. The mass of the robotic manipulator varies by  $\Delta m=10$ . Simulation results indicate that besides its good tracking performance, PID controller is resistant to the variation of parameters. Table 2 shows the sum of tracking error squares and the applied torque energy of the robotic manipulator. Also, this table shows the superiority of the PID controller over the sliding mode controller.

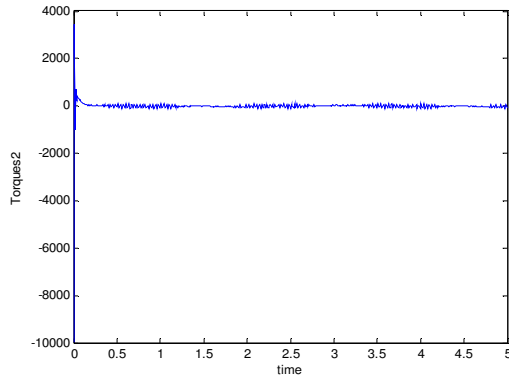


Figure-4d

Torque joint second robot by PID controller

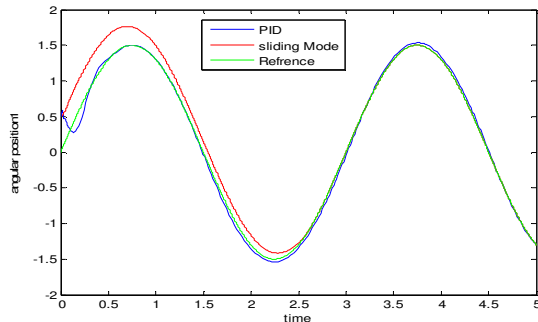


Figure-5a

Position first joint robot (robustness analyze)

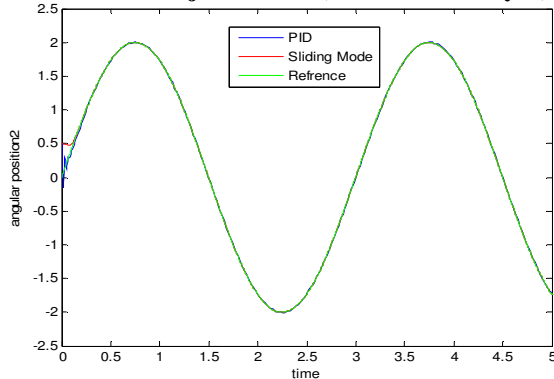


Figure-5b

Position second joint robot (robustness analyze)

## Conclusion

In this paper, a PID controller was designed for the nonlinear model of the robotic manipulator. Performance results of this controller were compared with the sliding mode controller. Besides the fact that PID controller has performed quite well in tracking, no chattering, which is the issue in sliding mode controllers, was observed. In fact, this paper indicates that it's not always beneficial to search for complicated controllers with different combinations. Instead, many systems can be controlled by a simple controller and expected to show an acceptable behavior.

Table-2

Sum of error squares and applied torque energy of the robotic manipulator in PID and sliding mode controllers

	First joint Error	Second joint Error	First joint Torque	Second joint Torque
PID Controller	6.7227	2.8178	1.6106e9	1.1162e9
Sliding Mode	290.9145	46.7418	1.6127e10	3.938e9

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