Research Journal of Recent Sciences ISSN 2277-2502
Vol. **1(8),** 17-22, August **(2012)** *Res.J.Recent Sci.*

Pareto Optimization of Vehicle Suspension Vibration for a Nonlinear Half-car Model Using a Multi-objective Genetic Algorithm

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Available online at: www.isca.in
Received 7th April 2012, revised 16th April 2012, accepted 24th April 2012

Abstract

In this paper, multi-objective genetic algorithm (MOGA) is used for Pareto optimization of a four degree of freedom vehicle vibration model. Vehicle suspension design must fulfill some conflicting criteria. Among those is ride comfort which is attained by reducing the sprung mass accelerations via suspension spring and damper. Moreover, good handling or road holding capability of a vehicle which is attained by minimize front and rear suspension deflection is a desirable property which requires stiff suspension and therefore is in contrast with a vehicle with ride comfort. Therefore, Multi-objective Genetic Algorithm (MOGA) is used for Pareto approach optimization of passive suspension system. The important conflicting objectives that have been considered in this work are, ride comfort and handling performance. Moreover, this approach returns the optimum answers in Pareto form that designer can, by making trade-offs, select desired answer. Finally, the simulation result shows that optimization of suspension settings will improve ride comfort and road holding capability simultaneously

Keywords: Vehicle vibration model, Pareto, genetic algorithm, multi-objective optimization.

Introduction

The vehicle suspension system is currently of great interest both academically and in the automobile industry worldwide[1]. Suspension is the term given to the system of springs, shock absorbers and linkages that connects a vehicle to its wheels[2]. Suspension systems can be classified as passive, semi-active, and active systems. The design of suspension systems involves a trade-off between ride comfort, suspension deflection, and tire deflection that in this case study is focused on optimization of passive suspension systems[3]. When designing vehicle suspension systems, it is well-known that spring and damper characteristics required for good handling on a vehicle are not the same as those required for good ride comfort. Any choice of spring and damper characteristic is therefore necessarily a compromise between ride comfort and handling[4].

There are two main parameters to work on during the design, the damping and stiffness of the suspension configuration. Soft springs result in better ride comfort, but cause poor road holding. A high damping ratio decreases the ride comfort but causes better road holding. Thus the designer has to make a compromise between road holding and ride comfort[5,6]. Furthermore, the necessity of trading off among the conflicting requirements of the suspensions in terms of comfort and road holding capability led to the use of multi- objective optimization techniques.

In fact, optimization in engineering design has always been of great importance and interest particularly in solving complex real-world design problems. In multi-objective optimization problems, there are several objectives or cost functions (a vector of objectives) to be optimized (minimized or maximized) simultaneously. These objectives often conflict with each other so that as one objective function improves, another deteriorates. Therefore, there is no single optimal solution that is best with respect to all the objective functions. Instead, there is a set of optimal solutions, well-known as Pareto optimal solutions, which distinguishes significantly the inherent natures between single- objective and multi-objective optimization problems[7]. A Genetic Algorithm is an adaptive search which is used for multi-objective optimization[8].

In this paper, a multi-objective genetic algorithm (MOGA) is used for multi-objective optimization of a four-degree of freedom vehicle vibration model. The conflicting objective functions that have been considered for minimization are, namely, acceleration of front and rear sprung mass, front and rear suspension deflection. The design variables used in the optimization of vibration are, namely, vehicle suspension stiffness coefficient ($k_{f1}$and k_{r1}), vehicle suspension damping coefficient (b_f and b_r) and front and rear tire stiffness($k_{f2}$and k_{r2}). Prominently, it is shown that a trade-off optimum design can be verified from those Pareto fronts obtained by multi-objective optimization process. Finally, the superiority of time domain vibration performance of such design point is shown in comparison with that given in the literature.

Material and Methods

Half-vehicle dynamics model: A four-degree of freedom vehicle with passive suspension, which is adopted from reference 6 is shown in figure-1. This model is composed of one sprung mass that joints to two unsprung masses (indicate tires). Moreover, the effect of degrees of freedom, linear motion (in vertical direction for sprung and unsprung masses) and rotating motion (pitching motion) for sprung mass, in terms of acceleration, velocity and movement, are considered in formulation of motion equations[6].

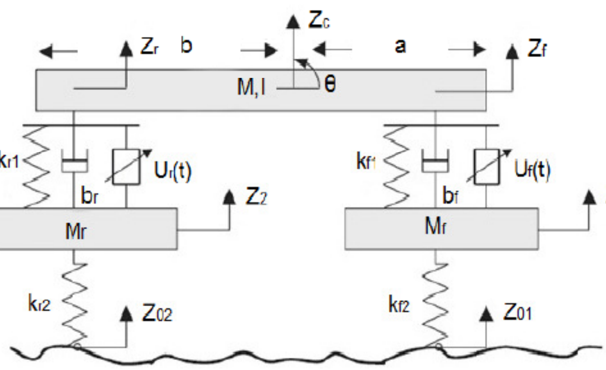

Figure-2
Half-car suspension vehicle model

Parameters M , m_f , m_r , I, a , b which denote the vehicle's fixed parameters are expressed as sprung mass, forward tire mass, rear tire mass, momentum inertia of sprung mass, forward and rear tires position in relation to the center of mass, respectively.

The differential equations of motion, with respect to the degrees of freedom, are derived by the use of Newton–Euler equations and can be written as follows:

$$M\ddot{z}_c = f_f + f_r, \quad I\ddot{\theta} = f_f a\cos(\theta) - f_r b\cos(\theta), \; m_f\ddot{z}_1 = -k_{f2}(z_1 - z_{01}) - f_f, m_r\ddot{z}_2 = -k_{r2}(z_2 - z_{02}) - f_r \quad (1)$$

$$f_f = k_{f1}(z_1 - z_c - a\sin(\theta)) + b_f(\dot{z}_1 - \dot{z}_c - a\dot{\theta}\cos(\theta))$$

$$f_r = k_{r1}(z_2 - z_c - b\sin(\theta)) + b_r(\dot{z}_2 - \dot{z}_c - b\dot{\theta}\cos(\theta))$$

where, z_c,z_1,z_2 and θ are vertical displacement of the central gravity of the sprung mass, vertical displacement of front tire , vertical displacement of rear tire and rotating motion (pitching motion),respectively. In addition, $\dot{z}_c$, $\dot{z}_1$ and $\dot{z}_2$ represent vertical sprung mass velocity, vertical front tire velocity and vertical rear tire velocity, respectively. $\ddot{z}_c$, $\ddot{z}_1$, $\ddot{z}_2$ and $\ddot{\theta}$ denote vertical sprung mass acceleration, vertical acceleration of the central gravity of the sprung mass, vertical acceleration of front tire, vertical acceleration of rear tire and rotating acceleration(pitch acceleration), respectively. Lastly, z_{01} and z_{02} represent the excitation via road disturbance, as shown in figure-2. Whereas the case study is related to passive suspension, the control signals $U_r(t)$, $U_f(t)$are considered zeros.

Multi-objective Pareto optimization: In most of the engineering problems, more than one objective function is important for the designer. Usually some conflicting objectives should be optimized by the designer at the same time. In such problems, in opposite to single objective optimization problems, in which there is only one optimum point for the problem, there are a set of optimum design vectors which are called Pareto front. The important characteristic of these solutions is that none of them are dominated by the other ones. The designer based on his or her needs chooses one of these solutions as the optimal one[9]. In general, Multi-objective optimization can be mathematically defined as: Find the vector $\boldsymbol{X}^* = \{\boldsymbol{x}_1^*, \boldsymbol{x}_2^*, \dots, \boldsymbol{x}_n^*\}^{\boldsymbol{T}}$ to optimize

$$F(X) = \{f_1(X), f_2(X), \dots, f_k(X)\}^T \quad (2)$$

Subject to m inequality constraints

$$g_i(X) \leq 0 \quad t = 1,2, \dots, m \quad (3)$$

and p equality constraints

$$h_i(X) = 0 \quad j = 1,2, \dots, p \quad (4)$$

Where $X^* \in R^n$ is the vector of decision or design variables, and $F(X) \in R^n$ is the vector of objective functions which each of them be either minimized or maximized. However, without loss of generality, it is assumed that all objective functions are to be minimized. Such multi-objective minimization based on Pareto approach can be conducted using some definitions.

Definition of Pareto dominance: A vector $\boldsymbol{U} = [\boldsymbol{u_1}, \boldsymbol{u_2}, \dots, \boldsymbol{u_n}]$, is dominance to vector $\boldsymbol{V} = [\boldsymbol{v_1}, \boldsymbol{v_2}, \dots, \boldsymbol{v_n}]$ (denoted by $\boldsymbol{U} < \boldsymbol{V}$)

If and only if :

$$\forall\, i \in \{1,2, \dots, k\}, u_i \leq v_i \;\wedge \exists\, j \in \{1,2, \dots, k\}: u_i < v_j \quad (5)$$

Definition of Pareto optimality: A point $X^* \in \Omega$ (Ω is a feasible region in R^n satisfying equation (3) and (4) is said to be Pareto optimal (minimal) if and only if there is not $X \in \Omega$ which can dominance to X^*. Alternatively, it can be readily restated as

$$\forall\, X \in \Omega, X \neq X^*, \exists\, i \in \{1,2, \dots, k\}: f_i(X^*) < f_i(X) \quad (6)$$

Definition of Pareto Set: *For a given Multi-objective optimization problem (MOP), a Pareto set* $\boldsymbol{P}^*$ *is a set in the decision variable space consisting of all the Pareto optimal vectors*

$$P^* = \{X \in \Omega \,|\, \nexists X' \in \Omega: F(X') < F(X)\} \quad (7)$$

Definition of Pareto front: *For a given MOP, the Pareto front* $\boldsymbol{PT}^*$ *is a set of vector of objective functions which are obtained using the vectors of decision variables in the Pareto set* $\boldsymbol{P}^*$, *that is*

$$\boldsymbol{PT}^* = (\boldsymbol{F}(\boldsymbol{X}) = (\boldsymbol{f_1}(\boldsymbol{X}), \boldsymbol{f_2}(\boldsymbol{X}), \dots, \boldsymbol{f_k}(\boldsymbol{X})): \boldsymbol{X} \in \boldsymbol{P}^* \quad (8)$$

In other words, the Pareto front PT^* is a set of the vectors of objective functions mapped from P^*. Genetic algorithm (GA) is one of the evolutionary algorithms. It uses direct values of functions and doesn't need to function's derivations. These and other properties of GA caused its comprehensive use in optimization problems[10].

The Pareto-based approach of NSGA-II has been recently used in a wide area of engineering MOPs because of its simple yet efficient non-dominance ranking procedure in yielding different level of Pareto frontiers. In this paper modified NSGA-II algorithm as a MO tool searches the definition space of decision variables and returns the optimum answers in Pareto form[11,12].

Results and Discussion

In this section, the multi-objective genetic algorithm (MOGA) is used for multi-objective design of vehicle model which has been shown in figure-3. Computer simulations are carried out to verify the effectiveness of the designed optimal suspension system. The corresponding ground displacement for the wheel is given by

$$z_r = \begin{cases} \frac{a}{2}(1-\cos(8\pi t)), & if \quad .5 \le t \le .75 \quad and \quad 3 \le t \le 3.25 \\ 0 & otherwise \end{cases}$$

Where a denotes bump amplitude. The road disturbance is shown in figure-2. It is supposed that the vehicle moves at constant velocity v=30 m/s over a road disturbance and It is further assumed that the rear tire follows the same trajectory as the front tire with a delay of $\Delta t = (a+b)/v$.

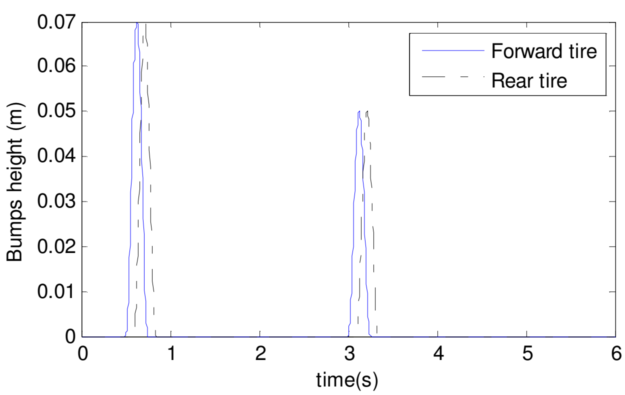

Figure-2
Typical road disturbance

The input values of fixed parameters are presented at table-1[6].

Table-1
The values of fixed parameters of the model

M	580 kg	$\boldsymbol{m_f}$	40 kg
I	910 kg.m^2	$\boldsymbol{m_r}$	30 Kg
$\boldsymbol{k_{f1}, k_{r1}}$	10000 N/M	$\boldsymbol{k_{f2}, k_{r2}}$	100000 N/M
$\boldsymbol{b_f, \ b_r}$	1000 N/M	a , b	1.25 m,1.45 m

In this paper, $5000 \le k_{f1} \le 15000$, $5000 \le k_{r1} \le 15000$, $500 \le b_f \le 2000$, $500 \le b_r \le 2000$, $50000 \le k_{f2} \le 150000$, $50000 \le k_{r2} \le 150000$ are observed as 6 design variables to be optimally found based on multi-objective optimization of 4 different objective functions that are considered defined as follow:

$$J_1 = \left\|\ddot{z}_c + a\ddot{\theta}\right\|_2 = \sqrt{\textstyle\sum_{i=1}^{n}(\ddot{z}_{ci} + a\ddot{\theta}_i)^2}$$
$$J_2 = \left\|\ddot{z}_c + b\ddot{\theta}\right\|_2 = \sqrt{\textstyle\sum_{i=1}^{n}(\ddot{z}_{ci} + b\ddot{\theta}_i)^2} \qquad (9)$$
$$J_3 = \|z_1 - z_c - \text{asin}(\theta)\|_2 = \sqrt{\textstyle\sum_{i=1}^{n}(z_{1i} - z_{ci} - \text{asin}(\theta_i))^2}$$
$$J_4 = \|z_2 - z_c + \text{b}\sin(\theta)\|_2 = \sqrt{\textstyle\sum_{i=1}^{n}(z_{2i} - z_{ci} + \text{b}\sin(\theta_i))^2}$$

Now these objective functions are considered in a Pareto optimization process to obtain some important trade-offs among the conflicting objectives, simultaneously. The evolutionary process of the multi-objective optimization is accomplished with a population size of 120 which has been chosen with crossover probability Pc and mutation probability Pm as 0.9 and 0.1, respectively. A total number of 116 non-dominated optimum design points have been obtained.

It is widely accepted that visualization tools are valuable to provide the decision maker a meaningful way to analyze Pareto set and select good solutions. For a 2-dimensnal problem it is normally easy to make an accurate graphical analysis of the Pareto set points, but for higher dimensions it becomes more difficult[13]. Therefore, the Level Diagrams method is used to visualize a Pareto front. In this method, each point of Pareto front must be normalized between 0 and 1 based on its minimum and maximum values

$$J_i^M = \max(J_i)\,, J_i^m = \min(J_i)\,, i = 1,2,3 \qquad (10)$$
$$\overline{J_i} = \frac{J_i - J_i^m}{J_i^M - J_i^m}$$

Provided that the origin of the n-dimensional space is considered as ideal point, the distance of the each Pareto front point is used to choose optimum points. In this work, Euclidean norm $\|\bar{J}\|_2 = \sqrt{\sum_{i=1}^{4}\bar{J}_i^2}$ is used for this purpose. Hence the point whose distance to the origin is the minimum, that is, the lowest value of $\|\bar{J}\|_2$ can be obtained as the most important trade-off point. The results of the 4-objective optimization process are shown in figure-3.

As it is shown, the point with the lowest vale of $\|\bar{J}\|_2$ has the low value of each objective function. To illustrate the result of the optimization process, 5 points are chosen of, which four of them have the minimum value of each objective function and the fifth one has the minimum value of $\|\bar{J}\|_2$ [14]. The values of the pertinent objective functions are given in table-2.

The time behavior of front and rear sprung mass acceleration of the trade-off design point E and the point proposed in reference 6 are shown for comparison in figures-4-5. It is obvious from these figures that the values of front and rear sprung mass acceleration of the design point obtained in this paper are better than that by the design point given in reference 6.

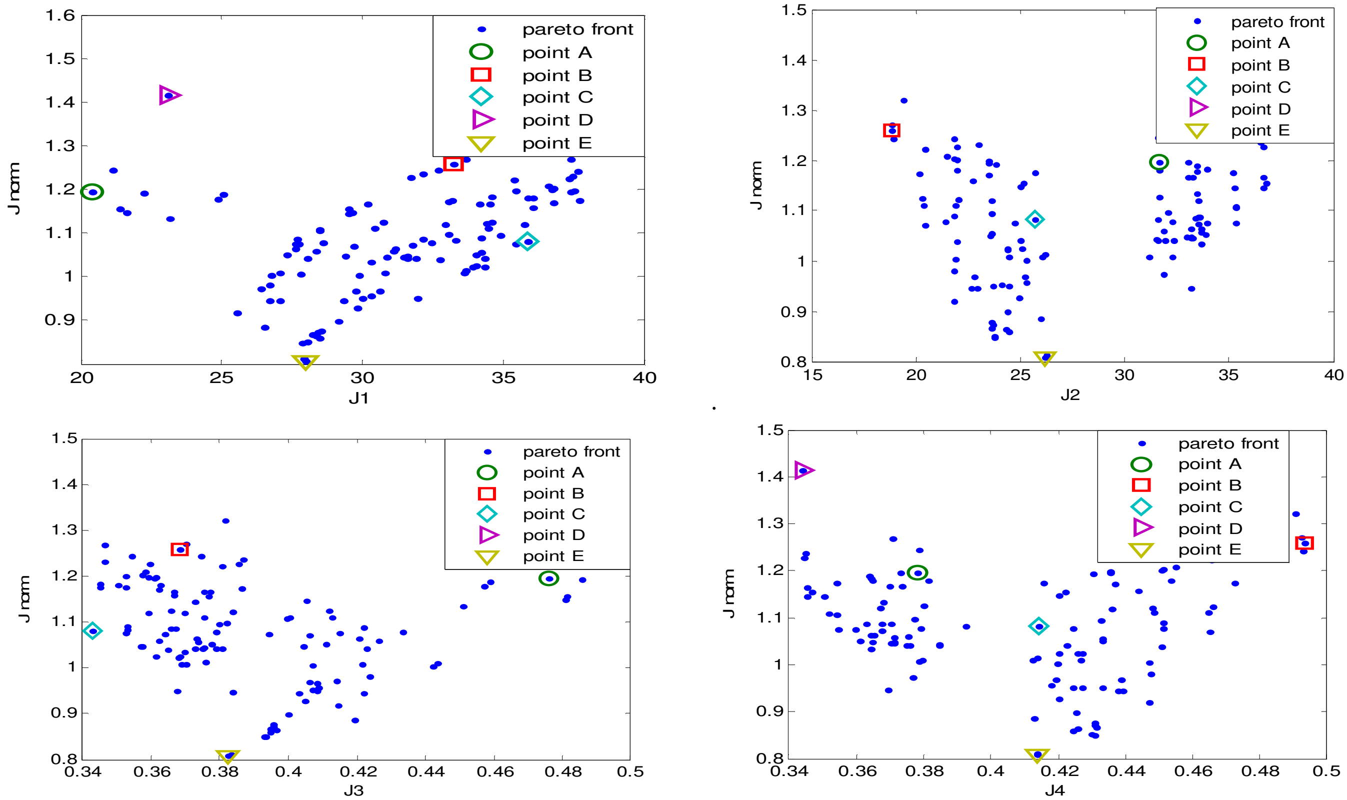

Figure-3
Euclidean norm Level Diagrams of Pareto front

Table-2
The values of objective functions of the best optimal point

Category	J_1	J_2	J_3	J_4	$\lVert\bar{J}\rVert_2$
min $\boldsymbol{J_1}$ (A)	20.37	31.65	0.4760	0.3780	1.1957
min $\boldsymbol{J_2}$ (B)	33.26	18.85	0.3685	0.4934	1.2593
min $\boldsymbol{J_3}$ (C)	35.89	25.72	0.3428	0.4142	1.0818
min $\boldsymbol{J_4}$ (D)	23.09	36.61	0.4853	0.3439	1.4148
min$\lVert\bar{J}\rVert_2$ (E)	27.97	26.16	0.3823	0.4137	0.8081

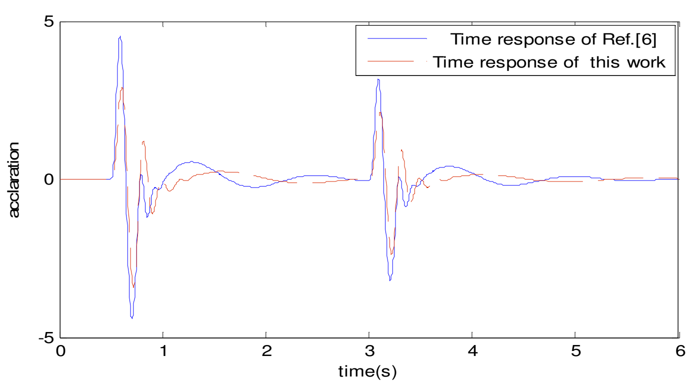

Figure-4
Time responses of front sprung mass acceleration

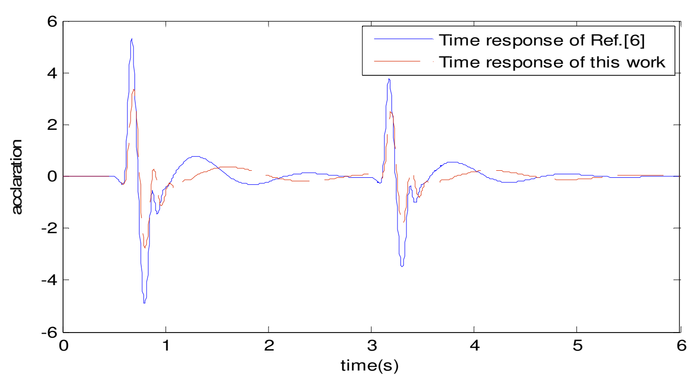

Figure-5
Time responses of rear sprung mass acceleration

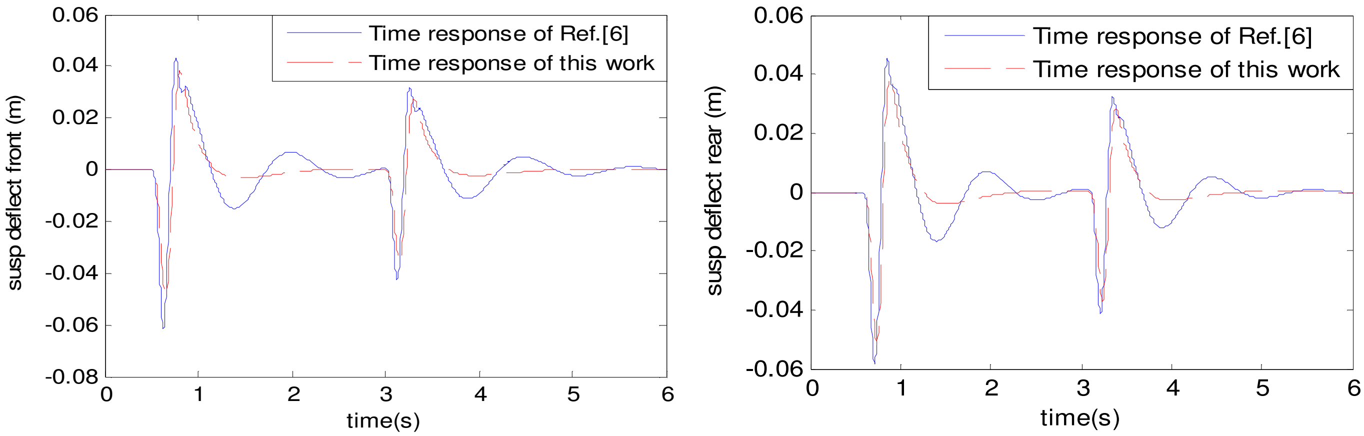

Figure-6
Time responses of front and rear suspension travel

Figure-6 depicts the front and rear suspension travel of the trade-off design point E and the point proposed in reference 6 for comparison purposes. The result shows that the front and rear suspension deflection of the design point obtained in this paper are better as compared to reference 6.

Conclusion

In this work, a multi-objective genetic algorithm has been used to optimally design vehicle vibration model. The objective functions which conflict with each other were selected as acceleration of front and rear sprung mass that are related to ride comfort and front and rear suspension deflection that are related to road holding ability. The multi-objective optimization of vehicle model led to the discovering of some important trade-offs among those objective functions. The superiority of the obtained optimum design points was shown in comparison with that reported in the literature. Such multi-objective optimization of vehicle model could unveil very important design trade-offs between conflicting objective functions which would not have been found otherwise. Therefore it is concluded that MOGA optimization improves the ride comfort while retaining the vehicle maneuverability characteristics, as compared to the suspension system that proposed in reference 6.

References

1. Rajeswari K. and Lakshmi P., GA Tuned distance based fuzzy sliding mode controller for vehicle suspension systems, *International Journal of Engineering and Technology,* **5**(**1**), 36-47 **(2008)**

2. Krishan K. and Aggarwal M.L. , A Finite Element Approach for Analysis of a Multi Leaf Spring using CAE Tools, *Res. J. Recent Sci.,* **1(2),** 92-96, **(2012)**

3. HoIou N.AL, Weaver O., Lahdhiri T. and Sung D**.,** Sliding Mode-based Fuzzy Logic Controller for a Vehicle Suspension System*, Proceedings of the American Control Conference J004 ,*San Diego, California June **(1999)**

4. Els P.S., Theron N.J., Uys P.E. and Thoresson M.J. **,** The ride comfort vs. handling compromise for off-road vehicles, *Journal of Terramechanics* ,**44** , 303–317, **(2007)**

5. Guclu R. and Yagiz N., Comparison of different control strategies on a vehicle using sliding mode control , *Iranian Journal of Science & Technology*, Transaction B, **28**, 413-422, **(2004)**

6. Hassanzadeh I., Alizadeh G., Shirjoposht N. P. and Hashemzadeh F. , A New Optimal Nonlinear Approach to Half Car Active Suspension Control , *IACSIT International Journal of Engineering and Technology* , **2**(**1**) , February, 78-84, **(2010)**

7. Nariman-Zadeh N. , Salehpour M. , Jamali A. and Hajiloo A. , Pareto optimization of a five-degree of freedom vehicle vibration model using a multi-objective uniform-diversity genetic algorithm (MUGA) , *Engineering Applications of Artificial Intelligence*, **23**(**4**) , 543–551, **(2010)**

8. Nikhil D., Rajesh A., Vijay M., Sandeep K., Mohit, Satyapal and Pardeep K., Thermodynamic Analysis of a Combined Heat and Power System, *Res. J. Recent Sci.,* Vol. **1(3),** 76-79, March **(2012)**

9. Khajavi M. N., Notghi B.and Paygane G., A Multi Objective Optimization Approach to Optimize Vehicle Ride and Handling Characteristics, *World Academy of Science, Engineering and Technology* **62**, 580-584, **(2010)**

10. Atashkari K., Nariman-Zadeh N., Golcu M., Khalkhali A.and Jamali A., Modeling And Multi-Objective Optimization Of A Variable Valve-Timing Spark-Ignition Engine Using Polynomial Neural Networks And Evolutionary Algorithms, *Energy Conversion and Management* ,**48** ,1029–1041, **(2007)**

11. Jamali A., Nariman-zadeh N., Darvizeh A., Masoumi A. and Hamrang S., Multi-objective evolutionary optimization of polynomial neural networks for modeling and prediction of explosive cutting process, *Engineering Applications of Artificial Intelligence*, **22**, 676–687, **(2009)**

12. Atashkari K., Nariman-zadeh N., Plilechi A., Jamali A. and Yao X., Thermodynamic Pareto Optimization of Turbojet Engines Using Multi-objective Genetic Algorithms, *International Journal of Thermal Sciences*, **44(11)**, 1061-1071, **(2005)**

13. Blasco X., Herrero J., sanchis J. and Martinez M., Decision Making Graphical Tool for Multi-objective Optimization Problem, *Springer- Verlag,* Berlin Heidelberg, 568-577, **(2007)**

14. Hajiloo A., Nariman-zadeh N., Rad A. and Samadi M., Pareto Optimum Design of Sliding Mode Controllers for Robotic Manipulators using a Multi-objective Uniform-diversity Genetic Algorithm (MUGA), *International Symposium on INnovations in Intelligent SysTems and Applications*, Kayseri & Cappadocia, TURKEY, 461-465, June **(2010)**