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Review Paper

Machine Repair Problem with Spares and N-Policy Vacation

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Abstract

In this paper we have taken a machine repairable system with spares and two repairmen where "the partial server vacation" is applied. In our system, the first repairman never takes vacations and always available for serving the failed units. The second repairman goes to vacation of random length when number of failed units is less than N. At the end of vacation period, this repairman returns back if there are N or more failed units/machine accumulated in the system. Otherwise this repairman goes for another vacation. Vacation time is exponentially distributed. By using of Markov process theory, we develop the steady state probabilities equations using transition diagram and solve these equations recursively. We present derivations of some queuing and reliability measures. A cost model is developed to determine the optimum value of N while the system availability is maintained at certain level. Sensitivity analysis is also investigated. In this paper we not only analyze the queuing problems but also analyze the reliability characteristics of the system.

Keywords: Repairable system, spares, vacation, availability, failure frequency, sensitive analysis.

Introduction

The machine repairable systems with spares have used in many situations. For example, to substitute when an operating machine fails. Another example of repairable system with spares can be found in operating room of a hospital, where standby power equipments are needed since the operating on a patient cannot be stopped when the power is breakdown. Similar examples can be found in many fields such as power stations, manufacturing systems and industrial systems. The machine repair problem with spares has been an area of interest for many researchers since long, instead of going to details of such earlier works, it's worthwhile to give a brief overview of some important contributions in recent past.

Hsieh and Wang[1] considered the reliability characteristics of a repairable system with m operating units, s warm spares and one removable repairmen in the facility, where the repairman apply the N-policy. They obtain the expressions of the reliability and the mean time to system failure. Jain, Rakhee and Maheshwari[2] extended the model of Hsieh and Wang[1] to analyze the repairable system in transient by incorporating reneging behavior of failed units. Wang and Sivazlian[3] considered the reliability characteristics of a repairable system with m operating units, swarm spares and R repairmen. They obtain the expressions of the reliability and the mean time to system failure. Wang and Ke[4] extend this model to consider the balking and reneging of failed units. They obtain the steady state availability and mean time to system failure. The above mentioned models assumes that the repairmen are always available, but in many real world repairable systems, this is not the situation, repairmen may become unavailable for a random period of time when there are no failed units in the system. In this random period, repairmen can perform some other task, which may reduce the burden on the system in terms of cost. However, there are a few works that take vacation into the consideration of repairmen in the machine repair models with spares.

Gupta[5] first studied machine interference problem with warm spares and server in which the server takes a vacation of random duration every time the repair facility becomes empty. They gave an algorithm to compute the steady-state probability distribution of the number of failed machines in the system.

Jain and Singh[6] studied a machine repair model with warm standbys, setup and vacation from the view point of queuing theory. They considered (N, L) switch-over policy for the two-repairmen. The first repairman turns on for repair only when N-failed units are accumulated and starts repair after a setup time. As soon as the system becomes empty, the repairman goes to vacation and returns back when this repairman finds the number of failed units in the system greater than or equal to threshold value N. the second repairman turns on when there are L failed units in the system and goes for a vacation if there are less than L failed units.

Ke and Wang[7] studied machine repair problem with two type spares and multi-server vacations. They solved the steady-state probabilities equations iteratively and derived the steady-state probabilities in matrix form. Jain and Upadhyaya[8] discussed threshold N-policy for degraded machining system with multiple type of spares and multiple vacations. Yu, Liu and Ma[9] studied steady-state Queue length analysis of a batch arrival queue under N-policy with single vacation and setup times. Zhang and Tian[10] gave analysis on queuing system with synchronous vacation of partial servers. Dequan, Wuyi and Hongjuan[11] perform analysis of machine repair system with warm spares and N-policy vacation.

In many practical multiple server systems, only some servers performs other jobs (takes vacation) when they becomes idle and the other servers are always available for serving the arriving units. In the queuing system, this type of vacation is called "the partial server vacation". This term motivates us to steady the machine repair system with spares and a "partial server vacation" policy.

Now in our studies we have taken a machine repairable system with spares and two repairmen where "the partial server vacation" is applied. In our system, the first repairman never takes vacations and always available for serving the failed units. The second repairman goes to vacation of random length when number of failed units is less than N. At the end of vacation period this repairman returns back if there are N or more failed units/machine accumulated in the system. Otherwise this repairman goes for another vacation. Vacation time is exponentially distributed. By using of Markov process theory, we develop the steady state probabilities equations with transition diagram and solve these equation recursively. We present derivations of some queuing and reliability measures. A cost model is developed to determine the optimum value of N while the system availability is maintained at certain level. Sensitivity analysis is also investigated. In this paper we not only analyze the queuing problems but also analyze the reliability characteristics of the system, while papers presented above only considered the queuing problems.

System Model

In this paper, we consider a machine repair system with m-operating units, s- spare units and two repairmen. The first repairman always available for servicing of failed units, while second repairman goes to vacations under some conditions. We call the first repairman "Repairman 1" and call the second repairman "Repairman 2". The assumptions of system model are as follows – i. For the execution of the system m-operating system are required. However, the system may also work in short mode, if all spare units are exhausted and there are less then m but more than k operating units in operation. In other words, the system breakdown if and only if L= s+m-k+1 or more units fail. The life time of operating machine and spare machine are exponentially distributed with rates of λ means that failure rate of both types are same. ii. As soon as operating unit fails, it replaced by a spare unit if available and it immediately sent for repair. When a failed unit is repaired, it is good as new one. The repaired unit goes to be operating side if there are less than m-operating units; otherwise the unit joins the standbys group. The repair time of Repairman 1 and repairman 2 are exponentially distributed with rates of μ_1 and μ_2, respectively. iii. Repairman 1 is always available for servicing of failed units. However, if there are less than N failed units, Repairman 2 goes to vacations of random length. On return from a vacation if Repairman 2 finds more than or equal to N failed units accumulated in the system, Repairman 2 will start to repair the failed units, otherwise Repairman 2 goes for another vacation. The vacation time is exponentially distributed with rate of θ. iv. The switch is perfect and switch-over time is instantaneous. When a spare moves into an operating state, its failure characteristic will be that of an operating machine.

Steady-state Analysis: In this section, we develop the difference equation of steady-state probabilities of the system by using Markov process and transition diagram. Then we derive the expressions of the steady state probabilities by recursive method and finally we give the explicit expression of some steady-state performance measures.

Let $L(t)$ be the number of failed units in the repair facility at time t and let

$$J(t) = \begin{cases} 0, & Repairman\ 2\ is\ on\ vacations \\ 1, & Repairman\ 2\ \ is\ not\ no\ vacation\ at\ time\ t \end{cases}$$

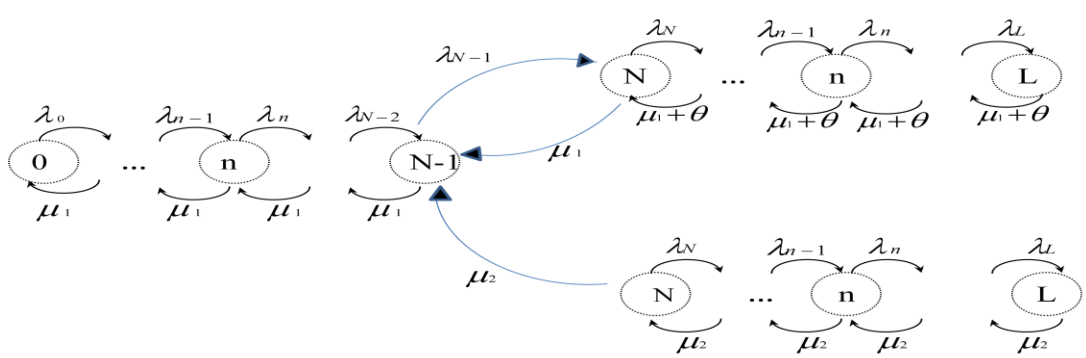
Then, $\{L(t), J(t)\}$ is Markov process with state space

$$E = \{(n,0) : n = 0,1,\ldots,L\} \cup \{(n,1) : n = N, N+1,\ldots, L\}$$

Define the steady- state probabilities of the system as follows:

$$P_{n,0} = \lim_{t\to\infty} P\{L(t) = n, J(t) = 0\}, \quad 0 \le n \le L,$$
$$P_{n,1} = \lim_{t\to\infty} P\{L(t) = n, J(t) = 1\}, \quad N \le n \le L$$

Transition state diagram



Then, the steady state probabilities equations governing the model are obtained as follows:

$$-\lambda_0 P_{0,0} + \mu_1 P_{1,0} = 0, \quad (1)$$

$$\lambda_{n-1}P_{n-1,0} - (\lambda_n + \mu_1)P_{n,0} + \mu_1 P_{n+1,0} = 0, \ 1 \le n \le N-2, \quad (2)$$

$$\lambda_{N-2}P_{N-2,0} - (\lambda_{N-1} + \mu_1)P_{N-1,0} + \mu_1 P_{N,0} + \mu_2 P_{N,1} = 0, \quad (3)$$

$$\lambda_{n-1}P_{n-1,0} - (\lambda_n + \mu_1 + \theta)P_{n,0} + \mu_1 P_{n+1,0} = 0, N \le n \le L-1, \quad (4)$$

$$\lambda_{L-1}P_{L-1,0} - (\mu_1 + \theta)P_{L,0} = 0, \quad (5)$$

$$\theta P_{N,0} - (\lambda_N + \mu_2)P_{N,1} + \mu_2 P_{N+1,1} = 0, \quad (6)$$

$$\theta P_{n,0} + \lambda_{n-1}P_{n-1,1} - (\lambda_n + \mu_2)P_{n,1} + \mu_2 P_{n+1,1} = 0, \ N+1 \le n \le L-1, \quad (7)$$

$$\theta P_{L,0} + \lambda_{L-1}P_{L-1,1} - \mu_2 P_{L,1} = 0, \quad (8)$$

With the normalizing condition

$$\sum_{n=0}^{L} P_{n,0} + \sum_{n=N}^{L} P_{n,1} = 1 \quad (9)$$

Where

$$\lambda_n = \begin{cases} (m+s-n)\lambda, & n = 0,1,\dots,L-1 \\ 0, & n = L \end{cases}$$

In order to solve the steady- state probability equations (1) – (9), we define

$$\varphi_n = \begin{cases} \left(\frac{1}{\mu_1}\right)\prod_{j=0}^{n-1}\lambda_j, & 1 \le n \le N-1 \\ \left(\frac{1}{\mu_1}\right)\prod_{j=N}^{n}\beta_j \prod_{j=0}^{N-2}\lambda_j, & N \le n \le L \end{cases} \quad (10)$$

Where $\beta_j, j = N, N+1, \dots, L,$ is defined iteratively as follows:

$$\beta_L = \frac{\lambda_{L-1}}{\mu_1+\theta}, \quad (11)$$

$$\beta_n = \frac{\lambda_{n-1}}{\mu_1+\theta\left[1+\sum_{i=n+1}^{L}\prod_{j=n+1}^{i}\beta_j\right]}, \quad N \le n \le L-1, \quad (12)$$

The following statement gives the solutions of steady-state probabilities equations (1) – (9)

Statement the steady-state probabilities are given by

$$P_{n,0} = \varphi_n P_{0,0}, \ 1 \le n \le L, \quad (13)$$

$$P_{n,1} = \frac{\theta}{\mu_2}\psi_n P_{0,0}, \ N \le n \le L, \quad (14)$$

And $$P_{0,0} = \left(1 + \sum_{n=1}^{L}\varphi_n + \frac{\theta}{\mu_2}\sum_{n=N}^{L}\psi_n\right)^{-1} \quad (15)$$

Where φ_n is defined by eq. (10) and ψ_n is defined iteratively as follows:

$$\psi_n = \sum_{i=N}^{L}\varphi_n, \quad (16)$$

$$\psi_n = \frac{\lambda_{n-1}}{\mu_2}\psi_{n-1} + \sum_{i=n}^{L}\varphi_i, \quad N+1 \le n \le L \quad (17)$$

Explanation of the above results: from eq. (1)

$P_{1,0} = \frac{\lambda_0}{\mu_1} P_{0,0}$ from eq. (2) after putting value of n = 1, 2, N-2 we get following recursion relations

$$P_{2,0} = \frac{\lambda_1}{\mu_1} P_{1,0}$$

$$P_{3,0} = \frac{\lambda_2}{\mu_1} P_{2,0}$$

……………

$$P_{N-1,0} = \frac{\lambda_{N-2}}{\mu_1} P_{N-2,0}$$

for n = N-1

$$P_{N,0} = \frac{\lambda_{N-1}}{\mu_1} P_{N-1,0}$$

from all above relations from (i) – (v) can be represent by a single recursive relation

$$P_{n,0} = \frac{\lambda_{n-1}}{\mu_1} P_{n-1,0} \quad 1 \le n \le N-1 \quad (18)$$

After putting all above relations (i) – (v) in Eq. (18) we get

$$P_{n,0} = \frac{\lambda_{n-1}\lambda_{n-2}\dots\lambda_0}{\mu_1\mu_1\dots\mu_1} P_{0,0} \quad 1 \le n \le N-1$$

$$P_{n,0} = \left(\frac{1}{\mu_1}\right)^n \prod_{i=0}^{n-1}\lambda_i \ P_{n-1,0} \quad 1 \le n \le N-1$$

$$P_{n,0} = \varphi_n P_{0,0}, \quad 1 \le n \le N-1 \quad (19)$$

Whereφ_n, for $1 \le n \le N-1$, is defined by eq. (10). From eq. (5)

$$P_{L,0} = \frac{\lambda_{L-1}}{\mu_1+\theta} P_{L-1,0} \quad P_{L,0} = \beta_L P_{L-1,0} \quad (20)$$

Where β_L is defined by eq. (11)

$$\lambda_{L-1}P_{L-1,0} - \mu_1 P_{L,0} = \theta P_{L,0}$$

From eq. (4)

$$\lambda_{n-1}P_{n-1,0} - (\mu_1 + \theta) P_{n,0} = \lambda_n P_{n,0} - \mu_1 P_{n+1,0},$$

$N \le n \le L-1$

By using Eq. (vi) recursively in (vii) we get

$$\lambda_{n-1}P_{n-1,0} - (\mu_1 + \theta) P_{n,0} = \theta \sum_{i=n+1}^{L} P_{i,0}$$

$N \le n \le L-1$ (21)

Recursively using eq. (20) in (21) we get

$$\lambda_{n-1}P_{n-1,0} - (\mu_1 + \theta) P_{n,0} = \theta \sum_{i=n+1}^{L}\prod_{j=n+1}^{i}\beta_j P_{n,0}$$

$$\lambda_{n-1}P_{n-1,0} = (\mu_1 + \theta(1 + \sum_{i=n+1}^{L}\prod_{j=n+1}^{i})P_{n,0}$$

$$P_{n,0} = \frac{\lambda_{n-1}}{\mu_1 + \theta\ (1 + \sum_{i=n+1}^{L}\prod_{j=n+1}^{i})} P_{n-1,0}$$

$$P_{n,0} = \beta_n P_{n-1,0}, \quad N \le n \le L-1 \quad (22)$$

Where β_n is defined by eq. (12)

From eq. (19), (20) and (22) we get eq. (13)

$$P_{n,0} = \varphi_n P_{0,0}, \ 1 \le n \le L,$$

From Eq. (2.7)

$$\mu_2 P_{n,1} - \lambda_{n-1}P_{n-1,1} = \mu_2 P_{n+1,1} - \lambda_n P_{n,1} + \theta P_{n,0},$$

$N+1 \le n \le L-1$

From eq. (8) we get

$$\mu_2 P_{n+1,1} - \lambda_n P_{n,1} = \theta P_{L,0}$$

From above, we get

$$\mu_2 P_{n,1} - \lambda_{n-1}P_{n-1,1} = \theta P_{L,0} + \theta P_{n,0},$$

$N+1 \le n \le L-1$ (23)

$$P_{n,1} = \frac{\lambda_{n-1}}{\mu_2} P_{n-1,1} + \frac{\theta}{\mu_2}\sum_{i=n}^{L} P_{i,0},$$

$N+1 \le n \le L-1$ (24)

After taking n = N+1 in eq. (24)

$$P_{N+1,1} = \frac{\lambda_N}{\mu_2} P_{N,1} + \frac{\theta}{\mu_2}\sum_{i=N+1}^{L} P_{i,0}$$

From above substitution in eq. (6) we get

$$\theta P_{N,0} - (\lambda_N + \mu_2)P_{N,1} + \mu_2\left(\frac{\lambda_N}{\mu_2} P_{N,1} + \frac{\theta}{\mu_2}\sum_{i=N+1}^{L} P_{i,0}\right) = 0$$

$$\theta P_{N,0} - \lambda_N P_{N,1} - \mu_2 P_{N,1} + \lambda_N P_{N,1} + \theta\sum_{i=N+1}^{L} P_{i,0} = 0$$

$$\theta P_{N,0} - \mu_2 P_{N,1} + \theta\sum_{i=N+1}^{L} P_{i,0} = 0$$

$$\mu_2 P_{N,1} = \theta P_{N,0} + \theta\sum_{i=N+1}^{L} P_{i,0}$$

$$P_{N,1} = \frac{\theta}{\mu_2}\sum_{i=N}^{L} P_{i,0} \quad (25)$$

Substituting eq. (13) into eq. (25)

$$P_{N,1} = \frac{\theta}{\mu_2}\sum_{i=N}^{L}\varphi_i P_{0,0}$$

$$P_{N,1} = \frac{\theta}{\mu_2}\psi_N P_{0,0}$$

ψ_N is defined in eq. (16)
then

$$P_{n-1,1} = \frac{\theta}{\mu_2}\psi_{n-1} P_{0,0}$$

Substituting eq. (13) and above into eq. (24)

$$P_{n,1} = \frac{\lambda_{n-1}}{\mu_2}\frac{\theta}{\mu_2}\psi_{n-1}P_{0,0} + \frac{\theta}{\mu_2}\sum_{i=n}^{L}\psi_i P_{0,0}$$

$$P_{n,1} = \frac{\theta}{\mu_2}\left(\frac{\lambda_{n-1}}{\mu_2}\psi_{n-1} + \frac{\theta}{\mu_2}\sum_{i=n}^{L}\psi_i\right) P_{0,0}$$

$$P_{n,1} = \frac{\theta}{\mu_2}\psi_n P_{0,0}\,, \qquad N \le n \le L$$

ψ_n is defined in eq. (17)
Substituting eq. (13) and (14) into eq. (9)

$$1+\sum_{n=1}^{L} P_{n,0} + \sum_{n=N}^{L} P_{n,1} = 1$$

$$1+\sum_{n=1}^{L} \varphi_n P_{0,0} + \sum_{n=N}^{L} \psi_n P_{0,0} = 1$$

$$\left(1+\sum_{n=1}^{L}\varphi_n + \sum_{n=N}^{L}\psi_n\right) P_{0,0} = 1$$

$$P_{0,0} = \frac{1}{(1+\sum_{n=1}^{L}\varphi_n + \sum_{n=N}^{L}\psi_n)}$$

$$P_{0,0} = \left(1+\sum_{n=1}^{L}\varphi_n + \sum_{n=N}^{L}\psi_n\right)^{-1}$$

By this, we proved given theorem.
Using steady state probabilities presented in above statement, we can easily obtain the following performance of queuing and reliability measure.

Some of the results are concluded from above: Reliability measure: The average number of failed units in the system is given by-

$$L_f = \sum_{n=1}^{L} nP_n$$

$$L_f = \sum_{n=1}^{N-1} nP_{n,0} + \sum_{n=N}^{L} nP_{n,1}$$

$$L_f = \sum_{n=1}^{N-1} n\varphi_n P_{0,0} + \sum_{n=N}^{L} n\psi_n P_{0,0}$$

$$L_f = \left(\sum_{n=1}^{N-1} n\varphi_n + \sum_{n=N}^{L} n\psi_n\right)P_{0,0} \tag{26}$$

The average number of units that function as spears is given by -

$$L_s = \begin{cases} \left(\sum_{n=N}^{s-1}(s-n)\varphi_n + \frac{\theta}{\mu_2}\sum_{n=N}^{s-1}(s-n)\psi_n\right)P_{0,0}, & N < s \\ \sum_{n=0}^{s-1}(s-n)\varphi_n\, P_{0,0}, & N > s \end{cases}$$

The probability that Repairman 1 is busy is given by $P_{0,0}$ = Probability that there are no failed units in the system $1 - P_{0,0}$ = Probability that there are failed units in the system

$$P_b^1 = 1 - P_{0,0} \tag{28}$$

The probability that Repairman 2 is busy is given by Probability that there are N and more failed units in the system

$$P_{n,1} = \frac{\theta}{\mu_2}\psi_n P_{0,0}$$

$$P_b^2 = \frac{\theta}{\mu_2}\sum_{n=N}^{L}\psi_n P_{0,0} \tag{29}$$

The steady-state availability of the system is given by as following: If failed units in system are $L = s+m-k+1$ or more then L than system is breakdown, in this case system is not available. So system availability-

$$A = 1 - \left(\varphi_L + \frac{\theta}{\mu_2}\psi_L\right)P_{0,0} \tag{30}$$

The steady-state failure frequency is given by

$$M = \lambda_{L-1}\left(\varphi_{L-1} + \frac{\theta}{\mu_2}\psi_{L-1}\right)P_{0,0} \tag{31}$$

Cost Model

We develop a steady-state expected cost function per unit time, and impose a constraint on the availability of the system in which N is a decision variable. Our objective is to determine the optimum N*, so that the cost is minimized and the availability of the system is maintained at a certain level.

According to Ke and Wang[7] research paper, let C_f be the cost per unit time of one failed unit in the repair facility, let C_s be the cost per unit time of one unit that functions as a spare, let C_b^1 be the cost per unit that Repairman 1 is busy, let C_b^2 be the cost per unit that Repairman 2 is busy, let C_I be the cost per unit time that Repairman 1 is idle, and C_v be the reward per unit time that Repairman 2 is on vacation.

Using the definition of the cost parameters listed above, the total expected cost function per unit time is given by

$$F(N) = C_f L_f + C_s L_s + C_b^1 P_b^1 + C_b^2 P_b^2 + C_I P_I - C_v P_v$$

Where $P_I = P_{0,0}$ is the probability that Repairman 1 is idle, $P_v = 1 - P_b^2$ is the probability that Repairman 2 is on Vacation, L_f, L_s, P_b^1 and P_b^2 are given by some of the results are concluded from statement given above.
In order to maintain the availability of the system at a certain level, we present the cost minimization problem as follows:

$$\underset{s.t\ A \ge A_0}{\text{Min}}\ F(N) = C_f L_f + C_s L_s + C_b^1 P_b^1 + C_b^2 P_b^2 + C_I P_I - C_v P_v$$

Where A is the steady state availability of the system given by eq. (30) and A_0 is the given level of the availability of the system as a system parameter. The analytic study of the behavior of the expected cost function would have been an arduous task to undertake since the decision variable N is discrete and repair in an expression which is highly non-linear and complex. Thus, one may use a heuristic approach to obtain the optimum value N* which is determined by satisfying inequalities as follows: $F(N^*-1) > F(N^*) < F(N^*+1)$
And $A \ge A_0$

Testing and Validation

In this paper, we provide an example to perform a sensitivity analysis for changes in the optimum value N* along with changes is specific values of the system parameter $\lambda,\ \mu_1,\ \mu_2\ and\ \ \theta.$

Example. We set m = 10, s = 6, k = 7 and $A_0 = 0.8$ the following cost elements are used: $C_f = 80, C_s = 50\ , C_b^1 = 70, C_b^2 = 60, C_I = 40, C_v = 70$

The numerical results of the optimum value N*, the optimum cost F (N*) and the availability of the system at the optimum value A (N*) are illustrated in table 1-3.

In table 1, we fix $\mu_1 = 6\ , \mu_2\ = 12\ , \theta = 4$ and vary the value of N* from 1 to 10, and choose different values of λ. Table 1 show that the optimum cost $F(N^*)$ increases significantly and the optimum value $A(N^*)$ decreases as λ increases. The number of working units is more than spares units. Hence, the cost due to failure of working units is larger than the cost due to failure of units as spares.

In table 2, we fix $\lambda = 1\ , \theta = 4$ and vary the value of N* from 1 to 10, and choose different values of(μ_1, μ_2). Table 2 shows that the optimum cost $F(N^*)$ decreases and the optimum value A(N^*) increases significantly as μ_1 increases, While the optimum cost $F(N^*)$ increases and the optimum value A(N*) decreases significantly as μ_2 increases. This is because that the number of failed units decreases as μ_1 increases. This results in the optimum cost decreasing as μ_1 increases. However, since Repairman 2 takes N policy vacation, the reward due to the vacation of Repairman 2 decreases as μ_2 increases. It results in the increasing of the optimum cost.

In table 3, we fix $\lambda = 1\ ,\ \mu_1 = 6\ , \mu_2\ = 12$ and vary the value of N from 1 to 10, and choose different values of θ. Table 3 shows that the optimum cost $F(N^*)$ decreases slightly as θ increases. This is because that the reward due to vacation of Repairman 2 increases as θ increases. This results in the decreasing of the optimum cost. From the last five columns of Table 3, we observe that $A(N^*)$ are the same even though θ varies from 4 to 8. Intuitively, this seems too insensitive to changes in θ decreases as λ increases. The number of working units is more than spares units. Hence, the cost due to failure of working units is larger than the cost due to failure of units as spares.

Table 1
The optimum cost, the optimum value and the system's availability at the optimum value $(\mu_1 = 6\ , \mu_2\ = 12\ , \theta = 4)$

(λ)	0.2	0.4	0.6	0.8	1	1.2	1.2
N*	9	9	8	8	8	7	6
F(N*)	163.42	345.15	415.68	438.47	455.80	526.19	574.37
A(N*)	.99	.99	.96	.91	.84	.80	.83

Table 2
The optimum cost, the optimum value and the system's availability at the optimum value $(\lambda = 1\ , \theta = 4)$

(μ_1, μ_2)	(4,12)	(6,12)	(8,12)	(10,12)	(12,12)	(6,10)	(6,14)
N*	7	8	8	9	8	8	7
F(N*)	496.39	455.80	447.90	447.83	410.30	496.49	433.59
A(N*)	.82	.84	.89	.93	.97	.81	.88

Table 3
The optimum cost, the optimum value and the system's availability at the optimum value $(\lambda = 1, \mu_1 = 6\ , \mu_2\ = 12\ )$

(θ)	1	2	3	4	5	6	7
N*	6	6	7	8	8	8	9
F(N*)	314.21	422.41	441.09	455.80	481.86	501.63	523.60
A(N*)	.81	.85	.85	.84	.85	.86	.84

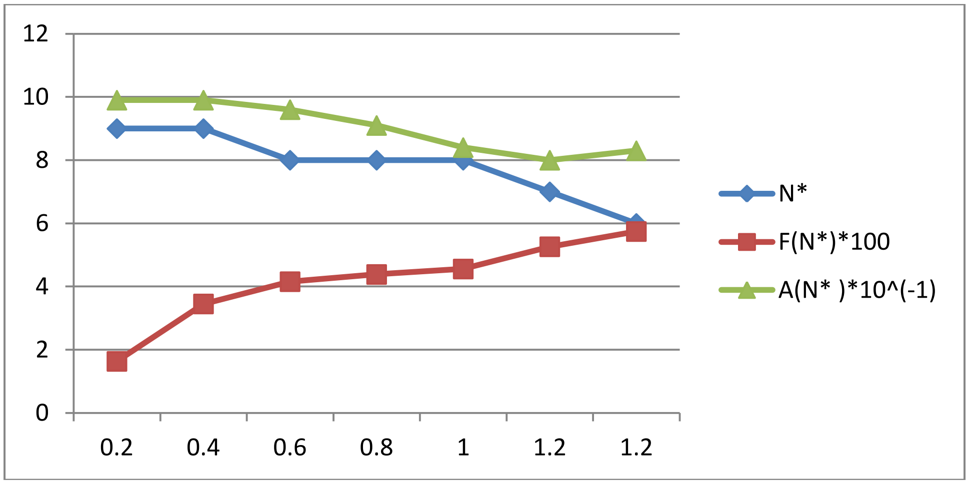


Figure 1

Figure no. 1 as for the table-1

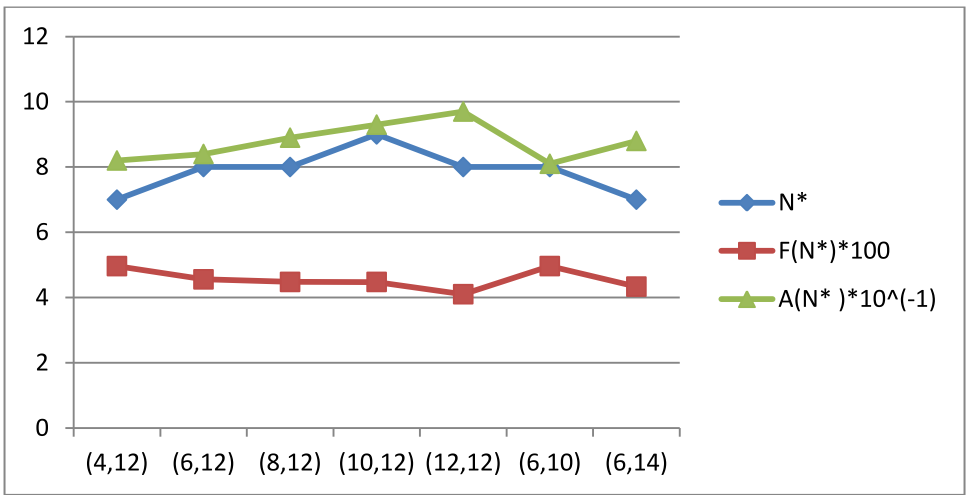


Figure 2

Figure no. 2 as for the table-2

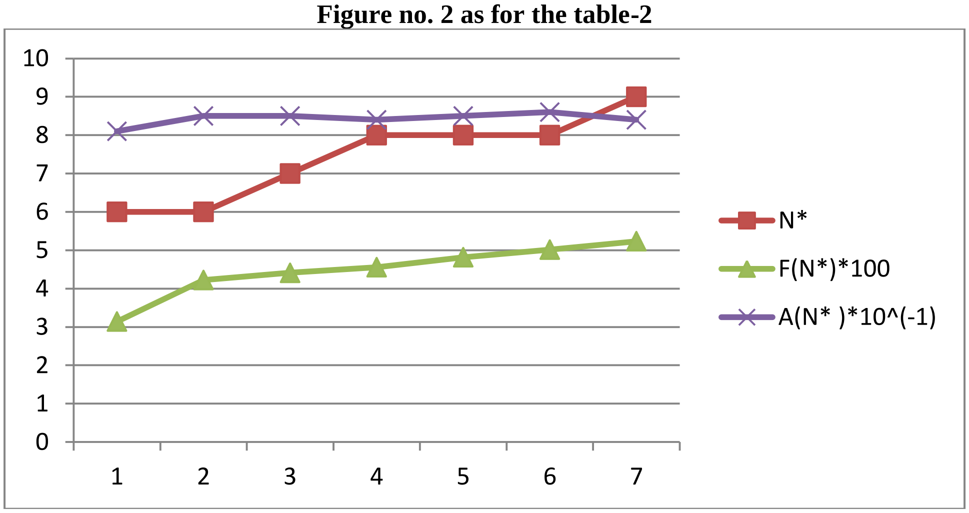


Figure 3

Figure no. 3 as for the table-3

Conclusion

In this paper, we have analyzed a machine repair system with warm spares and N-policy vacation of a repairman from view point of queuing and reliability. We derived the expressions of the steady-state probabilities iteratively. Some performance measures of queuing and reliability were obtained. We developed a cost model to determine the optimum N while the system availability is maintained at a certain level. Sensitivity analysis is also investigated using java program.

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