Hexagonal Geometrical Inclusion to Estimate Effective Thermal Conductivity (ETC) of Porous System and Suspension system Including the Effect of Natural Convection

Selvakumar B.¹, Prabhu Raja V.¹, NandhaKumar R.¹, Senthil Kumar A.P.¹, Vignesh M.S.¹, VivekSharma G.R.¹, Karthikeyan P.²

¹Department of Mechanical Engineering PSG College of Technology, Coimbatore, INDIA
²Department of Automobile Engineering PSG College of Technology, Coimbatore, INDIA

Available online at: www.isca.in
(Received 20th October 2011, revised 26th November 2011, accepted 10th December 2011)

Abstract:
In this present work, a numerical model of hexagonal geometrical inclusion is developed to estimate the Effective Thermal Conductivity (ETC) of the two-phase materials taking the natural convection into account. A 2-D Constant Cross Section of unit cell model is analyzed with 1-D and 2-D heat transfer using Computational Fluid Dynamics (CFD) software. Numerical analysis has been carried out by considering primary effects (Conductivity ratio, concentration) and secondary effects (natural convection) to estimate ETC of two phase materials. The developed hexagonal geometrical model also shows excellent agreement with the available experimental and analytical data.

Keywords: Effective Thermal Conductivity (ETC), Hexagonal Inclusions, Unit-cell approach, Two-phase materials, Natural Convection.

Introduction
The need of ETC estimation of two-phase materials such as ceramics, soils, foams, porous materials, fiber reinforced materials and composites are becoming increasingly important in the technological development. Two phase material have different physical and chemical characteristics. The performance of the two-phase material is dependent on the ETC of the materials. As it is tedious to conduct experiments for studying the effect of the parameters like concentration of the dispersed phase, conductivity of the material of the particles, grain size and shape, contact area, radiations, pressure effect on ETC, a numerical model and theoretical expression is needed to predict the value of ETC. Various mechanisms of heat transfer is observed in two phase systems like solid-solid conduction at the point of contact, conductive heat transfer through the gas medium, radiation and convection in interstitial voids. Radiation is negligible at low temperatures and convective heat transfer can be neglected for Grash of Number < 2500¹. Many research are trying to evaluate ETC of two phase materials by analytical and experimental methods, which is time consuming and costly. So, numerical models which are easy and huge availability of resources attract the researchers to predict the ETC numerically. The minimum and maximum bounds for predicting the thermal conductivity of the two phase system has been developed by Maxwell and phase inverted Maxwell models. The ETC must lie between these bounds for all ranges of concentration and conductivity ratio². The upper and lower limits to the conductivity of two-phase materials based on parallel and series resistance are available³.

Effective thermal conductivity of two phase material will be affected by the particle contact as well as the secondary parameters like thermal radiation, pressure dependence, particle flattening, shape and size distribution for cylindrical unit cell containing spherical inclusions⁴. It is assumed in this model that the deformation of the flux field is taken only as a function of concentration, not as a function of the conductivity ratio is an important deficiency in this model⁴. The effective thermal conductivity can be obtained algebraically for number of porous media by applying the lumped parameter method based on an electric resistance analogy⁵. This method consists of choosing a unit cell, dividing the geometry under consideration into solid, fluid or composite layers (consisting of both fluid and solid), and assuming one-dimensional conduction in the direction of temperature gradient in the unit cell. A comprehensive conductivity model by considering the primary parameters based on unit cell and field solution approaches can be used to predict the ETC of various binary metallic mixtures with a high degree of accuracy⁶. Analytically effective thermal conductivity of two phase materials of various materials with different geometrical inclusions can be estimated⁶.

The collocated parameter model based on the unit cell approach is more effective in estimating the effective thermal conductivity of the two-phase materials⁷. Empirical correlations based on a finite element analysis can also be used to predict the thermal conductivity of Fiber Reinforced Composite Laminates (FRCL) and the other constituents⁸. In this model, FRCL is cured at high pressures to prevent air voids. The collocated parameter models⁷ are used to evaluate the ETC of the two-phase materials including the effect of...
various inclusions in the unit cell. The algebraic equations are derived using unit cell based isotherm approach for two dimensional spatially periodic medium. The geometry of the medium was considered as matrix of touching and non-touching in-line square, circular, hexagon and octagon cylinders. The effect of Darcy number, fluid Prandtl number, and thermal conductivity ratio on the Nusselt number affects the effective thermal conductivity of two phase materials. The study is conducted by using this parameters assume uniform porosity, hence $\phi = 0.4$ and $\alpha = 2.5^{10}$. The prediction of thermal conductivity of packed beds with randomly distributed inclusions or pores based on the solutions of Laplace’s heat conduction equation is also possible. The 3-D formulation provides a means of predicting the fiber thermal conductivity in both of the longitudinal and transverse directions. The transverse thermal conductivity of continuous reinforced composites containing a random fiber distribution with imperfect interfaces can be estimated using finite-element analysis. The three-dimensional numerical finite-difference methods are used to estimate the thermal conductivity of a composite with two or more constituents. This model helps to understand how the relative quantities and distributions of the component materials, within a sample, affect the whole sample conductivity. In this paper, a numerical model has been developed based on the isotherm unit cell approach, considering the effect of primary parameters such as concentration, conductivity ratio, contact ratio and secondary parameters such as natural convection for predicting the ETC of two-phase materials. The validation of the model is carried by compared with available literature experimental data and conventional models for various types of two-phase materials.

Numerical Modeling for Hexagonal geometrical Inclusions Based on Primary (concentration, conductivity ratio) and Secondary Parameters (as natural convection) to Estimate ETC of Two-Phase Materials

The development of numerical model for estimating the ETC based on the material and structure is extremely important for thermal design and analysis of two-phase systems. The resistance method is basic for the formulation of numerical model. The main feature of the method is to assume one-dimensional heat conduction and two-dimensional heat conduction in a unit cell. The unit cell is divided into three parallel layers, namely, solids, fluid or composite layers shown in Fig. 1 for hexagonal inclusion. The ETC equation based on the analytical method for hexagon cylinder is as follows.

**Figure-1**

Two-dimensional spatially periodic two-phase system
(a) Hexagon cylinder, (b) Unit cell of Hexagon cylinder for 1-D heat transfer, (c) Unit cell of Hexagon cylinder for 2-D heat transfer
The assumption in this work is that the two phase media is stagnant and thermal conductivity is considered to be stagnant thermal conductivity. The Business approximation is applied in the numerical modeling to take density difference into account. Two phase medium is considered to be isotropic, saturated with single phase fluid and the fluid is always in local thermal equilibrium with the solid matrix. All the calculations in the present study have been carried out using a commercial solver called FLUENT by considering the equation-2.1.

**Boundary Conditions:** Wall boundary conditions:
The velocity components at the wall are set to a no slip boundary condition. The components of velocity (u, v) are zero on the impermeable wall initially at time t=0.

\[ u, v = 0: \quad \text{...(2.2)} \]

Surface of heater is subjected to a constant heat flux condition.

\[ q'' = \text{constant} \quad \text{...(2.3)} \]

Outer wall is subjected to adiabatic condition,

\[ \frac{\partial T}{n} = 0 \quad \text{...(2.4)} \]

Interface Conditions: The interface conditions between the fluid and solid medium using continuity in temperature, pressure, and heat flux.

**Grid independence Study and Validation test:** Grid independence tests were conducted for the case of a unit cell model of 2-D with 1D and 2D heat transfer analysis. Hexagonal cylinder with conductivity ratio \( \alpha = 20 \) and concentration \( \nu = 0.1 \), is taken for study of grid independence. In this regard, three iterations have been carried out for this case. The mesh size was changed from very coarse to very fine, (14400, 90000, and 146500) when performing the iterations. In each of these three runs, the average temperature throughout the length of wall was obtained and plotted as shown in figure-2. The results show that the variation of 14400 and 90000 cells is nearly 2.5%, whereas the variation between 90000 and 146500 cells is less than 1%. So the mesh size of 90000 was used in rest of the iterations. The validation test is carried out by simulating a simple non-linear 1D analysis along the thickness of the unit cell to obtain the temperature distribution. This solution agrees with the analytical result. The meshed surface of Hexagonal cylinder is shown in the figure-3.
Comparison between Numerical (with convection) and Analytical ETC/K, for Conductivity ratio $\alpha = 20$ for Hexagon cylinder

Comparison between Numerical (with convection) and Analytical ETC/K, for Conductivity ratio $\alpha = 100$ for
Results and Discussion

Influence of concentration for Hexagonal geometrical inclusions including the effect of Natural Convection: The effect of concentration (υ) on the non-dimensional thermal conductivity of two-dimensional geometry’s for conductivity ratio’s (α = 20, 100) have been investigated taking the effect of natural convection into account.

Hexagon cylinder: The hexagonal model lies between parallel and series lines for conductivity ratio (α) = 20,100 and contact ratio (λ) = 0 to 0.2 as shown in figure-4 and figure-5. It also lies between Maxwell and Inverted-Maxwell models. This shows that the numerical modeling have good agreement with analytical model. Hexagonal model is not applicable beyond υ = 0.65 and 0.8 respectively, due to anisotropic shape property. It is the limitation of the conductivity of two-dimensional geometry’s for conductivity ratio’s (υ = 20, 100) have been investigated taking the effect of natural convection into account.

From the tables 1-2, numerical estimated one-dimensional (1-D) and two-dimensional (2-D) ETC with convection are compared with the available literature experimental values for hexagon inclusions. For porous systems, 2-D heat transfer shows 11.92% of deviation when compared to 12.38% deviation of 1-D heat transfer, i.e. more volume of fluid in the porous system causes, natural convection to occur. So in this case 2-D heat transfer is preferred. For suspension systems, the percentage of deviation is less for 1-D heat transfer (8.18%) when compared to 2-D transfer (12.07%) due to high value of conductivity ratio.

Table-1
Comparison of results of numerical models with experimental data for hexagon cylinder porous system

<table>
<thead>
<tr>
<th>No.</th>
<th>Sample (solid/fluid phase)</th>
<th>k_s W/mK</th>
<th>k_f W/mK</th>
<th>α (k_s/k_f)</th>
<th>υ</th>
<th>k_exp W/mK</th>
<th>λ (c/a)</th>
<th>k_hex W/mK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Glass sphere/air</td>
<td>1.099</td>
<td>0.024</td>
<td>20.24</td>
<td>0.74</td>
<td>0.227</td>
<td>0.85</td>
<td>0.04</td>
</tr>
<tr>
<td>2</td>
<td>Glass sphere/air</td>
<td>1.099</td>
<td>0.024</td>
<td>20.24</td>
<td>0.74</td>
<td>0.227</td>
<td>0.85</td>
<td>0.04</td>
</tr>
<tr>
<td>3</td>
<td>Silica sphere/water</td>
<td>12.41</td>
<td>0.586</td>
<td>20.24</td>
<td>0.74</td>
<td>0.227</td>
<td>0.85</td>
<td>0.04</td>
</tr>
<tr>
<td>4</td>
<td>Stainless steel/ethanol</td>
<td>20.864</td>
<td>0.337</td>
<td>61.23</td>
<td>0.45</td>
<td>0.207</td>
<td>0.65</td>
<td>0.04</td>
</tr>
<tr>
<td>5</td>
<td>Glass sphere/iso-octane</td>
<td>1.064</td>
<td>1.044</td>
<td>7.368</td>
<td>0.57</td>
<td>0.406</td>
<td>0.65</td>
<td>0.04</td>
</tr>
<tr>
<td>6</td>
<td>Lead shots/helium</td>
<td>34.348</td>
<td>1.147</td>
<td>23.66</td>
<td>0.62</td>
<td>0.17</td>
<td>0.65</td>
<td>0.04</td>
</tr>
<tr>
<td>7</td>
<td>Lead shots/hydrogen</td>
<td>34.348</td>
<td>1.179</td>
<td>191.88</td>
<td>0.62</td>
<td>0.17</td>
<td>0.65</td>
<td>0.04</td>
</tr>
<tr>
<td>8</td>
<td>Lead shots/water</td>
<td>34.348</td>
<td>1.627</td>
<td>54.78</td>
<td>0.62</td>
<td>0.17</td>
<td>0.65</td>
<td>0.04</td>
</tr>
<tr>
<td>9</td>
<td>Zirconia powder/air</td>
<td>2.002</td>
<td>0.021</td>
<td>95.28</td>
<td>0.47</td>
<td>0.12</td>
<td>0.65</td>
<td>0.04</td>
</tr>
<tr>
<td>10</td>
<td>Lead/water</td>
<td>33.764</td>
<td>0.586</td>
<td>57.618</td>
<td>0.6</td>
<td>4.329</td>
<td>0.37</td>
<td>0.13</td>
</tr>
<tr>
<td>11</td>
<td>Zirconia powder/air</td>
<td>2.002</td>
<td>0.03</td>
<td>66.7</td>
<td>0.58</td>
<td>0.23</td>
<td>0.32</td>
<td>0.13</td>
</tr>
<tr>
<td>12</td>
<td>Zirconia powder/air</td>
<td>2.002</td>
<td>0.03</td>
<td>66.7</td>
<td>0.64</td>
<td>0.281</td>
<td>0.45</td>
<td>0.13</td>
</tr>
<tr>
<td>13</td>
<td>Zirconia powder/air</td>
<td>2.002</td>
<td>0.03</td>
<td>66.7</td>
<td>0.7</td>
<td>0.364</td>
<td>0.75</td>
<td>0.13</td>
</tr>
<tr>
<td>14</td>
<td>Glass beads/air</td>
<td>1.201</td>
<td>0.028</td>
<td>42.89</td>
<td>0.65</td>
<td>0.22</td>
<td>0.75</td>
<td>0.24</td>
</tr>
<tr>
<td>15</td>
<td>Glass beads/benzene</td>
<td>1.201</td>
<td>0.14</td>
<td>8.579</td>
<td>0.65</td>
<td>0.5</td>
<td>0.75</td>
<td>0.24</td>
</tr>
<tr>
<td>16</td>
<td>Quartz sand/water</td>
<td>5.003</td>
<td>0.62</td>
<td>8.069</td>
<td>0.676</td>
<td>2.331</td>
<td>0.75</td>
<td>0.24</td>
</tr>
<tr>
<td>17</td>
<td>Glass beads/air</td>
<td>1.091</td>
<td>0.029</td>
<td>37.621</td>
<td>0.6</td>
<td>0.18</td>
<td>0.75</td>
<td>0.24</td>
</tr>
<tr>
<td>18</td>
<td>Micro beads/air</td>
<td>1.046</td>
<td>0.026</td>
<td>40.231</td>
<td>0.65</td>
<td>0.193</td>
<td>0.75</td>
<td>0.24</td>
</tr>
<tr>
<td>19</td>
<td>Micro beads/soltrol</td>
<td>1.046</td>
<td>0.133</td>
<td>7.865</td>
<td>0.639</td>
<td>0.452</td>
<td>0.75</td>
<td>0.24</td>
</tr>
<tr>
<td>20</td>
<td>Wassau sand/n-heptane</td>
<td>8.374</td>
<td>0.129</td>
<td>64.915</td>
<td>0.485</td>
<td>0.722</td>
<td>0.75</td>
<td>0.24</td>
</tr>
<tr>
<td>21</td>
<td>Ottawa sand/helium</td>
<td>8.374</td>
<td>0.147</td>
<td>56.996</td>
<td>0.64</td>
<td>1.323</td>
<td>0.75</td>
<td>0.24</td>
</tr>
<tr>
<td>22</td>
<td>Wassau sand/helium</td>
<td>8.374</td>
<td>0.147</td>
<td>56.996</td>
<td>0.41</td>
<td>0.598</td>
<td>0.75</td>
<td>0.24</td>
</tr>
<tr>
<td>23</td>
<td>Miami silt loam/air</td>
<td>2.932</td>
<td>0.023</td>
<td>127.48</td>
<td>0.458</td>
<td>0.169</td>
<td>0.75</td>
<td>0.24</td>
</tr>
<tr>
<td>24</td>
<td>Miami silt loam/air</td>
<td>2.932</td>
<td>0.023</td>
<td>127.48</td>
<td>0.552</td>
<td>0.221</td>
<td>0.75</td>
<td>0.24</td>
</tr>
<tr>
<td>25</td>
<td>Glass/air</td>
<td>1.13</td>
<td>0.026</td>
<td>43.462</td>
<td>0.6</td>
<td>0.176</td>
<td>0.75</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Average Deviation: 12.38 | 11.92
Table-2
Comparison of results of numerical models with experimental data for hexagon cylinder suspension system

<table>
<thead>
<tr>
<th>No.</th>
<th>Sample (solid/liquid phase)</th>
<th>$k_s$ W/m K</th>
<th>$k_f$ W/m K</th>
<th>$\alpha$ (k/k_s)</th>
<th>$\nu$</th>
<th>$k_{ex}$ W/m K</th>
<th>$\lambda_e$ (c/a)</th>
<th>$b_{ex}$ W/m K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Graphite/Water</td>
<td>160.03</td>
<td>0.66</td>
<td>241.01</td>
<td>0.05</td>
<td>0.83</td>
<td>0.009</td>
<td>0.92</td>
</tr>
<tr>
<td>2</td>
<td>Graphite/Water</td>
<td>160.03</td>
<td>0.66</td>
<td>241.01</td>
<td>0.11</td>
<td>1.13</td>
<td>0.02</td>
<td>1.26</td>
</tr>
<tr>
<td>3</td>
<td>Graphite/Water</td>
<td>160.03</td>
<td>0.66</td>
<td>241.01</td>
<td>0.17</td>
<td>1.44</td>
<td>0.02</td>
<td>1.52</td>
</tr>
<tr>
<td>4</td>
<td>Graphite/Water</td>
<td>160.03</td>
<td>0.66</td>
<td>241.01</td>
<td>0.24</td>
<td>1.92</td>
<td>0.02</td>
<td>2.05</td>
</tr>
<tr>
<td>5</td>
<td>Selenium/polypropylene glycol</td>
<td>5.19</td>
<td>0.14</td>
<td>37.09</td>
<td>0.1</td>
<td>0.18</td>
<td>0.03</td>
<td>0.19</td>
</tr>
<tr>
<td>6</td>
<td>Selenium/polypropylene glycol</td>
<td>5.19</td>
<td>0.14</td>
<td>37.09</td>
<td>0.2</td>
<td>0.22</td>
<td>0.03</td>
<td>0.25</td>
</tr>
<tr>
<td>7</td>
<td>Selenium/polypropylene glycol</td>
<td>5.19</td>
<td>0.14</td>
<td>37.09</td>
<td>0.3</td>
<td>0.32</td>
<td>0.1</td>
<td>0.36</td>
</tr>
<tr>
<td>8</td>
<td>Selenium/polypropylene glycol</td>
<td>5.19</td>
<td>0.14</td>
<td>37.09</td>
<td>0.4</td>
<td>0.42</td>
<td>0.13</td>
<td>0.45</td>
</tr>
<tr>
<td>9</td>
<td>Aluminum/water</td>
<td>204.24</td>
<td>0.66</td>
<td>310.86</td>
<td>0.06</td>
<td>0.76</td>
<td>0.005</td>
<td>0.77</td>
</tr>
<tr>
<td>10</td>
<td>Aluminum/water</td>
<td>204.24</td>
<td>0.66</td>
<td>310.86</td>
<td>0.12</td>
<td>0.97</td>
<td>0.007</td>
<td>1.05</td>
</tr>
<tr>
<td>11</td>
<td>Aluminum/water</td>
<td>204.24</td>
<td>0.66</td>
<td>310.86</td>
<td>0.18</td>
<td>1.4</td>
<td>0.007</td>
<td>1.32</td>
</tr>
<tr>
<td>12</td>
<td>Aluminum/water</td>
<td>204.24</td>
<td>0.66</td>
<td>310.86</td>
<td>0.21</td>
<td>1.81</td>
<td>0.02</td>
<td>2.02</td>
</tr>
<tr>
<td>13</td>
<td>Graphite/Water</td>
<td>190.94</td>
<td>0.56</td>
<td>286.37</td>
<td>0.16</td>
<td>1.19</td>
<td>0.02</td>
<td>1.24</td>
</tr>
<tr>
<td>14</td>
<td>Zinc sulphate/lard</td>
<td>0.61</td>
<td>0.2</td>
<td>3.11</td>
<td>0.38</td>
<td>0.31</td>
<td>0.001</td>
<td>0.34</td>
</tr>
<tr>
<td>15</td>
<td>Zinc sulphate/lard</td>
<td>0.61</td>
<td>0.2</td>
<td>3.11</td>
<td>0.56</td>
<td>0.35</td>
<td>0.001</td>
<td>0.38</td>
</tr>
<tr>
<td>16</td>
<td>Marble/Vaseline</td>
<td>2.98</td>
<td>0.19</td>
<td>16.09</td>
<td>0.6</td>
<td>0.75</td>
<td>0.002</td>
<td>0.79</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subscripts:</th>
<th>eff Effective, exp Experimental, num Numerical, hex Hexagon</th>
</tr>
</thead>
</table>

**Conclusion**

The effects of concentration, conductivity ratio on the ETC of hexagon geometrical inclusions have been investigated with the effect of natural convection. The numerical models are also compared with available experimental data. The results show that for the porous system, 2-D heat flow is more accurate than 1-D heat flow to estimate ETC. In emulsion system, 1-D heat flow is better than 2-D heat flow.

**Nomenclature:** α Length of the square, hexagon and octagon cylinders or Diameter of the circular cylinder, A Wall area (m²), ε Width Of the connecting plate in the square, circular, hexagon and octagon cylinders, Cp Specific heat Capacity, kJ/kg K, CCSwc Constant Cross Section with convection, CR Contact Ratio, Devi Deviation, ETC (A) Analytical Effective Thermal Conductivity (W/m K), ETC (E) Empirical Effective Thermal Conductivity (W/m K), ETC (NWC) Numerical Effective Thermal Conductivity with Natural Convection (W/m K), $k_{eff}$ Effective thermal conductivity of two-phase materials, (W/m K), $k_f$ Fluid or continuous thermal conductivity, (W/m K), $k_s$ Solid or dispersed thermal conductivity, (W/m K), $K_{eff}$ Equivalent thermal conductivity of a composite layer, (W/m K), l Length of the unit cell, (m), $\eta$ Adiabatic Index $\gamma'$ Heat Flux, (W/m²), T Temperature, (K), u X component of velocity (m/s), v Y component of velocity (m/s), WC With Natural Convection.

**Greek Symbols:** $\alpha$ Conductivity ratio (k/k_s), $\varepsilon$ Length Ratio (a/l), $\Phi$ Ratio of equivalent thermal conductivity of a composite layer to the fluid or continuous thermal conductivity ($k_{eff}$ / $k_f$), $\lambda$ Contact ratio (c/a), $\nu$ Concentration, $\zeta$ Connecting plate height to the length of the unit cell in the circular cylinder (6/l)

**References**


