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Empirical Correlation of Various Inclusions on the Effect of Primary and Secondary Parameters for Estimation of Effective Thermal Conductivity (ETC) of Two Phase Materials

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Abstract:

In this present work, an Empirical Correlation is developed to estimate the Effective Thermal Conductivity (ETC) of the two-phase materials for various inclusion shapes based on unit cell. The coefficients of the correlation can be given as functions of concentration and contact ratio. The validity of the correlation is verified through Finite Element Analysis (FEA).

Keywords: Effective thermal conductivity (ETC); Concentration; Conductivity ratio; Unit-cell approach; Inclusions; Two-phase materials

Introduction

Estimation of effective thermal conductivity of two phase materials (ceramics, solids, foams, emulsion systems, porous and suspension system, solid-solid mixtures, fibre reinforced materials and composites) becomes increasingly important in microelectronic cooling, space craft industries, catalytic reactors, heat recovery process, heat exchangers, solar collectors and nuclear reactors. Numerous models were developed to find out the effective conductivity of the mixtures, but one of the major limitations of the models is its suitability for specific applications. The prediction of thermal conductivity of Fiber Reinforced Composite Laminates (FRCL) and the other constituents can be obtained by the correlation based on FEA[1]. In this model, FRCL is cured at high pressures to prevent air voids. The transverse thermal conductivities of unidirectional fiber composites with and without thermal barrier can be estimated analytically based on the electrical analogy technique and on the cylindrical filament-square packing array unit cell model (C-S model)[2]. A practical two-dimensional (2-D) thermal circuit method based on series and parallel arrangement are used to estimate effective thermal conductivity. But, this model does not tackle the issue of FRCL structures[3]. A 3-D finite element model gives estimation of effective thermal conductivity more accurately[4]. The three dimensional (3-D) series and parallel thermal resistance network can be used to estimate effective thermal conductivity which gives better result compared to 2-D model[5]. Using numerical modeling the thermal conductivity and mechanical properties of carbon-fiber reinforced composites with different fiber cross section can also be estimated[6]. The thermal diffusivity of unidirectional fiber-reinforced composites (UFRC) material can be determined by using modulated photo-thermal techniques[7]. Numerical trials on ideal composite having perfect bond between fiber and matrix are conducted under four different boundary condition combinations representing the periodicity temperature field[8]. The computer programs are available to help designers to find the optimum solution of materials and structure to produce composites of a specific effective thermal conductivity. This computer program includes package of calculation modules corresponding to the analytical solutions for different structure models[9]. Analytically effective thermal conductivity of two phase material of various geometrical inclusions can also be determined[10].

Finite Element Model, Heat Transfer analysis and Boundary Condition for Various Inclusions:

Finite Element model: The unit cell of various inclusion shapes (square, hexagon, octagon and circular cylinder) has been modeled parametrically by using the commercial software. The numerical modeling has been carried out for various inclusions by considering the primary (conductivity ratio, concentration) and secondary parameters (contact resistance, various shapes). A thermal solid element SOLID 90 is used for discretization of the unit cell. The element has 20 nodes with a single degree of freedom, temperature, at each node. The 20-node elements have compatible for different shapes and suitable for curved boundaries.

Boundary Condition: One face of the unit cell is subjected to constant temperature and the opposite face is subjected to convective thermal environment. All other faces are kept as adiabatic in order to achieve 1D heat transfer. The boundary condition imposed on the unit cell is shown in the figure-1.

Convergent test and software validation has been carried out. Heat transfer analysis has been performed and the average surface temperature on the convection wall of the unit cell is obtained. Once the temperature of the convective side is known, the effective thermal conductivity across the two walls can be calculated by using the following simple heat balance equation.

$$h \times A \times (T_{wall2} - T_{conv}) = \frac{k_{eff} \times A \times (T_{wall1} - T_{wall2})}{L} \quad \text{……..1}$$

Several simulations were done for a wide spectrum of possible variation in the concentrations, conductivity ratios and contact ratios for all inclusion shapes and shown in tables-1,2,3, and 4.

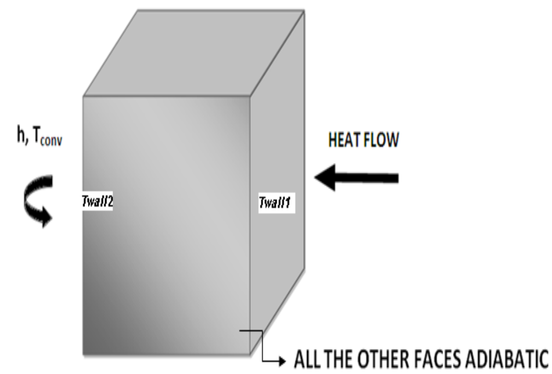


Figure-1
Thermal boundary condition applied on the unit cell

Table-1
Simulation results of the Effective Thermal Conductivity for Square cylinder

λ	A	ν = 0.1	ν = 0.2	ν = 0.3	ν = 0.4	ν = 0.5	ν = 0.6
0	5	1.201778	1.466074	1.8150715	2.290078	2.965074	3.986394
	20	1.280179	1.685639	2.29327752	3.279881	5.12193	9.70284
	100	1.306659	1.765363	2.48497521	3.73944	6.412145	15.83542
	500	1.31231	1.782847	2.52842986	3.849435	6.756749	18.13638
	800	1.312849	1.784516	2.53260367	3.860114	6.790996	18.38704
	1000	1.313029	1.785073	2.53399885	3.863688	6.802491	18.47215
0.01	5	1.213951	1.486188	1.8434394	2.325745	3.003256	4.010572
	20	1.372144	1.856056	2.56947863	3.700776	5.728391	10.36853
	100	1.83812	2.796546	4.23091077	6.57824	11.13546	24.1649
	500	4.015592	7.113782	11.5848924	18.73687	32.52185	71.30988
	800	5.640492	10.33841	17.0628297	27.77054	48.34486	105.5779
	1000	6.723128	12.48745	20.7126899	33.78849	58.88239	128.3634
0.02	5	1.224585	1.503991	1.86779031	2.355059	3.031571	4.024297
	20	1.452161	2.004831	2.80044534	4.029141	6.140903	10.70829
	100	2.306054	3.702855	5.70100501	8.81208	14.39701	28.33002
	500	6.423321	11.83965	19.319491	30.69963	50.71558	97.7523
	800	9.503488	17.93078	29.5018811	47.04344	77.76775	148.9182
	1000	11.55642	21.99095	36.2884888	57.93563	95.79312	182.9796
0.05	5	1.252162	1.549905	1.92906019	2.425026	3.093794	4.050006
	20	1.656623	2.382104	3.36717354	4.785992	7.011983	11.32403
	100	3.494756	6.003068	9.29639301	13.9541	21.20913	35.49742
	500	12.53715	23.82024	38.2444176	58.21942	88.44829	142.5382
	800	19.31205	37.17437	59.9317514	91.36546	138.7436	222.1734
	1000	23.82825	46.07669	74.3888234	113.4602	172.267	275.2291
0.07	5	1.267579	1.575782	1.96316752	2.462603	3.12563	4.062136
	20	1.769303	2.59242	3.67688146	5.183367	7.446029	11.6093
	100	4.145397	7.275603	11.2540549	16.63656	24.54091	38.69688
	500	15.87906	30.43705	48.5261692	72.5339	106.7879	162.1418
	800	24.6727	47.80047	76.460501	114.4119	168.3603	254.1851
	1000	30.53476	59.37571	95.0824155	142.3283	209.4028	315.5184
0.1	5	1.287024	1.609373	2.00719767	2.510683	3.165495	4.077301
	20	1.909667	2.862458	4.07220227	5.684796	7.980285	11.96091
	100	4.950309	8.903665	13.7416907	19.99747	28.58428	42.51662
	500	20.00757	38.89103	61.5783468	90.43011	128.9601	185.3914
	800	31.29408	61.37518	97.4397663	143.2178	204.1503	292.1089
	1000	38.81807	76.3643	121.346592	178.4078	254.2726	363.2309

Table - 2
Simulation results of the Effective Thermal Conductivity for Hexagon cylinder

λ	A	ν = 0.1	ν = 0.2	ν = 0.3	ν = 0.4	ν = 0.5	ν = 0.6
0	5	1.129474	1.285332	1.473142604	1.703532	1.992481	2.363975
	20	1.178269	1.407197	1.706005288	2.111725	2.691111	3.576791
	100	1.194685	1.450186	1.793167623	2.277042	3.004818	4.206151
	500	1.198233	1.459557	1.812566178	2.314772	3.079082	4.363825
	800	1.198568	1.460449	1.814418103	2.318399	3.08628	4.379297
	1000	1.19868	1.460747	1.815036858	2.319611	3.088688	4.384482
0.01	5	1.135693	1.295331	1.486886327	1.721293	2.013906	2.387907
	20	1.224361	1.487453	1.826781717	2.285068	2.92956	3.892979
	100	1.459542	1.929599	2.540677742	3.388728	4.613478	6.56538
	500	2.53184	3.92988	5.693291108	8.110305	11.58006	17.22927
	800	3.327214	5.422943	8.043027385	11.62233	16.75494	25.13483
	1000	3.85644	6.417944	9.60871369	13.96212	20.20234	30.40059
0.02	5	1.141101	1.304277	1.499347709	1.737108	2.032794	2.40836
	20	1.264669	1.558879	1.9345538	2.435734	3.131522	4.153329
	100	1.694665	2.361485	3.211153033	4.357678	5.978796	8.49127
	500	3.73128	6.178197	9.204339388	13.21959	18.87034	27.7158
	800	5.249012	9.034354	13.6862317	19.84062	28.49416	42.05134
	1000	6.259917	10.93814	16.67349377	24.25333	34.90779	51.60422
0.05	5	1.155943	1.328834	1.533158564	1.779582	2.082228	2.460363
	20	1.373183	1.751956	2.221652303	2.829129	3.643024	4.784396
	100	2.318808	3.519679	4.986331999	6.868706	9.406565	13.05047
	500	6.91614	12.19014	18.4856421	26.45262	37.13317	52.43219
	800	10.35439	18.68731	28.60000309	41.1204	57.89206	81.90158
	1000	12.64587	23.01848	35.34241442	50.89794	71.72956	101.5446
0.07	5	1.164701	1.343515	1.553271486	1.804525	2.110606	2.489561
	20	1.436162	1.865744	2.389542414	3.055151	3.928235	5.123558
	100	2.677723	4.19775	6.017094248	8.302213	11.29752	15.45534
	500	8.747305	15.69912	23.86638189	33.98871	47.16846	65.39288
	800	13.29001	24.32005	37.24408449	53.2361	74.04073	102.7881
	1000	16.31786	30.06708	46.16211963	66.06683	91.95403	127.7154
0.1	5	1.176292	1.36334	1.580462846	1.838075	2.148437	2.528085
	20	1.518701	2.017848	2.613653825	3.354591	4.301737	5.559461
	100	3.144896	5.098324	7.385011464	10.18934	13.75289	18.50112
	500	11.12951	20.35431	30.9891858	43.89301	60.18097	81.74957
	800	17.1088	31.7915	48.68480455	69.15623	94.97571	129.1385
	1000	21.09442	39.41605	60.48151661	85.99766	118.1709	160.7287

Table – 3
Simulation results of the Effective Thermal Conductivity for Octagon cylinder

λ	A	ν = 0.1	ν = 0.2	ν = 0.3	ν = 0.4	ν = 0.5	ν = 0.6
0	5	1.126626	1.27914476	1.462833	1.688205	1.972097	2.342012
	20	1.170767	1.389734639	1.674364	2.059713	2.614204	3.488804
	100	1.184876	1.42672524	1.749335	2.201878	2.887988	4.065337
	500	1.187853	1.434614333	1.765617	2.233577	2.951336	4.207859
	800	1.188134	1.435350647	1.767199	2.236597	2.95744	4.221785
	1000	1.188229	1.435613293	1.767687	2.237623	2.959496	4.22646
0.01	5	1.13163	1.287101358	1.473642	1.702283	1.989274	2.361882
	20	1.206837	1.452106498	1.767848	2.19651	2.808506	3.770193
	100	1.38592	1.788833905	2.315289	3.065336	4.185315	6.129169
	500	2.184179	3.276821172	4.6853	6.714157	9.745179	15.26846
	800	2.773205	4.384966597	6.450345	9.424547	13.86447	22.02474
	1000	3.164576	5.123229584	7.626369	11.22995	16.60786	26.52383
0.02	5	1.135742	1.293887682	1.483137	1.714486	2.003949	2.37756
	20	1.23739	1.506270986	1.850296	2.314295	2.972137	3.984993
	100	1.563264	2.114078871	2.825742	3.821144	5.290628	7.727589
	500	3.084185	4.965176708	7.355548	10.69422	15.62875	24.01096
	800	4.213877	7.09717435	10.7447	15.82969	23.3453	36.13371
	1000	4.965009	8.516855144	12.99862	19.24517	28.47548	44.21559
0.05	5	1.147319	1.313027261	1.509572	1.747865	2.04318	2.418478
	20	1.321634	1.656214471	2.075586	2.628485	3.392608	4.514493
	100	2.045117	3.007425417	4.211713	5.819778	8.101755	11.59962
	500	5.531328	9.591071299	14.59061	21.20645	30.60454	45.08342
	800	8.133771	14.52517015	22.3676	32.7273	47.41093	70.11674
	1000	9.868353	17.81030886	27.54851	40.4076	58.67182	86.82155
0.07	5	1.154362	1.32474183	1.525649	1.767906	2.066161	2.441712
	20	1.372226	1.747020528	2.210633	2.813269	3.631246	4.802731
	100	2.332206	3.545751436	5.038158	6.988736	9.6856	13.6781
	500	6.992104	12.37661015	18.8985	27.34538	39.01565	56.32857
	800	10.4751	18.99326771	29.2874	42.59709	61.00026	88.26215
	1000	12.797	23.40511406	36.20945	52.76274	75.6245	109.5423
0.1	5	1.164043	1.340934435	1.547858	1.79539	2.097139	2.472451
	20	1.441088	1.871482383	2.395265	3.062996	3.94646	5.172657
	100	2.721	4.28086934	6.162952	8.559585	11.76144	16.30444
	500	8.971889	16.17413254	24.75288	35.58923	50.01737	70.50344
	800	13.64976	25.08901667	38.68497	55.83733	78.68635	111.1119
	1000	16.7666	31.03131489	47.97754	69.35088	97.79204	138.1676

Vol. **1(1),** 22-32, Jan. **(2012)**

Table – 4
Simulation results of the Effective Thermal Conductivity for Circular cylinder

λ	A	ν = 0.1	ν = 0.2	ν = 0.3	ν = 0.4	ν = 0.5	ν = 0.6
0	5	1.126529	1.278865	1.4621957	1.687065	1.970425	2.340322
	20	1.17051	1.388925	1.6723531	2.055594	2.606727	3.478962
	100	1.184546	1.425638	1.7465802	2.195997	2.876522	4.048967
	500	1.187499	1.433466	1.7626892	2.227238	2.938856	4.189566
	800	1.187778	1.434209	1.764223	2.230228	2.944866	4.203311
	1000	1.187871	1.434457	1.7647352	2.231227	2.946876	4.207916
0.01	5	1.138058	1.298048	1.489436	1.722973	2.015517	2.392892
	20	1.252925	1.535645	1.8982915	2.385464	3.081939	4.157201
	100	1.651895	2.288483	3.1224643	4.283063	6.049802	9.054614
	500	3.532657	5.86917	8.90935	13.14262	19.72853	31.36158
	800	4.931408	8.547833	13.23899	19.76586	29.94786	48.01716
	1000	5.862705	10.33322	16.12489	24.18024	36.75874	59.11715
0.02	5	1.148133	1.315042	1.5134239	1.754315	2.053615	2.435089
	20	1.324165	1.663547	2.0924887	2.6621	3.457505	4.644418
	100	2.056338	3.045224	4.3076484	6.025144	8.529321	12.55038
	500	5.585652	9.78429	15.08797	22.28848	32.89489	50.29831
	800	8.222448	14.83333	23.165701	34.47185	51.14058	78.54433
	1000	9.97668	18.19773	28.548002	42.58718	63.29607	97.38054
0.05	5	1.173343	1.358813	1.5752294	1.833736	2.147723	2.53577
	20	1.498776	1.985798	2.5770841	3.329761	4.319577	5.68312
	100	3.038804	4.935448	7.2298639	10.16674	14.09247	19.69637
	500	10.58667	19.54093	30.274936	43.95216	62.26064	88.59472
	800	16.23747	30.49367	47.543161	69.27665	98.36569	140.2252
	1000	20.00378	37.78888	59.069281	86.16312	122.4347	174.6533
0.07	5	1.186747	1.383103	1.6099143	1.878115	2.199931	2.591227
	20	1.589684	2.160377	2.8402655	3.685841	4.767508	6.200985
	100	3.545175	5.949164	8.7981858	12.34083	16.91617	23.12076
	500	13.16375	24.76031	38.395692	55.28066	77.08917	106.7682
	800	20.369	38.86419	60.592284	87.48221	122.2041	169.4686
	1000	25.17034	48.26569	75.382984	108.9383	152.2754	211.2779
0.1	5	1.202578	1.413663	1.6544844	1.935713	2.268013	2.664396
	20	1.695086	2.376096	3.1706373	4.133892	5.326287	6.840122
	100	4.126892	7.190848	10.748451	15.04664	20.383	27.23581
	500	16.11899	31.13757	48.4774	69.34508	95.22248	128.5004
	800	25.10637	49.09638	76.77421	110.0697	151.3388	204.4193
	1000	31.09353	61.06327	95.630551	137.2046	188.7487	255.0404

Regression Analysis: Regression Analysis has been conducted to correlate the numerically predicted values of the Effective Thermal Conductivity (ETC) via the finite element simulation results. In this regression analysis, MATLAB software is used to calculate the constants. The constants has been obtained in the correlation between ETC and conductivity ratio as the function of concentration and contact ratio.

Empirical Correlation: Using the tables 1-4, a curve is plotted between Non dimensional thermal conductivity (ETC/K_f) and conductivity ratio. Regression analysis tool available in the MATLAB software is used for curve fitting. The regression analysis result shows that the relationship between Non dimensional thermal conductivity (ETC/K_f) and conductivity ratio for all inclusion shapes (square, hexagon, octagon and circular).

$$\left(\frac{ETC}{K_f}\right)_i = a_i \times (\alpha)^{b_i} + c_i \qquad \ldots\ldots 2$$

For six contact ratios with a minimum of coefficient of determination ($R^2 > 0.99$) value, the constants a_i's, b_i's and c_i's (i = 1, 2...6 representing different concentrations) for each contact ratios are functions of the concentrations. The empirical forms of these constants which give the corresponding value for a_i, b_i and c_i for each case of contact ratio are found to be of a polynomial form of degree 5 with $R^2 = 1$.

$$a_i = f_5 * \nu^5 + f_4 * \nu^4 + f_3 * \nu^3 + f_1 * \nu + f_0$$
$$b_i = g_5 * \nu^5 + g_4 * \nu^4 + g_3 * \nu^3 + g_2 * \nu^2 + g_1 * \nu + g_0$$
$$b_i = h_5 * \nu^5 + h_4 * \nu^4 + h_3 * \nu^3 + h_2 * \nu^2 + h_1 * \nu + h_0$$

Again, the constants f_i's (i = 0,1,2…5), g_i's (i = 0,1,2…5) and h_i's (i = 0,1,2…5) are function of the contact ratio (λ) present in the two phase system. The empirical form for these constants which combines these six different contact ratio values are found to be of fifth order polynomials with $R^2 = 1$.

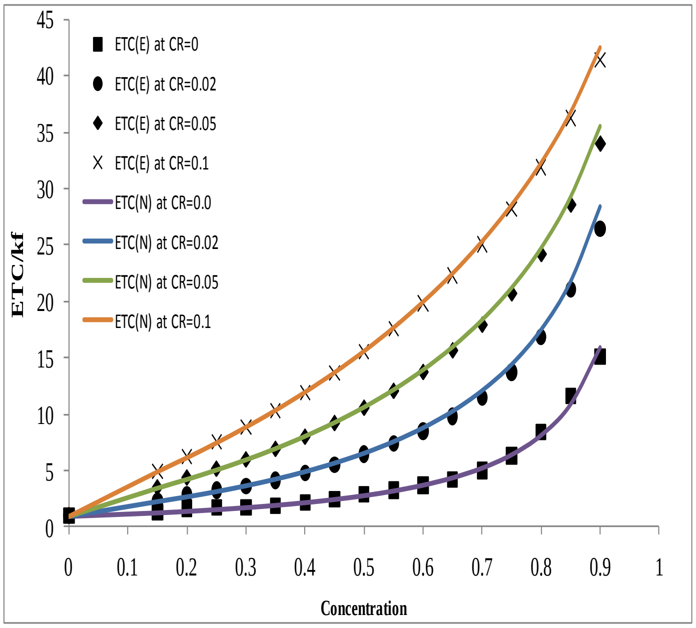
Finally, the general empirical expression which predicts the ratio of the ETC of the two phase system to the fluid conductivity (K_f) as a function of α, υ and λ is taking the following form:

$$(ETC/K_f) = \{f_5(\lambda)*\upsilon^5 + f_4(\lambda)*\upsilon^4 + f_3(\lambda)*\upsilon^3 + f_2(\lambda)*\upsilon^2 + f_1(\lambda)*\upsilon + f_0(\lambda)\}* (\alpha)^{\{g_5(\lambda)*\upsilon5+g_4(\lambda)*\upsilon4+g_3(\lambda)*\upsilon3+g_2(\lambda)*\upsilon2+g_1(\lambda)*\upsilon+g_0(\lambda)\}} + h_5(\lambda)*\upsilon^5+h_4(\lambda)*\upsilon^4+h_3(\lambda)*\upsilon^3+h_2(\lambda)*\upsilon^2+h_1(\lambda)*\upsilon+ h_0(\lambda) \quad \ldots\ldots 3$$

The value of the coefficients f_i's, g_i's and h_i's is different for each inclusion shapes.

Results and Discussion

Effect of concentration on ETC: Several analytical models were proposed to estimate the ETC of two-phase materials. But the series and parallel models are the minimum and maximum bounds for predicting the thermal conductivity of the two-phase system. The ETC must lie between these bounds for all ranges of concentration and conductivity ratio. These are the most restrictive bounds proposed and every model should incorporate these bounds as a minimum and maximum. Non-dimensional thermal conductivity of a two-phase system mainly depends on concentration, conductivity ratio, size, shape and thermal contact between solid-solid and solid-fluid interface. The effect of concentration (υ) on the non-dimensional thermal conductivity of two-dimensional (square, hexagon and octagon cylinders) shapes has been studied. The model has been compared with the standard models such as Analytical, Maxwell and Hashin-Shtrikman, parallel and series resistances respectively. The effect of concentration on non-dimensional thermal conductivity of two-dimensional spatially periodic two-phase material for various inclusions at different conductivity ratios is shown in figure-2 and figure-3 respectively. The ETC of the two-phase materials increases as the concentration increases. The numerically estimated ETC of the two-phase material lies between Parallel and Series lines for all conductivity ratios and contact ratio (λ) = 0 to 0.2. The advantage of numerical model is that it can be used effectively to predict the ETC for conductivity ratios less than one. The hexagon shaped inclusion model is applicable for concentration varying from 0 to 0.65. For concentrations above 0.65, the physical model is not possible because of the anisotropic shape property of the hexagon. Similarly the octagon shaped inclusion model is not possible for the concentrations above 0.8, because of the anisotropic shape.



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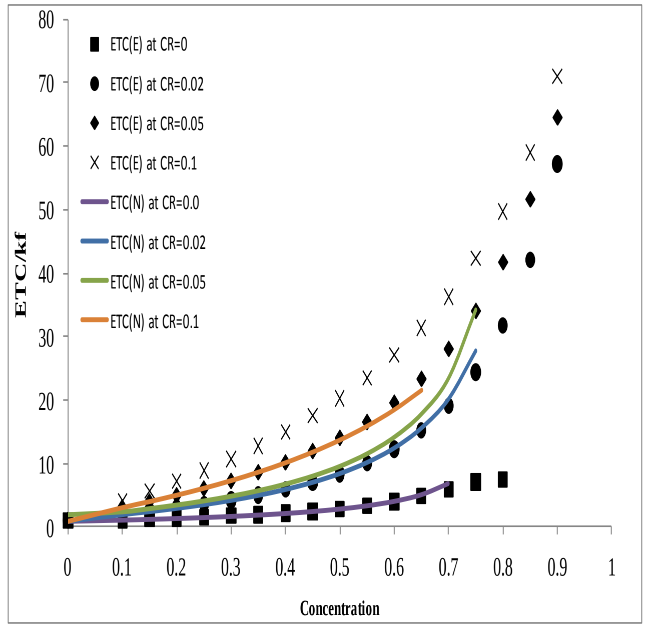

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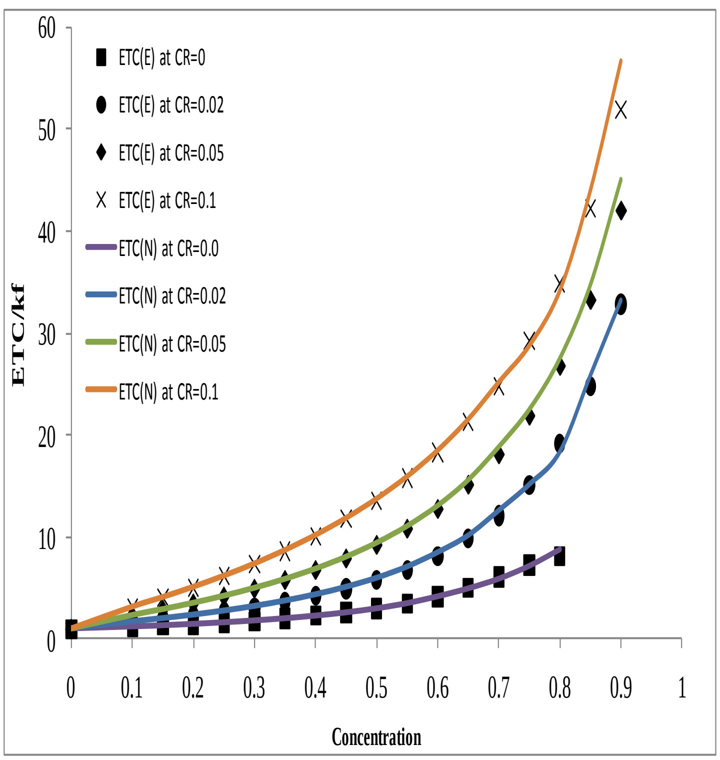

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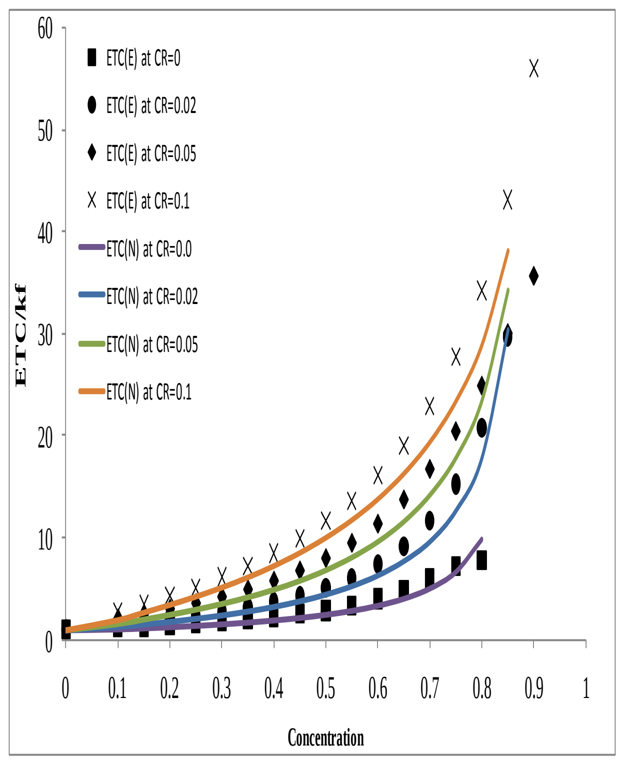
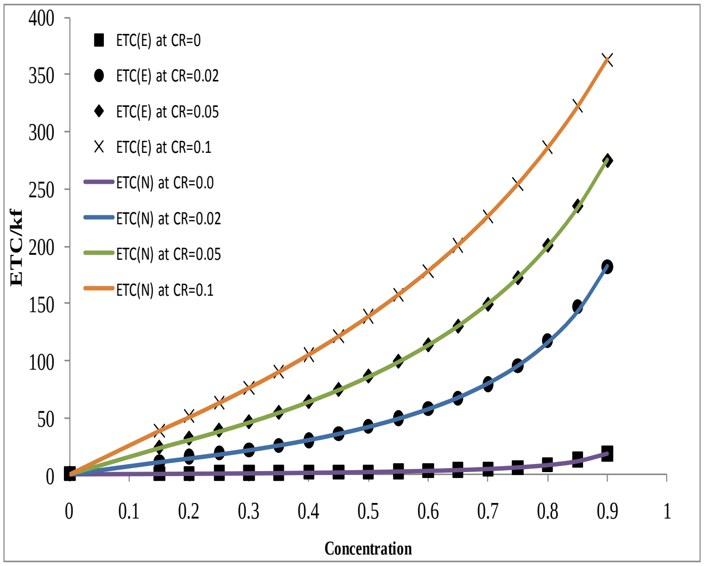


(d)

Figure- 2

Comparison between Numerical and Empirical ETC/K_f for Various Inclusions for Conductivity ratio = 100 (a) Square cylinder, (b) Circular cylinder, (c) Hexagon cylinder, (d) Octagon cylinder



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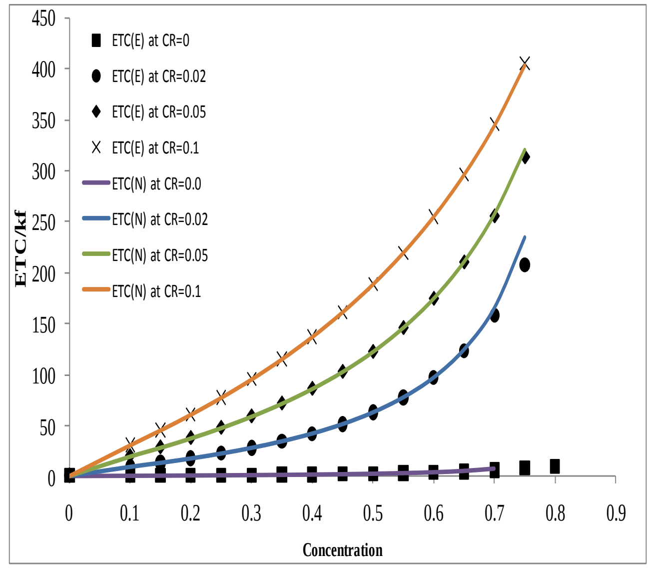

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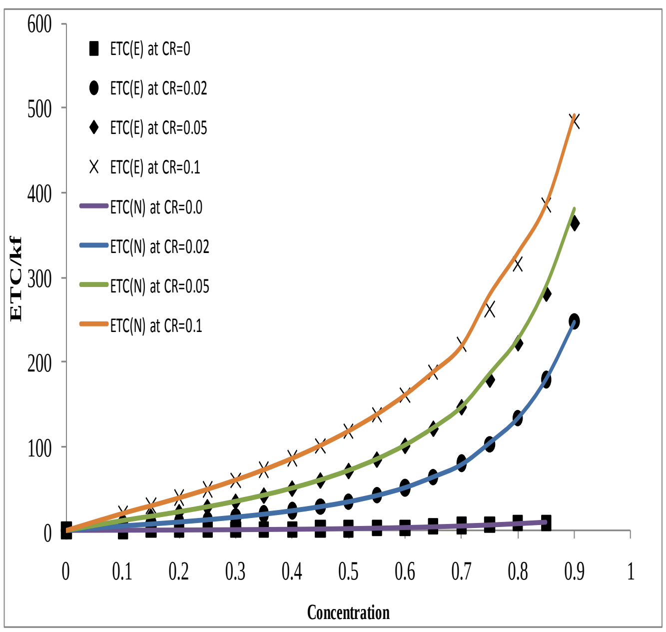

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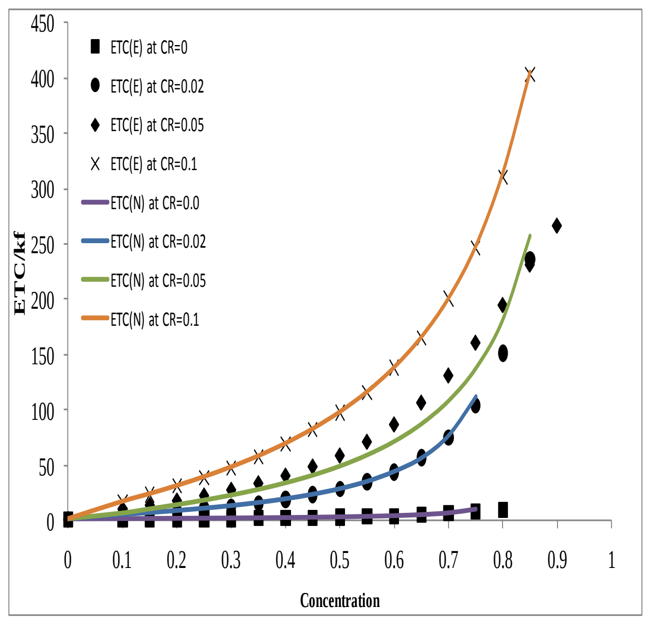
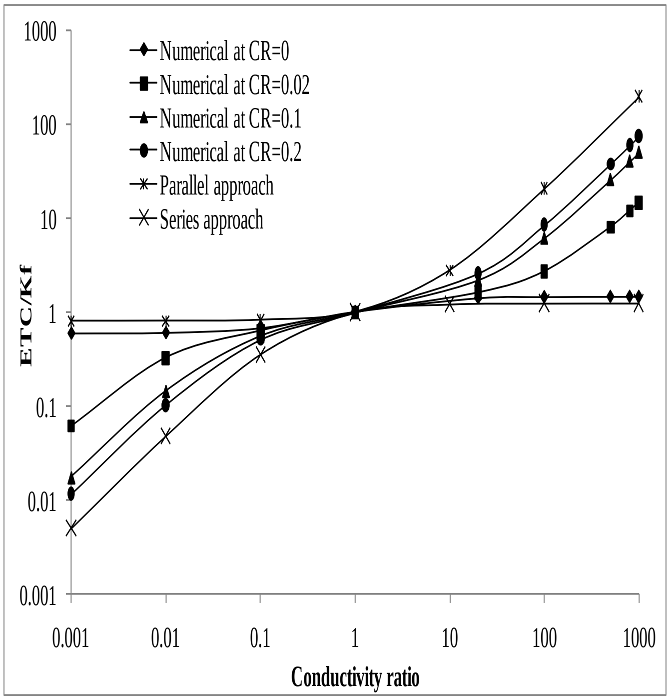


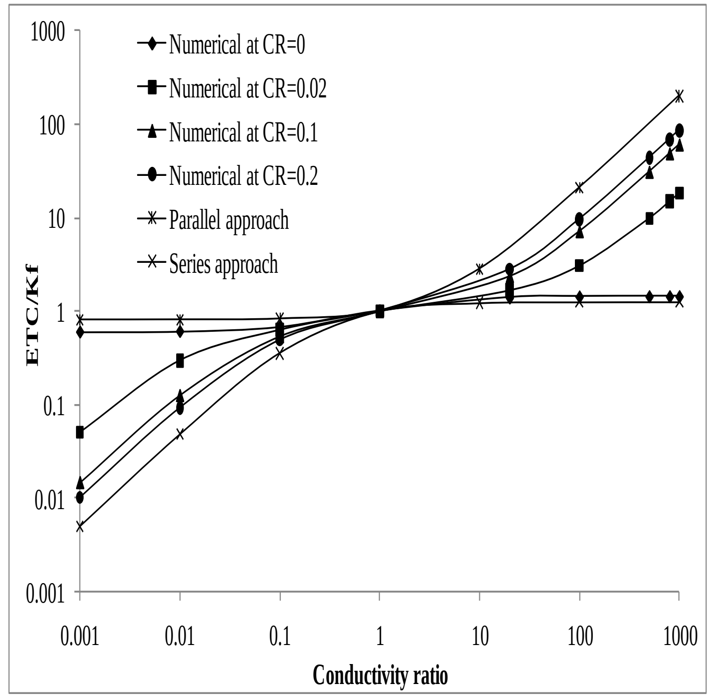
(d)

Figure-3
Comparison between Numerical and Empirical ETC/K_f for Various Inclusions for Conductivity ratio = 1000
(a) Square cylinder, (b) Circular cylinder, (c) Hexagon cylinder, (d) Octagon cylinder

Effect of conductivity ratio on ETC: The variation of non-dimensional thermal conductivity with conductivity ratio (α) for low (υ = 0.2), medium (υ = 0.4) and high concentration (υ = 0.6) of the two-dimensional spatially periodic medium for various contact ratios are respectively shown in figure-4 and figure-5. The predicted non-dimensional thermal conductivity increases with the conductivity ratio and contact ratios, for conductivity ratios greater than one. Similarly the predicted non-dimensional thermal conductivity decreases with the decrease in conductivity ratio and increase in contact ratios, for conductivity ratios lesser than one. For lower (υ = 0.2) and medium (υ = 0.4) concentrations, the deviation between all four inclusion models is negligible.



(a)



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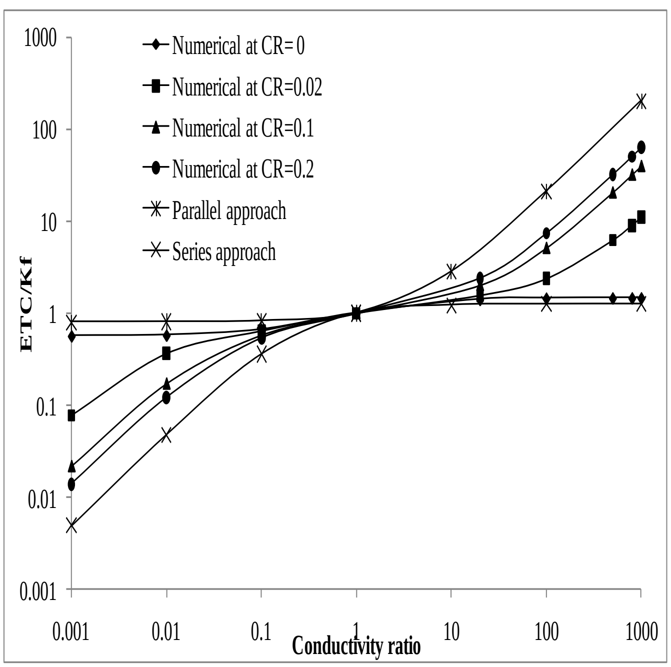

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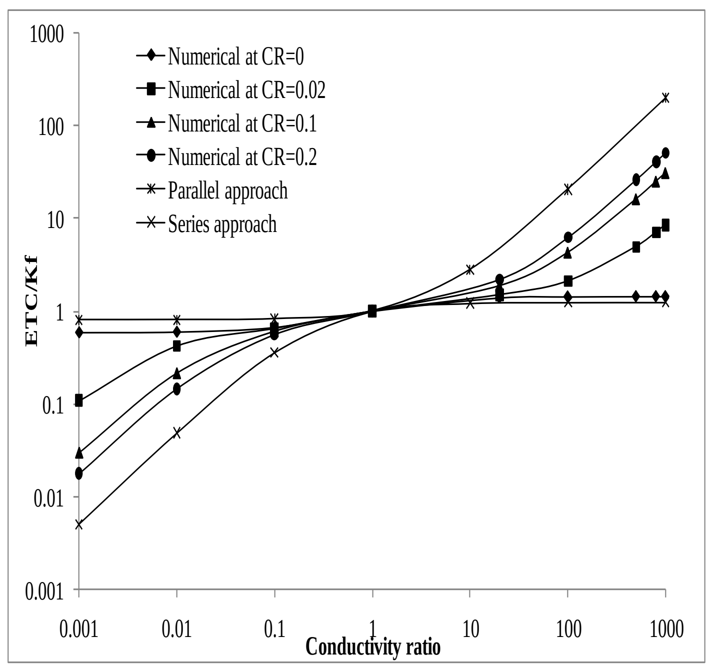

(d)

Figure – 4
Effect of conductivity ratio on non-dimensional thermal conductivity for concentration = 0.2 for (a) Square cylinder, (b) Circular cylinder, (c) Hexagon cylinder, (d) Octagon cylinder

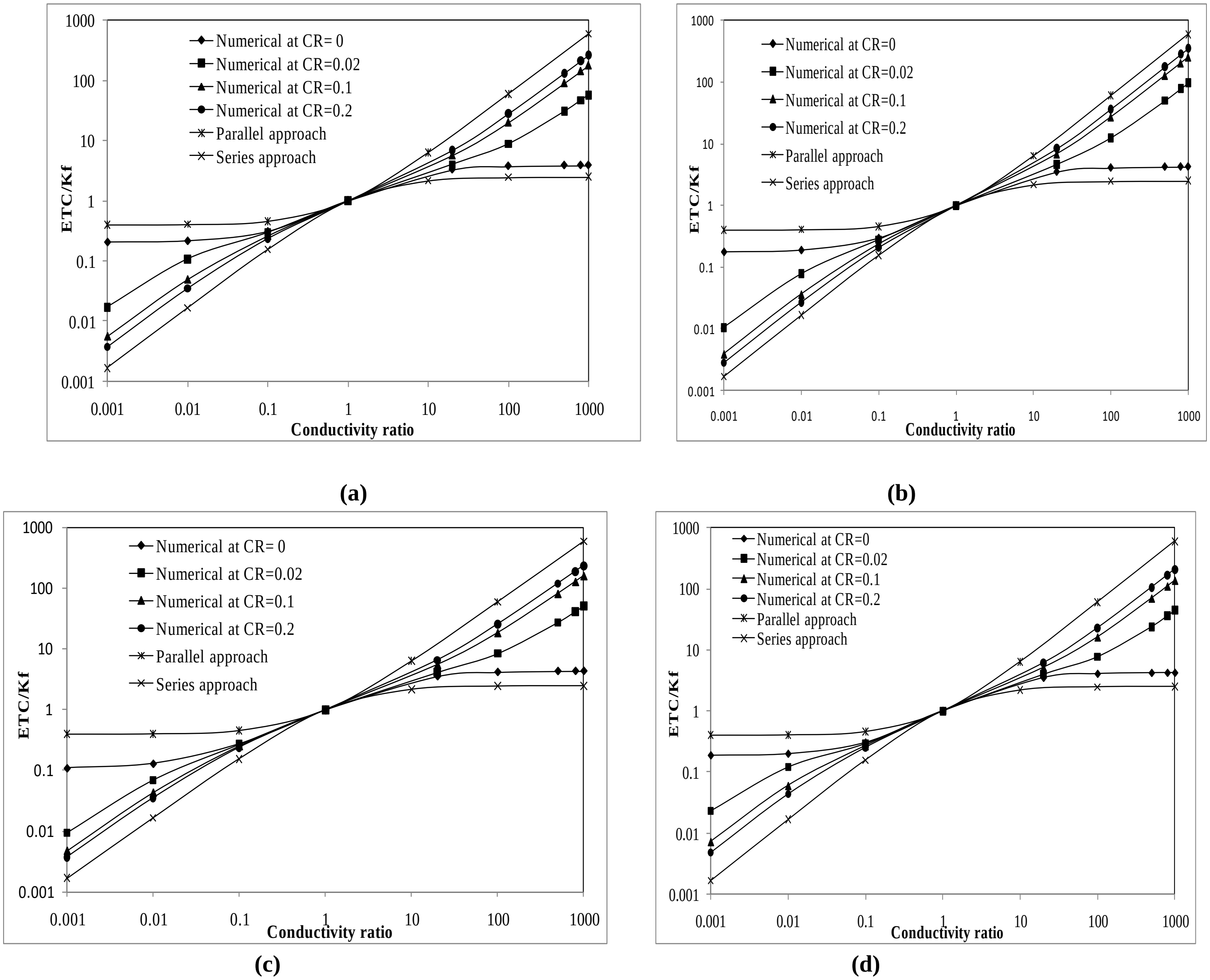

Figure – 5
Effect of conductivity ratio on non-dimensional thermal conductivity for concentration = 0.6 for (a) Square cylinder, (b) Circular cylinder, (c) Hexagon cylinder, (d) Octagon cylinder

Prediction of ETC for conductivity ratios lesser than one: The developed empirical correlation (equation-3) cannot be used to predict the effective thermal conductivity when conductivity ratio is less than one ($\alpha < 1$), because the numerical results of the effective thermal conductivity at $\alpha < 1$ is not considered in the regression analysis. The reason for that is the effect of concentration and contact ratio on the effective thermal conductivity is opposite when $\alpha < 1$ i.e. the effective thermal conductivity increases with the increase in concentration and contact ratio for $\alpha > 1$, whereas for $\alpha < 1$ the effective thermal conductivity decreases with the increase in concentration and contact ratio. The $\log_{10}$ ETC value of parallel model (which is the upper limit for ETC) at α is equal to the $\log_{10}$ ETC value of series model (which is the lower limit for ETC) at $1/\alpha$, which means there exist a relationship between ETC_{α} and $ETC_{(1/\alpha)}$. Therefore it is sufficient to find out ETC at $\alpha > 1$, from which ETC at $\alpha < 1$ can be found out easily by using the following equation

$\log_{10} ETC_{(1/\alpha)} - \log_{10} ETC_{(\alpha)} = 0.06$ at concentration = 0.2 (4)

$- \log_{10} ETC_{(1/\alpha)} - \log_{10} ETC_{(\alpha)} = 0.04$ at concentration = 0.4 (5)

$- \log_{10} ETC_{(1/\alpha)} - \log_{10} ETC_{(\alpha)} = 0.03$ at concentration = 0.6 (6)

The above equations can be used effectively for all ranges of conductivity ratio and contact ratio.

Conclusion

A very simple empirical formula has been developed to predict the ETC of two-phase materials using the conductivity ratio, concentration and the contact ratio.

The simulation covers a wide range of conductivity ratio and a wide range of concentration. The accuracy of predicting the ETC using the developed correlation is very high ($R^2 > 0.99$). The correlation would be of direct use in industrial applications of two-phase materials due to its simplicity. The model is tested at different conductivity ratios (5 to 2500) and various concentrations (0.05 up to 0.95). The ETC of the two-phase materials can be accurately predicted throughout the spectrum using the simple correlation. The effect of concentration and conductivity ratio on the ETC of two phase materials has been studied and it is found from the numerical results that ETC increases with the increase in Concentration and conductivity ratio for conductivity ratios greater than one. But for conductivity ratios lesser than one, ETC decreases with the increase in concentration. The influence of concentration and conductivity ratio on contact ratio has been studied and it is found that for two-phase materials with higher concentration, the contact ratio is higher because of more interaction between dispersed particles. For higher conductivity ratio materials the contact ratio is found to be very less compared to lower thermal conductivity ratio materials, because the dispersed particles with very high thermal conductivity have the tendency to attract the heat flux lines towards it and very negligible contact between them is sufficient to transport the heat.

Nomenclature: A Wall area (m^2) , h Heat transfer coefficient ($W/m^2.K$),$T_{\mathbf{conv}}$ bulk temperature of the fluid at the convection side (K),$T_{\mathbf{wall1}}$ fixed wall temperature (k), $T_{\mathbf{wall2}}$convective wall temperature
(k)

Greek Symbols: α Conductivity ratio (k_s/k_f), υ Concentration, λ Contact ratio

Subscripts: ETC/K_f Non-dimensional thermal conductivity, ETC (A) Analytical Effective Thermal Conductivity (W/m K), ETC (N) Numerical Effective Thermal Conductivity (W/mK), ETC (E) Empirical Effective Thermal Conductivity (W/m K), ETC_α Effective Thermal Conductivity at α (W/m K), $ETC_{(1/\alpha)}$ Effective Thermal Conductivity at $1/\alpha$ (W/m K).

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