Analysis of an N-policy M/M/1 queueing system with server start up, unreliable server and balking

P. Jayachitra
Department of Mathematics, Hindusthan Institute of technology, Coimbatore, Tamil Nadu, India
jayaasasi_akshu@rediffmail.com

Available online at: www.isca.in, www.isca.me
Received 24th December 2017, revised 30th January 2018, accepted 5th February 2018

Abstract

This paper is concerned with the optimal control of a non reliable and removable server in an N-policy M/M/1 queueing system with server start up and balking. When the queue length reaches N (N≥1), the server is immediately turned on but is temporarily unavailable to serve the waiting units. The server needs some startup time before providing the service. It is assumed that server breakdowns according to a Poisson process and the repair time have an exponential distribution. Arriving customers may balk with a certain probability and may depart without getting service due to impatience. Lack of service occur where the server is busy or due to sudden breakdown. Explicit expression for the number of units in the system are obtained and also derived various system measures. The total expected cost function is developed to determine the optimal threshold N at a minimum cost. The sensitivity analysis is performed to examine the effect of various parameters in the system.

Keywords: N-policy, server breakdowns, M/M/1 queueing system, server startup, probability generating function, balking.

Introduction

Many practical queueing system especially those with balking and reneging have been widely applied to many real life problems such as the situations involving impatient telephone switch board customers, the hospital emergency room handling critical patients, the inventory system with storage of perishable goods, etc. Queueing system with balking, reneging or both have been studied by many researchers. Haight\(^1\) first considered an M/M/1 queue with balking. An M/M/1 queue with customers reneging was also proposed by Haight\(^2\). The combined effects of balking and reneging in an M/M/1/N queue have been investigated by Ancker and Gafarian\(^3,4\). Barre\(^5\) carried out one of the early work on reneging where he considered deterministic reneging with single server Markovian arrival and service rates. Abou-El-Ate and Harri\(^6\) considered multiple server queueing system M/M/C/N with balking and reneging. Wang and Chang\(^7\) extended this work to study an M/M/R/N queueing system with balking, reneging and server breakdowns. Multi-server Markovian queue system with balking and reneging was discussed by Choudhury and Medhi\(^8\). M/M/1 queueing system with the customers impatience in the form of random balking was analyzed by Manoharan and Jose\(^9\). This paper provides an adequate service for its customers with tolerable waiting, whenever customer impatience becomes sufficiently strong and customers leave before being served.

The concept of the N policy was first introduced by Yadin and Naor\(^10\). Wang\(^11\) first proposed a management policy for Markovian queueing systems under the N policy with server breakdowns. Wang\(^12\) and Wang et al.\(^13\) extended the model proposed by Wang\(^11\) to M/Ek/1 and M/H2/1 queueing systems, respectively. They developed the analytic closed form solutions and provided a sensitivity analysis. Jayachitra and James Albert\(^14\) presented an elaborated survey on various queueing models under N-policy. Jayachitra and James Albert\(^15\) discussed optimal control of an N-policy M/Ek/1 queueing system with server startup and breakdown. Also the authors\(^16,17\) studied an Erlangian model under server breakdowns and multiple vacations and provided a cost model to determine the optimal operating policy at minimum cost. However to the best of our knowledge for M/M/1 queueing system with server startup, N-policy, unreliable server and balking, there is no literature which takes customers impatience into consideration. This motivates us to study a queueing system with N-policy, server startup, breakdowns and balking.

Thus in the present paper we consider an M/M/1 queueing system with server start up, N-policy, unreliable server and balking. The model can be realized in many real life situations.

The article is organized as follows. A full description of the model is given in section 2. The steady state analysis of the system state probabilities is performed through the generating functions in section 3. Steady state results on the expected number of customers in different states of the server are given in section 4. Special cases of this model are discussed in section 5. In section 6 the characteristic features of the system are investigated. Optimal control policy is explained in section 7. In section 8 sensitivity analysis is carried out through numerical illustration. Finally in section 9 conclusions and future scope of the study are presented.
The main objectives of the analysis carried out in this paper for the optimal control policy are: i. To establish the steady state equations and obtain the steady state probability distribution of the number of customers in the system in each state. ii. To derive expressions for the expected number of customers in the system when the server is in idle, startup, busy and breakdown states respectively. iii. To formulate the total expected cost functions for the system, and determine the optimal value of the control parameter N. iv. To carry out sensitivity analysis on the optimal value of N and the minimum expected cost for various system parameters through numerical experiment.

**Model description**

For the purpose of analytical investigation, we consider the model with the following assumptions: i. The arrival is poisson process with parameter \( \lambda \). Arriving customers at the server form a single waiting line and is served in the order of their arrivals. The server may serve only one unit at a time and the service times are exponentially distributed with mean \( 1/\mu \). ii. The server may breakdown at any time with a Poisson breakdown rate \( \alpha \). The breakdown time is exponentially distributed with mean \( 1/\theta \). While the server is in operation and found to be broken down where \( n=1,2,3\ldots \).

The steady state equations are given as follows

\[
(\lambda b_1 + \theta) P_{0,n} = \lambda b_1 P_{0,n-1} \quad n > N
\]  
\[
(\lambda b_1 + \mu + \alpha) P_{1,1} = \mu P_{1,2} + \beta P_{2,1}
\]  
\[
(\lambda b_1 + \mu + \alpha) P_{1,n} = \lambda b_1 P_{1,n-1} + \mu P_{1,n+1} + \beta P_{2,n} \quad (2 \leq n \leq N-1)
\]  
\[
(\lambda b_1 + \mu + \alpha) P_{n,n} = \lambda b_1 P_{n,n-1} + \mu P_{n,n+1} + \beta P_{2,n} + \alpha P_{n,n} \quad (n \geq N)
\]

\[
(\lambda b_1 + \beta) P_{2,1} = \alpha P_{1,1}
\]  
\[
(\lambda b_1 + \beta) P_{2,n} = \lambda b_1 P_{2,n-1} + \alpha P_{1,n} \quad (n \geq 2)
\]

Solving equations (1),(2),(3) and (4) recursively, we finally get

\[
P_{0,n} = \begin{cases} 
P_{0,0}, & 1 \leq n \leq N-1 \\
R^{n-(N-1)} P_{0,0}, & n \geq N
\end{cases}
\]

where \( R = \frac{\lambda b_0}{\lambda b_1 + \theta} \).

**Probability generating function**: The technique of using the probability generating function may be applied in a recursive manner from equation (1) to (9) to obtain the analytic solution of \( P_{0,0} \) in neat closed form expression. Define the probability generating function of \( H_d(z) \), \( H_1(z) \), \( Q(z) \) and \( R(z) \) respectively as follows:

\[
H_d(z) = \sum_{n=0}^{N} z^n P_{0,n}
\]

\[
H_1(z) = \sum_{n=N}^{\infty} z^n P_{0,n}
\]

\[
Q(z) = \sum_{n=1}^{\infty} z^n P_{1,n}
\]

\[
R(z) = \sum_{n=1}^{\infty} z^n P_{2,n}
\]

Applying algebraic manipulation technique to equation (2) and using in equation (11), we get the following

\[
H_d(z) = \frac{1 - z^N}{1 - z} P_{0,0}
\]

Using equation (10) in equation (12) we get

\[
H_1(z) = \frac{Rz^N}{1 - Rz} P_{0,0}
\]
Multiplying equation (3) by \( z \) and (4) by \( z^n \) and summary over \( n \) we get

\[
H_1(z) = \frac{\lambda b_0 z}{\lambda b_1 (1 - z) + \theta} P_{0,0}
\]  
(17)

Equating (16) and (17) we get

\[
R_N^N = \frac{\lambda b_0 z^N}{1 - R(z)} P_{0,0}
\]  
(17a)

Multiplying equation (1) by \( z \) and (5),(6) and (7) by \( z^n \) and summary over \( n \) we get

\[
\left[ \lambda b_1 z^2 - \left( \lambda b_1 + \mu + \alpha \right) z + \mu \right] Q(z) + \beta z R(z) = \left[ \lambda b_0 z^2 - \frac{\theta R(z) N^+}{1 - R(z)} \right] P_{0,0}
\]  
(18)

Applying same procedure over the equations (8) and (9) to obtain the following equations

\[
R(z) = \frac{\alpha}{\lambda b_1 + \beta - \lambda b_1 z} Q(z)
\]  
(19)

Substituting equation (19) in equation (18) we get

\[
Q(z) = \left[ \lambda b_0 + \beta - \lambda b_1 z \right] \left[ \frac{\lambda b_1 z^2 - (\lambda b_1 + \mu + \alpha) z + \mu}{\lambda b_1 z^2 + (\lambda b_1 + \mu + \alpha) z + \mu} \right] \left[ \lambda b_0 z^2 - \frac{\theta R(z) N^+}{1 - R(z)} \right] P_{0,0}
\]  
(20)

**Probability that the empty system:** We evaluate the probability \( P_{0,0} \) using normalizing condition. For this purpose we evaluate \( H_0(1), H_1(1), Q(1) \) and \( R(1) \) from equations (15),(16),(20) and (19) respectively as

\[
H_0(1) = NP_{0,0}
\]  
(21)

\[
H_1(1) = \frac{\lambda b_0}{\theta} P_{0,0}
\]  
(22)

\[
Q(1) = \left[ \frac{\rho b_0 \left( N + \lambda b_0 \right)}{1 - \rho b_1 \left( 1 + \frac{\alpha}{\beta} \right)} \right] P_{0,0}
\]  
(23)

\[
R(1) = \frac{\alpha}{\beta} \left[ \frac{\rho b_0 \left( N + \lambda b_0 \right)}{1 - \rho b_1 \left( 1 + \frac{\alpha}{\beta} \right)} \right] P_{0,0}
\]  
(24)

Let \( G(Z) \) represent the number of customers in the system.

\[
G(Z) = H_0(z) + H_1(z) + Q(z) + R(z)
\]  
(25)

Now using the normalizing condition \( G(1) = 1 \), we obtain the value of probability

\[
P_{0,0} = \frac{1 - \rho b_1 \left( 1 + \frac{\alpha}{\beta} \right)}{\left( N + \lambda b_0 \right) - \rho b_1 \left( 1 + \frac{\alpha}{\beta} \right) (b_1 - b_0)}
\]  
(26)

Let \( P_0, P_1, P_2 \) and \( P_3 \) be the probability that the server is in idle, startup, busy and break down states respectively. Then,

\[
P_0 = H_0(1) \quad \text{(i)}
\]

\[
P_1 = H_1(1) \quad \text{(ii)}
\]

\[
P_2 = Q(1) \quad \text{(iii)}
\]

\[
P_3 = R(1) \quad \text{(iv)}
\]

**Expected number of customers in the system**

We define the expected number of customers in the system as follows.

\[
L_i = \text{the expected number of customers in the system when the server is idle.}
\]

\[
L_s = \text{the expected number of customers in the system when the server is doing pre service (start up work).}
\]

\[
L_w = \text{the expected number of customers in the system when the server is working.}
\]

\[
L_d = \text{the expected number of customers in the system when the server is broken down.}
\]

\[
L_N = \text{Expected number of customer in the system.}
\]

Using the probability generating functions expected number of units in the system at different states are presented below.

\[
L_i = H_0'(1) = \frac{N(N-1)}{2} P_{0,0}
\]  
(27)

\[
L_s = H_1'(1) = \frac{\lambda b_0}{\theta} \left[ N + \lambda b_0 \right] P_{0,0}
\]  
(28)

\[
L_w = Q'(1)
\]

\[
L_d = R'(1) = \frac{\alpha}{\beta} L_w + Q(1) \left( -\frac{\alpha b_0}{\beta} \right)
\]  
(30)

The expected number of customers in the system is given by

\[
L_N = H_0'(1) + H_1'(1) + Q'(1) + R'(z)
\]
Special Cases

In this section we present some existing results in the literature which are special cases of our model.

Case (i): If \( \Theta = \infty, \alpha = 0, \beta = \infty, b_1 = 1, b_0 = 1 \) and \( N = 1 \), our model represents the ordinary \( M/M/1 \) queueing system with reliable case. When \( N = 1, \Theta = \infty, \alpha = 0, b_1 = 1, b_0 = 1 \) and \( \beta = \infty \), expression (20) for \( Q(z) \) reduces to the special case stated in the expressions (2.14) of Gross and Harris \(^{18}\).

Case (ii): If \( \Theta = \infty, b_1 = 1 \) and \( b_0 = 1 \) our model represents the \( N \)-policy \( M/M/1 \) queueing system with unreliable case. When \( \Theta = \infty, b_1 = 1 \) and \( b_0 = 1 \) expression (20) for \( Q(z) \) and (19) for \( R(z) \) reduces to a special case of expression (18) and (19) of Wang \(^{11}\) respectively.

Case (iii): If \( \Theta = \infty, \alpha = 0, \beta = \infty, b_1 = 1, b_0 = 1 \) and \( N = 1 \), equation (26) reduces to the result of probability that the empty system in the ordinary \( M/M/1 \) queueing model.

Other System Characteristic

Let I, S, B, and D denote the length of the server off, startup, busy and breakdown periods respectively. Applying the memory less property of the Poisson process, we find that the mean length of the idle period is

\[
E(I) = \frac{N}{\lambda b_0}
\]

The expected length of the idle period, startup period, busy period and breakdown period are denoted by \( E(I) \), \( E(S) \), \( E(B) \) and \( E(D) \) respectively. The expected length of the busy cycle is given by

\[
E(C) = E(I) + E(S) + E(B) + E(D)
\]

From equations (21) to (24) we obtain the long run fraction of time for the server is idle, start up, busy and broken down respectively.

\[
\frac{1}{E(C)} = \frac{\lambda b_0}{P_{0}}
\]

Optimal Control policy

We develop the total expected cost function per unit time for the \( N \)-policy \( M/M/1 \) Queueing system with server startup, unreliable server and balking in which \( N \) is a decision variable. With the cost structure being constructed, the objective is to determine the optimal operating policy so as to minimize this function. Let

- \( C_h \) = holding cost per unit time for each customer present in the system.
- \( C_k \) = Cost per unit time for keeping the server idle.
- \( C_f \) = Set up cost per busy cycle.
- \( C_d \) = breakdown cost per unit time
- \( C_s \) = startup cost per unit time for preparatory work of the server before starting the service.

\[
E(I) = H_0(I) = NP_{0}\]

\[
E(S) = H_s(I) = \frac{\lambda b_0}{\theta} P_{0}
\]

\[
E(B) = Q(1) = \frac{\rho b_0}{1 - \left[ \frac{\rho}{\beta} \left( 1 + \frac{\alpha}{\beta} \right) (b_1 - b_0) \right]}
\]

\[
E(D) = R(1) = \frac{\alpha}{\beta} Q(1)
\]
$C_h =$ cost per unit time for the server keeping on and in operation.

$C_k =$ Cost per unit time when a customer balks.

\[ T_{\text{cost}}(N) = C_h N + C_0 \frac{E(I)}{E(C)} + C_s \frac{E(S)}{E(C)} + C_b \frac{E(B)}{E(C)} + C_d \frac{E(D)}{E(C)} \]  

(38)

\[ C_i \left[ \lambda (1-h_0) \frac{E(I)}{E(C)} + \lambda (1-h_1) \frac{E(S)}{E(C)} + \lambda (1-h_2) \frac{E(B)}{E(C)} + \lambda (1-h_3) \frac{E(D)}{E(C)} \right] \]

Using equations (31), (33)-(37), the results of (38) can be explicitly expressed which is a very long and complex formula for $T_{\text{cost}}(N)$ . We obtain the optimal value $N$ which minimizes the cost function $T_{\text{cost}}(N)$ , by differentiating it with respect to $N$ and setting the result to be zero.

i.e, \[ \frac{\partial}{\partial N} T_{\text{cost}}(N) = 0 \]  

(39)

The solution $N$ to (39) may not be an integer and the optimal positive integer value of $N$ is one of the integers surrounding $N^*$ which gives a smaller cost $T_{\text{cost}}$ . Here it should be pointed out explicitly that the solution really gives the minimum value and \[ \frac{\partial^2}{\partial N^2} T_{\text{cost}}(N) = 0 \] at $N=N^*$ is greater than zero when the values of system parameters satisfy suitable conditions. However it is quite tedious to present the explicit expression. Therefore we will perform the numerical illustrations to demonstrate that the function is really convex and the solution gives a minimum.

**Sensitivity Analysis**

In the course of analysis, sensitivity analysis has been carried out to find the optimum value of $N$ (ie., $N^*$) and the minimum cost based on changes in the system parameters using MATLAB. In order to arrive at the conclusions, the following arbitrary values of the system parameters are considered.

- Case 1: $C_h=5$, $C_i=10$, $C_b=200$, $C_i=125$, $C_f=500$, $C_k=10$
- Case 2: $C_h=5$, $C_i=20$, $C_b=200$, $C_i=250$, $C_i=500$, $C_k=15$
- Case 3: $C_h=5$, $C_i=50$, $C_b=400$, $C_i=350$, $C_i=500$, $C_i=1000$, $C_k=40$
- Case 4: $C_h=10$, $C_i=100$, $C_b=500$, $C_i=400$, $C_i=600$, $C_i=1000$, $C_k=70$
- Case 5: $C_h=50$, $C_i=100$, $C_b=500$, $C_i=500$, $C_i=800$, $C_i=1500$, $C_k=100$

**Table-1:** The optimal value of $N$ and its minimum expected cost for $\lambda = 0.3$, $\mu = 1.2$, $\beta = 3.0$, $b_0=0.4$, $b_1=0.2$

<table>
<thead>
<tr>
<th>Case</th>
<th>(Ω,α)</th>
<th>(0.4,0.2)</th>
<th>(0.6,0.2)</th>
<th>(0.8,0.2)</th>
<th>(0.5,0.1)</th>
<th>(0.5,0.2)</th>
<th>(0.5,0.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>N*</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>50.997</td>
<td>49.783</td>
<td>49.663</td>
<td>48.890</td>
<td>49.491</td>
<td>50.091</td>
</tr>
<tr>
<td>(ii)</td>
<td>N*</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>65.113</td>
<td>64.946</td>
<td>63.032</td>
<td>63.615</td>
<td>64.765</td>
<td>65.913</td>
</tr>
<tr>
<td>(iii)</td>
<td>N*</td>
<td>9</td>
<td>9</td>
<td>8</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>72.227</td>
<td>72.090</td>
<td>70.211</td>
<td>70.306</td>
<td>71.447</td>
<td>72.585</td>
</tr>
<tr>
<td>(iv)</td>
<td>N*</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>82.343</td>
<td>80.461</td>
<td>79.887</td>
<td>78.758</td>
<td>79.892</td>
<td>81.025</td>
</tr>
<tr>
<td>(v)</td>
<td>N*</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>94.192</td>
<td>90.872</td>
<td>90.662</td>
<td>90.780</td>
<td>91.895</td>
<td>93.009</td>
</tr>
</tbody>
</table>
### Table-2: The optimal value of N and its minimum expected cost for $\lambda=0.3$, $\Theta=0.2, \alpha=0.05, b_0=0.4, b_1=0.2$

<table>
<thead>
<tr>
<th>Case</th>
<th>$(\mu, \beta)$</th>
<th>$(1.0,3.0)$</th>
<th>$(1.2,3.0)$</th>
<th>$(1.4,3.0)$</th>
<th>$(1.4,2.0)$</th>
<th>$(1.4,4.0)$</th>
<th>$(1.4,6.0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>N*</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>55.168</td>
<td>53.874</td>
<td>52.946</td>
<td>53.956</td>
<td>53.846</td>
<td>53.843</td>
</tr>
<tr>
<td>(ii)</td>
<td>N*</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>71.432</td>
<td>69.442</td>
<td>67.972</td>
<td>69.649</td>
<td>69.321</td>
<td>69.249</td>
</tr>
<tr>
<td>(iii)</td>
<td>N*</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>11</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>80.470</td>
<td>77.616</td>
<td>75.547</td>
<td>75.786</td>
<td>75.428</td>
<td>75.309</td>
</tr>
<tr>
<td>(iv)</td>
<td>N*</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>86.724</td>
<td>83.940</td>
<td>81.927</td>
<td>82.163</td>
<td>81.809</td>
<td>80.611</td>
</tr>
<tr>
<td>(v)</td>
<td>N*</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>111.592</td>
<td>109.705</td>
<td>107.853</td>
<td>108.060</td>
<td>107.750</td>
<td>107.644</td>
</tr>
</tbody>
</table>

### Table-3: The optimal value of N and its minimum expected cost for $\beta=3, \mu=1.2, \alpha=0.05, b_0=0.4, b_1=0.2$.

<table>
<thead>
<tr>
<th>Case</th>
<th>$(\lambda, \Theta)$</th>
<th>$(0.2,0.4)$</th>
<th>$(0.4,0.4)$</th>
<th>$(0.6,0.4)$</th>
<th>$(0.4,0.1)$</th>
<th>$(0.4,0.3)$</th>
<th>$(0.4,0.5)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>N*</td>
<td>8</td>
<td>10</td>
<td>11</td>
<td>10</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>53.874</td>
<td>63.232</td>
<td>71.740</td>
<td>82.362</td>
<td>63.232</td>
<td>57.924</td>
</tr>
<tr>
<td>(ii)</td>
<td>N*</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>11</td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>69.422</td>
<td>80.445</td>
<td>90.525</td>
<td>101.934</td>
<td>80.449</td>
<td>74.375</td>
</tr>
<tr>
<td>(iii)</td>
<td>N*</td>
<td>10</td>
<td>11</td>
<td>15</td>
<td>13</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>77.766</td>
<td>90.811</td>
<td>102.790</td>
<td>115.349</td>
<td>90.811</td>
<td>83.666</td>
</tr>
<tr>
<td>(iv)</td>
<td>N*</td>
<td>9</td>
<td>11</td>
<td>12</td>
<td>11</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>84.059</td>
<td>97.910</td>
<td>110.699</td>
<td>121.593</td>
<td>97.980</td>
<td>91.623</td>
</tr>
<tr>
<td>(v)</td>
<td>N*</td>
<td>7</td>
<td>9</td>
<td>10</td>
<td>9</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>109.703</td>
<td>130.855</td>
<td>149.594</td>
<td>181.397</td>
<td>130.855</td>
<td>115.232</td>
</tr>
</tbody>
</table>

### Table-4: The optimal value of N and its minimum expected cost for $\lambda = 0.3, \mu = 1.2, \beta = 3.0, \alpha=0.05, \Theta=0.4$

<table>
<thead>
<tr>
<th>Case</th>
<th>$(b_0, b_1)$</th>
<th>$(0.4,0.3)$</th>
<th>$(0.45,0.3)$</th>
<th>$(0.5,0.3)$</th>
<th>$(0.4,0.2)$</th>
<th>$(0.4,0.3)$</th>
<th>$(0.4,0.4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>N*</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>48.928</td>
<td>51.203</td>
<td>53.450</td>
<td>49.779</td>
<td>48.928</td>
<td>47.092</td>
</tr>
<tr>
<td>(ii)</td>
<td>N*</td>
<td>5</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>73.580</td>
<td>77.015</td>
<td>80.399</td>
<td>74.236</td>
<td>73.580</td>
<td>72.949</td>
</tr>
<tr>
<td>(iii)</td>
<td>N*</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>8</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>138.820</td>
<td>144.435</td>
<td>149.959</td>
<td>139.197</td>
<td>134.820</td>
<td>130.484</td>
</tr>
<tr>
<td>(iv)</td>
<td>N*</td>
<td>6</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>220.275</td>
<td>228.966</td>
<td>237.548</td>
<td>229.445</td>
<td>220.275</td>
<td>211.158</td>
</tr>
<tr>
<td>(v)</td>
<td>N*</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{cost}}(N^*)$</td>
<td>427.812</td>
<td>461.951</td>
<td>495.795</td>
<td>435.099</td>
<td>427.812</td>
<td>420.686</td>
</tr>
</tbody>
</table>
The optimal value of $N$ and its minimum expected cost $T_{\text{cost}}(N^*)$ for the above five cases are shown in Table-1 for various values of ($\Theta, \alpha$). It is observed that: i. $T_{\text{cost}}(N^*)$ decreases as $\Theta$ increases and $T_{\text{cost}}(N^*)$ increases as $\alpha$ increases. ii. $N^*$ decreases as $\Theta$ increases. Intuitively $N^*$ is insensitive to the changes in $\alpha$.

The optimal value of $N$ and its minimum expected cost $T_{\text{cost}}(N^*)$ for the above five cases are shown in Table-2 for various values of ($\mu, \beta$). From Table-2 we find that i. $N^*$ increases when $\mu$ changes from 1.0 to 1.4. and $N^*$ remains unchanged when $\beta$ changes from 2.0 to 6.0. ii) $T_{\text{cost}}(N^*)$ decreases as $\mu$ and $\beta$ increases respectively. The optimal value of $N$ and its minimum expected cost $T_{\text{cost}}(N^*)$ for the above five cases are shown in table (3) for various values of ($\lambda, r$). From Table-3 we find that i. $N^*$ increases when $\lambda$ increases and $N^*$ decreases when $\Theta$ increases. ii. $T_{\text{cost}}(N^*)$ increases when $\lambda$ increases and $T_{\text{cost}}(N^*)$ decreases as $\Theta$ increases. From Table-4 we find that i. $N^*$ slightly increases when $b_0$ increases from 0.4 to 0.5 and $N^*$ slightly decreases when $b_1$ changes from 0.2 to 0.4. ii) $T_{\text{cost}}(N^*)$ increases when $b_0$ increases and $T_{\text{cost}}(N^*)$ decreases when $b_1$ increases.

Over all we conclude that: i. $\alpha$ and $\beta$ do not affect $N^*$, ii. $\Theta$, $b_0$ and $b_1$ rarely affect $N^*$. iii. $\lambda$ and $\mu$ affect $N^*$ significantly, iv. $C_h$ and $C_f$ have much stronger effect on $N^*$. 

![Figure-1: Arrival rate $\lambda$ Vs Total cost $T_c$ by varying service rate $\mu$.](image1)

![Figure-2: Service rate $\mu$ Vs Total cost $T_c$ by varying repair rate $\beta$.](image2)
Figure-1 displays the correlation between the arrival rate \( \lambda \) and the total cost \( T_c \) by varying service rate \( \mu \). We can observe that as the service rate goes on increasing, the total cost goes on decreasing. In Figure-2 the total cost goes on decreasing as we go on increasing the repair rate \( \beta \). In Figure-3 the total cost goes on increasing as we go on increasing the breakdown rate \( \alpha \).

**Conclusion**

Optimal strategy analysis of an N-policy M/M/1 queuing system with server startup, unreliable server and balk is studied. Some of the system performance measures have been derived. A cost function has been formulated to determine the optimal value of N. Sensitivity analysis has been carried out through numerical illustrations. These numerical values will be useful in analyzing practical queueing system and make decision The present study can be extended by considering that the service time, breakdown time and repair time follow general distribution.

**References**

