Heat Conduction and the $H$ - Function

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Abstract

This paper will put an insight into an application to certain products containing the $H$ - Function in boundary value problems. The results established in this paper are general in nature and hence encompass several cases of interest.


Introduction

The $H$ - Function is defined and represented in the following manner\(^1\).

$$H_{p,q}[z] = \frac{1}{2\pi i} \int_L \phi(\xi) d\xi (z \neq 0)$$

(1)

Where,

$$\phi(\xi) = \prod_{j=1}^{m} \Gamma(b_j - \beta_j \xi) \prod_{j=m+1}^{n} \Gamma(1 - a_j + \alpha_j \xi)^{A_j} \prod_{j=1}^{\alpha} \Gamma(1 - b_j + \beta_j \xi) \prod_{j=n+1}^{\beta} \Gamma(a_j - \alpha_j \xi)^{B_j}$$

The nature of contour $L$, sufficient conditions of convergence of defining integral (1) and the behavior and other details about the $H$ - Function can be seen in paper\(^1\)\(^-\)\(^5\).

The following formulas will be required in our investigation:

$$\frac{\pi}{2} \int_0^{\beta} \cos nx \, dx = \frac{\pi \cos \frac{n\pi}{2}}{2} \frac{\Gamma(\beta + n + 1)}{\Gamma(\beta + n + 1)}$$

(2)

Main Result

Theorem: Prove that

$$\prod_{j=1}^{m} \frac{(a_j, \alpha_j; A_j)_{1,n+1,p}}{(b_j, \beta_j; B_j)_{1,m+1,q}^{n+1,p}}$$

Proof of the Theorem: To obtain the result (3) we first express $H$ function occurring on the L.H.S. of equation (3) in terms of contour integral using equation (1) and Interchanging the order of integration and summation then we apply the formula (2) and interpret the resulting contour integral as $H$ function we arrive at the right hand side of (3) after a little simplification.

A Boundary Value Problem

We consider a problem on heat conduction in a square plate under certain boundary conditions. If a square plate has its faces and its edges $x = 0$ and $x = \pi$ ($0 < y < \pi$) insulated. Its edges $y = 0$ and $y = \pi$ ($0 < x < \pi$) are kept at temperature zero and $f(x)$ respectively, then its steady temperature $u(x,y)$ is given by Churchill\(^6\).

$$\frac{\pi}{2} \int_0^{\beta} \cos nx \, dx = \frac{\pi \cos \frac{n\pi}{2}}{2} \frac{\Gamma(\beta + n + 1)}{\Gamma(\beta + n + 1)}$$

(3)
Now, we shall consider the problem of determining $u(x, y)$, where

$$u(x, y) = \frac{a_0}{2\pi} y + \sum_{n=1}^{\infty} a_n \frac{\sinh xy}{\sinh nx} \cos nx$$  \hspace{1cm} (4)$$

$$a_n = \frac{2\pi}{\pi} f(x) \cos nx dx$$  \hspace{1cm} (5)$$

$n = 0, 1, 2, \ldots$

We shall consider the problem of determining $u(x, y)$, where

$$u(x, a) = f(x) = (\sin x)^p$$

Combining (6) and (7) and making the use of the integral (3), we derive

$$a_n = 2^{2-p} \cos \left(\frac{n\pi}{2}\right)$$

Putting the value of $a_n$ from (7) in (4), we get the following required solution of the problem

$$u(x, y) = \frac{a_0}{2\pi} y + \sum_{n=1}^{\infty} 2^{2-p} \frac{\cos\left(\frac{n\pi}{2}\right)}{\sinh nx} \cos nx$$

$$\frac{m}{n} \sum_{p=0}^{\infty} \left[ \left( \left( \frac{1}{2} \right)^{p} \right) \left( \left( \frac{1}{2} \right)^{q} \right) \right]$$

Provided the condition stated with (3) are satisfied.

**Conclusion**

It may be pointed out here that the $H$ function is very general in nature and a fruitful nature of $H$ has its particular cases a number of important functions can also be obtained but we do not record them here.

**References**


