ON Q-Fuzzy Ideal and Q-Fuzzy Quotient Near-Rings

Bhimraj Basumary and Gopi Kanta Barthakur

1Research Scholar, Department of Mathematical Sciences, Bodoland University, Kokrajhar, Assam, INDIA
2Department of Mathematics, G. K. B. College, Morigaon, Assam, INDIA

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Abstract

In this paper, we shall study Q-fuzzy ideal and Q-fuzzy quotient near-ring and investigate some of there properties and we prove some characterizations of a near-ring in terms of Q-fuzzy quotient near-ring and Q-fuzzy ideal.

Keywords: Q-fuzzy ideal, Q-fuzzy quotient near-ring.

Introduction

Zadeh introduced fuzzy set in 1965. The idea of the fuzzy ideal in near-ring is discussed by Zaid. Solarairaju et al. introduce the new structures of Q-fuzzy groups. On the other hand Muhammad Akram introduces the 'T-fuzzy Ideals and quotient near-ring. In this paper, we shall study quotient near-rings via Q-fuzzy ideals and study some of their properties. Generally in this work we follow a paper published by Muhammad Akram to prove theorems.

Preliminaries

Definition: A near-ring is a set R which is non empty with two binary operation “+” and “.” Which holds the conditions, (R, +) is group, (R, .) is semi group, and multiplicative is distributive with respect to addition.

Definition: Let us consider a non empty set A. Then a function µ : R → [0, 1] is called Q-level in A. For t in [0, 1] the set µt = { x | µ(x) ≥ t } is called Q-level subset of A.

Definition: A function µ : G×Q→[0, 1] is called Q-fuzzy set in G, where Q be a set and G be group respectively.

Definition: Consider a function f from a set A to B and a Q-fuzzy set µ in A. Then µ is a Q-fuzzy subset of B defined by

f(µ)(y, q) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu(x, q) : f^{-1}(y) \neq \emptyset \\ 0 : otherwise \end{cases}

Definition: Let Im(λ) denote the image set of λ. Let λ be a Q-fuzzy set in a set R. For t in [0, 1] the set λt = { x | xR, q∈Q; λ(x, Q) ≥ t } is called Q-level subset of λ.

Definition: Consider µ a Q-fuzzy set in a near-ring R, then µ is Q-fuzzy sub near-ring of R if it holds the conditions

1. µ(x–y, q) ≥ µ(x, q)
2. µ(xy, q) ≥ µ(y, q)
3. µ((x+z)y–xy, q) ≥ µ(z, q)

Definition: A Q-fuzzy subnear-ring µ in R is called Q-fuzzy ideal if

1. µ(x+y, q) ≥ µ(x, q)
2. µ(xy, q) ≥ µ(y, q)
3. µ((x+z)y–xy, q) ≥ µ(z, q)

Theorem: If we consider a onto homomorphism f : A → B of near-rings, Consider µ be a Q- fuzzy ideal in A, we get, then a Q- fuzzy ideal (f(µ)) in B.

Proof: Consider v, w be two elements in the set B. Since f is onto homomorphism, then as Muhammad Akram we are clear to show

{ b – c | b, c ∈ (v), c ∈ (w) } is subset of { x | x f (v–w) }.

Now as definition 1.9 of f(µ)(x, q) we have

f(µ)(v–w, q) = \sup_{x \in f^{-1}(v–w)} \mu(x, q)

≥ \sup_{b \in f^{-1}(v)} \mu(b – c, q)

≥ \min \{ \sup_{b \in f^{-1}(v)} \mu(b, q), \sup_{c \in f^{-1}(w)} \mu(c, q) \}

= \min \{ f(µ)(v, q), f(µ)(w, q) \}

Now following definition 1.9

f(µ)(vw, q) = \sup_{x \in f^{-1}(vw)} \mu(x, q)

≥ \sup_{b \in f^{-1}(v)} \mu(bc, q)

≥ \min \{ \sup_{b \in f^{-1}(v)} \mu(b, q), \sup_{c \in f^{-1}(w)} \mu(c, q) \}

f(µ) is Q-fuzzy sub near-ring.

Now also we have

f(µ)(v+w–v, q) = \sup_{x \in f^{-1}(v+w–v)} \mu(x, q)
Consider (x+A), (y+A) be two elements of R/A, now following definition 2.4 and from definition 2.6 we are clear to show
Ψ((x+A), q) = sup_{x \in R/A} \mu(x, q) is a \emph{Q-fuzzy} \textsuperscript{3} ideal \textsuperscript{6} of R/A.

Since, we have \mu(0, q) = \mu(s, q).

Also from definition \textsuperscript{2.12} \mu(a+s, q) \geq \mu(a, q).

Thus, we have \mu(a+s, q) = \mu(a, q), for all s \in A.

Then, we have \mu(a+s, q) = \mu(a, q), for all s \in A.

Hence the corresponding \mu \mapsto \Psi is one to one.

Let \Psi be \emph{Q-fuzzy} \textsuperscript{3} ideal \textsuperscript{6} of R/A. Consider \mu as a \emph{Q-fuzzy} \textsuperscript{3} set in R so that for all “a” in A \mu(a, q) is equal to \Psi(a+A, q).

Now, for x, y \in R, we have from definition 2.6 and from theorems 3.1 and 3.2 it follows
\mu(x, y, q) = \Psi((xy)+A, q) 
= \Psi((x+\Lambda)-(y+A), q) 
\geq \min \{ \Psi((x+A), q), \Psi((y+A), q) \} 
= \min \{ \Psi((x+\Lambda)-(y+\Lambda), q) \}

Thus \mu is \emph{Q-fuzzy} \textsuperscript{3} ideal \textsuperscript{6} of R. Clearly \mu(a, q) is equal to \Psi(a+A, q) which equal to \Psi(A, q), for all a in A. This indicates that \mu(0, q) is equal to \mu(s, q) for all s \in A.

\textbf{Theorem 3.4} Let us consider A be an \emph{ideal} \textsuperscript{6} of a near-ring \textsuperscript{5} R.

We can have then a \emph{Q-fuzzy} \textsuperscript{3} ideal \textsuperscript{6} of R so that \mu(0, a) is t and \lambda \textsuperscript{3} is called Q-level \textsuperscript{1} subset of \lambda.

\textbf{Proof:} Following definition 2.6 and theorems 3.1, 3.2, 3.3 the proof is straight forward \textsuperscript{2}.

\textbf{Theorem 3.5} Consider a \emph{Q-fuzzy} \textsuperscript{3} ideal \textsuperscript{6} \mu of a near-ring \textsuperscript{5} R also \mu(0, a) is t.

Then \Psi is a \emph{Q-fuzzy} \textsuperscript{3} ideal \textsuperscript{6} of R/\lambda \textsuperscript{3}, where \Psi is constructed as \Psi(x+\lambda \textsuperscript{3}, q) = \mu(x, q) for all x \in R and \lambda \textsuperscript{3} is called Q-level \textsuperscript{1} subset of \lambda.

\textbf{Proof:} Similarly following definition 2.6 and theorems 3.1, 3.2, 3.3 and 3.4 proof is straight forward \textsuperscript{3}.

\textbf{Conclusion}

In this paper, we have defined Q-fuzzy subnear-ring, Q-fuzzy ideal. With the help of Q-fuzzy subnear-ring and Q-fuzzy ideal, we have discussed on Q-fuzzy quotient near-ring and proved some theorems on Q-fuzzy quotient near-ring. We hope that this work will help for further work of fuzzy set.

\textbf{References}