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Review Paper

A survey on fractional order PID controller

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Abstract

There are a numerous authentic vibrant systems which are enhanced by considering a non-integer system which is related to the fractional calculus. Integer order differentiation and integration form the basis of previous calculation. The system representation using the method of fractional calculus is an influential instrument that has changed the view of the system modeling. A distinguish and numerous research related to fractional order controllers application in different areas of engineering and science, risen to various study perspectives of analysis, design, tuning and implementation of the fractional order controllers. The distinguish characteristic of fractional order control is that it is a generalization of classical control theory. FOPID controllers are more ample than the previously used IOPID controllers. FOPID controllers are comprehensively used by various technocrats to accomplish the most vigorous recital of the models. Fractional order controllers provide two extra parameters for tuning than the classical PID controllers, which enhance the overall performance of the system. The FOPID controllers are less receptive to the uncertainty of the parameter which may exist in the controller & controlled system.

Keywords: Fractional calculus, FOPID controller, FOPID tuning, IOPID, FOTF and FOC.

Introduction

In modern era, fractional order systems have engrossed lots of awareness in different fields and have amplified the interest in analysis and implementation of the non-integer order controllers. Most of the non-integer order controllers outperform the classical integer order controllers. The applications where FOC has been used broadly can be found in plentitude in the control system literature[1]. These mathematical phenomenon let detail explanation of an object more precisely than the classical integer order methods. Previously integer order models were used because the solution methods for the fractional calculus were not available, but nowadays there are numerous solutions for estimation of fractional derivative, integral and therefore fractional calculus is useful for various applications[2]. The fractional order is an abstract notion, can be realized only through approximate estimation. Control system theory indulges both fractional order controller and fractional order dynamic system which has to be controlled, but commonly in practice is fractional order controller because plant models are generally integer order models. The main aim of using fractional order models is to accomplish a robust performance of controller despite the presence of the qualms of the plant model, high frequency noise and load disturbances[3].

In most of the controllers proper tuning is very critical and it is very intricate and tricky task. In case of FOPID controllers five parameters have to be tuned for enhancement of system performance. Many modern researchers have planned several methods for tuning the $PI^{\lambda}D^{\mu}$ controllers. A few software tools are also available using which approximation, optimizing and tuning of FOPID controllers can be conveniently done. Valerio and Sa da Costa have proposed a Ziegler Nichols type empirical rule for tuning of controllers[17].

The paper is organized as follows. Section 2 describes the history and development of FOPID. Section 3 describes the basic essential definitions of fractional order calculus Section 4 states objective of research work. Section 5 briefs description of software tools used in implementation and analysis of fractional order controllers. In section 6 and 7 conclusion and references are discussed.

History and Devolvement of Fractional Order Controller

Fractional calculus aims at further improvement of the conventional process of derivation and integration to include other than integer orders. The initial impetus to the concept of the non-integer differentiation was given by L' Hospital in a letter written to Leibnitzin in 1695, from then onwards numerous well-known mathematicians such as Laplace, Euler, Fourier, Abel, and Laurent efficiently working on the concept of fractional calculus. In the nineteenth century with the help of Liouville, Grünwald, Letnikov and Riemann entire theory appropriate for current mathematical development has been distinguished. At present, fractional calculus is an unshakable theory with well framed mathematical foundation. The major cause for the dispersion of fractional calculus is that, this tool describes more exactly about physical systems[13]. It was Prof. Oustaloup who firstly introduced the fractional-order Controllers (FOC). He urbanized the three diverse version of the CRONE controller such that first, second and third generation controllers were developed[20]. I. Podlubny presented the first report on fractional order PID controller[15]. He coined a concept of fractional order $PI^{\lambda}D^{\mu}$ controller which is the extension of the classical PID controllers, where a PID controller structure with an integrator of order λ and a differentiator of order μ was introduced. The $PI^{\lambda}D^{\mu}$ controller is extra bendy and provides better prospects to alter the dynamic properties of fractional order systems in contrast with the classical PID controller[7].

In 1995 and 1996 J. Machado proposed algorithms to adopt the time domain which befits for z transform analysis and digital implementation. This study represents a first stage towards the development of motion control systems depend on the theory of FDI'S (Fractional derivatives and integrals). In 1961, Manabe introduced the frequency and transient response of the non-integer integral and its application to control systems and then after by Barbosa, Tenreiro and Ferreira in 2003. A frequency domain approach was also studied by Vinagre et al[4] by using fractional-order PID controllers. Several approximation methods for continuous models and discrete models were studied and compared these methods in both time and frequency domains for implementing fractional order controllers by fractional order operators[4]. An optimization method to tune the FOPID controller has been used in such a manner that predefined design specifications are fulfilled. Further research activities led to the progress of new successful tuning techniques for fractional order controllers by conservatory of the classical control theory.

These references[11, 20] provide a more bendable tuning plan using which the desired controls with respects to classical controllers can be achieved easily. Based on precise phase and gain margins with a minimum integral squared error (ISE) criterion an optimal FOPID controller is designed[5], a novel approximation method is projected which is extra accurate in the low and high frequency range compared with the well-established Oustaloup's approximation method for a fractional order differentiator. In reference[17], first and second sets Ziegler-Nichols tuning rules for FOPID controllers were proposed. Another approach is achieved in reference[10] optimization method to tune the controller and auto-tuning method for FOPID controller using the relay test has been proposed.

A fractional order controller was implemented by using PSO method[5]. Optimal problems of FOPID are solved by genetic algorithms (GA) and can be obtained for better results with PSO in contrast with conventional methods. A similar approach has been adopted for tuning of FOPID controller using integral performance index criteria with PSO. In 2010 New tuning methods for FOPID and set point weighting of FOPID are proposed [24]. Application of fractional calculus in control system is numerous and researcher's interest is increasing day by day in relevant field. Through well known tuning methods of FOPID controllers are obtainable with numerous examples to verify the success of the methods[23]. A model reduction method and an overt FOPID controller tuning rule for high order are proposed and by simulation showing the effectively in many dynamic systems[25].

Definitions of Fractional Order Calculus

The following definitions of fractional calculus are worn broadly in the area of control system.

Grunwald-Letnikov Definition: According to Grunwald-Letnikov (GL) definition the expression used is as given below:

$${}_aD_t^{\alpha} f(t) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t-jh)$$

Where: $w_j^{\alpha} = (-1)^j \binom{\alpha}{j}$ represents the coefficients of the polynomial $(1-z)^{\alpha}$. The coefficients can also be obtained recursively from

$w_0^{\alpha} = 1$, $w_j^{\alpha} = \left(1 - \frac{\alpha+1}{j}\right) w_{j-1}^{\alpha}$ j =1,2,…

Riemann-Liouville Definition: The Riemann-Liouville definition is defined as

$${}_aD_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau) d\tau$$

Where: $0 < \alpha < 1$ and a is the initial time instance, often assumed to be zero, i.e., a = 0. The differentiation is then denoted as $D_t^{-\alpha}$f(t)

Caputo's Definition: Caputo's definition is given by

$${}_0D_t^{\alpha} y(t) = \frac{1}{\Gamma(1-\gamma)} \int_0^t \frac{y^{(m+1)}(\tau)}{(t-\tau)^{\gamma}} d\tau$$

Where: $\alpha = m + \gamma$, m is a integer, and $0 < \gamma \le 1$. Similarly Caputo's Fractional Order integration is defined as

$${}_0D_t^{\gamma} y(t) = \frac{1}{\Gamma(-\gamma)} \int_0^t \frac{y(\tau)}{(t-\tau)^{\gamma+1}} d\tau \text{ , } \gamma<0$$

Cauchy's Definition: This definition is a general extension of the integer-order Cauchy formula

$$D^{\gamma} f(t) = \frac{\Gamma(\gamma+1)}{2\pi j} \int_C \frac{f(\tau)}{(\tau-t)^{\gamma+1}} d\tau$$

Where: C is the smooth curve encircling the single-valued function f(t).

Objective of research work: In this proposed work, there are three objectives: i. Filter Approximation method, ii. Model Reduction Techniques for fractional order transfer functions. iii. Tuning of FOPID Controllers.

Filter Approximation method: The evaluation of fractional differential equation when compare to integer order equation is difficult. Fractional order operator $G(s)=S^{\alpha}$ is represented by an integer order transfer function which is infinite dimensional. When fractional order transfer function has to be simulated or implemented and thus converts into an integer order transfer function with same close behavior of that function. There are many estimate methods for a FOTF which can realized by two ways: i. Analog realization, ii. Digital realization.

Oustaloup recursive approximation method is the best method of approximation. A generalized oustaloup filter is given by

$$G_f(s) = K\prod_{-N}^{N}\frac{s+w'_k}{s+w_k}$$

Where the poles gain and zero of the filter can be evaluated from

$$w'_k = w_b\left(\frac{w_h}{w_b}\right)^{\frac{k+N+1/2(1-\gamma)}{2N+1}};$$

$$K = w_h^{\gamma} \quad ; w_k = w_b\left(\frac{w_h}{w_b}\right)^{\frac{k+N+1/2(1+\gamma)}{2N+1}}$$

The differentiation order is γ and the order of the filter is 2N+1. The valid frequency range $[w_b, w_h]$ of fractional order transfer function fits very well within the filter design.

Model Reduction Techniques for fractional order transfer functions: If Oustaloup recursive filter is used for approximation of the FOTF, the obtained approximated integer order model system is very high order model. Thus a low order approximation to the original problem can be found using the optimal model reduction method in the form of

$$G_{(r|m,\tau)} = \frac{\beta_1 s^r+\cdots+\beta_r s+\beta_{r+1}}{s^m+\alpha_1 s^{m-1}+\cdots\alpha_{m-1}s+\alpha_m}e^{-\tau s}$$

An objective functions for minimizing the H_2 - norm of the derivation between the transfer functions of higher order and approximated system can be defined

$$J = \min_{\varphi}\left\|\widehat{G}(s) - G_{(r|m,\tau)}(s)\right\|2$$

Where φ is the set of parameters to be optimized

$$\Phi = [\beta_1, \beta_2, \ldots \beta_r, \alpha_1\alpha_2 \ldots \alpha_m, \tau]$$

For an evaluation of the criterion J, the deferred term in the reduced order model can be further approximated by a rational function $G_{r/m}(s)$ using the pade approximation technique. Thus, the revised criterion can then be defined by

$$J = \min_{\varphi}\left\|\widehat{G}(s) - G_{(r|m,\tau)}(s)\right\|2$$

The H_2–norm computation can be evaluated recursively using an optimization algorithm.

Tuning of FOPID Controllers: Many contemporary researchers have proposed several tuning strategies to tune FOPID controllers in both frequency and time domain analysis. Fractional order PID controllers have five parameters for tuning that means fractional order controllers provide two parameters extra for tuning than the classical PID controllers, which improves the performance of overall system. A review of tuning method for FOPID was presented by Valerio and Costa[18]. According to them analytical, numerical and Rule based tuning methods are considered.

C. Zhao and D. Xue illustrated the advanced performance obtained by using fractional order controllers by using a FOPID controller to control a class of fractional order system and executed with two examples[22]. B.M. Vinagre and C.A. Monje presented two methods to tune the FOPID controller, first one is based on optimization of specifications for five parameters to tune and the second one is based on auto tuning method which permits litheness and undeviating parameter selection of the fractional-order Controller, using the relay test[19]. Implementation of an auto tuning FOPID controller using a PLC was proposed by C.A. Monje and B.M. Vinagre, by means of this method, the gain crossover frequency and phase margin specifications are satisfied, collectively with the iso-damping characteristics of the time response of the system, assuring the robustness of the system to plant gain variations[8]. Y Luo proposed and designed two fractional order proportional integral controllers (FO-PI and FO-[PI]) for a class of fractional order systems[7]. S Das proposed tuning methodology of FOPID controllers using time and frequency domain for the control of higher order processes[4]. The Ziegler Nichols based tuning rules for FOPID controllers were proposed by Valerio and Costa. The first rule is based on tuning an integer PID controller which supposes the plant to have an S shaped unit step response. If the unit step response is other than S shaped[17] the method cannot be applied. Padula and Visioli proposed a new set of tuning rules for standard PID and FOPID controllers based on the disparagement of the integrated absolute error with a restriction on the maximum sensitivity[14]. C. Yeroglu and N. Tan proposed two methods for tuning of $PI^{\lambda}D^{\mu}$ controller, first one is based on Ziegler–Nichols and Astrom Hagglund method together and second one is related with the robust FOPID controller to control first order systems with parameter vagueness structure[21]. Tuning of fractional PI controllers for fractional order system models with and without time delays has been proposed in the literature. The proposed strategy is based on minimal cost of a quadratic cost function and demonstrated through three fractional order dynamic models. H. Li, Y. Luo and Y. Chen have proposed a new tuning method for FOPD for a class of typical second order plants is a simple practical, systematic and can have optimum performance and robustness[6].

Frequency domain design specification for robust fractional order PID tuning: C. Monje proposed a tuning method for a FOPID controller to tune so that the system satisfies different characteristics related robustness against plant unpredictable's, load disturbances, and high-frequency noise. In this design method specifications which is related to sensitivity functions, phase margin and robustness constraints were considered. Design specifications are formulated as follows:

Phase margin *specification*

$$\left|C(jw_{cg})G(jw_{cg})\right|_{db} = 0\ \mathrm{dB}$$

Gain crossover frequency specification

$$\mathrm{Arg}\left(C(jw_{cg})G(jw_{cg})\right) = -\pi + \varphi_m$$

Robustness of the plant against gain variations

$$\left(\frac{d\arg(C(jw_{cg})G(jw_{cg})}{dw}\right) w = w_{cg} \quad = 0$$

Reject of high frequency noise

$$\left|T(jw) = \frac{C(jw)G(jw)}{1 + C(jw)G(jw)}\right|_{db} \leq A\ \mathrm{dB\ at\ frequency}$$

and $w \geq w_t \frac{rad}{s}$

Rejection of a output disturbance

$$\left|S(jw) = \frac{1}{1 + C(jw)G(jw)}\right|_{db} \leq B\ \mathrm{dB\ at\ frequency}$$

and $w \geq w_s \frac{rad}{s}$

No steady state error: For obtaining the properly tuned FOPID controller above design specification should be fulfilled and five unknown parameter be solved through these non-linear equations.

Time domain based optimization tuning method

The time domain based optimal control tuning for FOPID controllers has an optimal set of controllers which optimize in suitable time domain integral performance indices namely[16],

$$\text{Integral of square error ISE} = \int_0^\infty e^2(t)dt$$

$$\text{Integral of absolute error IAE} = \int_0^\infty e(t)dt$$

$$\text{Integral of time square error ITSE} = \int_0^\infty te^2(t)dt$$

$$\text{Integral of time absolute error ITAE} = \int_0^\infty t|e(t)|dt$$

The integral performance indices are extensively used to measure system performance of a designed control system. For a fixed structured controller these indices are used for optimal tuning. The optimized unknown parameters of the FOPID controller are obtained by minimizing an integral performance index. Optimal tuning for FOPID controllers using an iterative optimization based on a non-linear function minimization is proposed in reference[11]. Through simulation result of the illustrative examples the effectiveness of the controllers can be observed. The tuning method for FO-PI controller mainly for first order plus delay systems by Y Chen et al and further used to expand tuning rules for the FOPDT class of dynamic systems. These urbanized tuning rules for FO-PI are not only suitable for FOPDT but also appropriate for further general class of plants.

Software for fractional PID Controller

For fractional order system analysis, simulations and implementation various software tool are available.

Ninteger: Ninteger code was accomplished by D. Valerio and J. Costain reference[17]. A control system and an optimization is being required by this tool box. Characteristics of this toolbox include: i. There are about 30 methods of Approximation of fractional order derivatives and integrals. ii. Fractional models have 3 identification methods. iii. GUI for controller design. iv. Analysis Functions for FOS.

FOMCON Toolbox: In 2013, A. Tepljakov reported his work on "Fractional-order Controller Design and Digital Implementation using FOMCON Toolbox for MATLAB". For a particular controller design and hardware realization suite of tools of the FOMCON ("Fractional-order Modeling and Control") toolbox for MATLAB is used. These days, use of this tool box is gaining importance amidst researchers of FOC. The main characteristics of this toolbox are i. Analysis Module using which Bode Plot, Nyquist Plot and Nicholas Plot can be obtained, ii. System Identification module for time and frequency domains, iii. Control Design Module which enables fractional order PID Design, Tuning and Optimization, iv. Fractional Order System Implementation Module for Continuous and Discrete Approximation.

FOPID toolbox: "Fractional Order PID Controller (FOPID)-Toolbox"[26], a paper was published by N. Lachhab in 2013. FOPID toolbox for the design of robust FOPID controllers was discussed in this paper. A novel non-smooth optimization technique was used. This toolbox has various features: i. To analyze various design, ii. To obtain Nicholas & Nyquist and plots. iii. To compute a fractional PID controller. iv. To obtain Step response. v. To tune computed parameters. vi. To obtain closed & open loop Bode Plot. vii. To compare and perform analysis of different approximation methods for the FOPID.

FOTF: (FOTF) is a toolbox urbanized for FOPID controller and systems by Xue. FOTF was designed and developed for time and frequency domain study, as well as constancy dimensions. It also provides delay in the FOTF. The FOPID was implemented, designed, tuned and optimized in Simulink.

Conclusion

In this paper a simplified and concise overview on fractional order PID controllers was presented. Fractional order PID controllers have five parameters for tuning that means fractional order controllers provide two extra more parameters for tuning than the classical PID controllers, which boost the overall performance of the system. The fractional order controllers are found which give robust performance. Designing fractional order controllers and algorithms for various engineering problems is still a topic of great interest for researchers.

References

1. Shah P. and Agashe S. (2016). Review of fractional PID controller. *Mechatronics*, 38, 29-41.

2. Chen Y., Petras I. and Xue D. (2009). Fractional order control-A tutorial. In American Control Conference, ACC'09, IEEE, 1397-1411.

3. Monje C.A., Chen Y., Vinagre B.M., Xue D. and Feliu-Batlle V. (2010). Fractional-order systems and controls: fundamentals and applications. Springer Science & Business Media.

4. Vinagre B.M., Podlubny I., Hernandez A. and Feliu V. (2000). Some approximations of fractional order operators used in control theory and applications. *Fractional calculus and applied analysis*, 3(3), 231-248.

5. Cao J.Y. and Cao B.G. (2006). Design of fractional order controllers based on particle swarm optimization. *Industrial Electronics and Applications, 2006 1ST IEEE Conference on,* IEEE, 1-6.

6. Das S., Saha S., Das S. and Gupta A. (2011). On the selection of tuning methodology of FOPID controllers for the control of higher order processes. *ISA transactions*, 50(3), 376-388.

7. Polubny I. (1999). Fractional-order systems and PIλDμ controller. *IEEE Trans. Automatic Control*, 44, 208-214.

8. Li H., Luo Y. and Chen Y. (2010). A fractional order proportional and derivative (FOPD) motion controller: tuning rule and experiments. *IEEE Transactions on control systems technology*, 18(2), 516-520.

9. Luo Y., Chen Y.Q., Wang C.Y. and Pi Y.G. (2010). Tuning fractional order proportional integral controllers for fractional order systems. *Journal of Process Control*, 20(7), 823-831.

10. Monje C.A., Vinagre B.M., Santamaria G.E. and Tejado I. (2009). Auto-tuning of Fractional Order PIλ Dμ Controllers using a PLC. *Proceedings of the 14 IEEE international conference on Emerging technologies & factory automation (IEEE Press Piscataway, NJ, USA)*, 1095, 1-7.

11. Monje C.A., Vinagre B.M., Chen Y.Q., Feliu V., Lanusse P. and Sabatier J. (2004). Proposals for fractional PID-tuning. *Proceedings of The First IFAC Symposium on Fractional Differentiation and its Applications (FDA04),* 115-120.

12. Monje C.A., Vinagre B.M., Feliu V. and Chen Y. (2008). Tuning and auto-tuning of fractional order controllers for industry applications. *Control engineering practice*, 16(7), 798-812.

13. Padula F. and Visioli A. (2015). Advances in robust fractional control. Springer Science & Business Media.

14. Padula F. and Visioli A. (2011). Tuning rules for optimal PID and fractional-order PID controllers. *Journal of process control*, 21(1), 69-81.

15. Podlubny I. (1994). Fractional-order systems and fractional-order controllers. *Institute of Experimental Physics, Slovak Academy of Sciences, Kosice*, 12(3), 1-18.

16. Tavazoei M.S. (2010). Notes on integral performance indices in fractional-order control systems. *Journal of Process Control*, 20(3), 285-291.

17. Valério D. and da Costa J.S. (2006). Tuning of fractional PID controllers with Ziegler–Nichols-type rules. *Signal Processing*, 86(10), 2771-2784.

18. Valerio D. and da Costa J.S. (2010). A review of tuning methods for fractional PIDs. In *4th IFAC Workshop on Fractional Differentiation and its Applications, FDA*, 10.

19. Vinagre B.M., Monje C.A., Calderón A.J. and Suárez J.I. (2007). Fractional PID controllers for industry application.A brief introduction. *Journal of Vibration and Control*, 13(9-10), 1419-1429.

20. Xue D. and Chen Y. (2002). A comparative introduction of four fractional order controllers. *Intelligent Control and Automation, 2002.Proceedings of the 4th World Congress on*, IEEE, 4, 3228-3235.

21. Yeroglu C. and Tan N. (2011). Note on fractional-order proportional-integral-differential controller design. *IET control theory & applications*, 5(17), 1978-1989.

22. Zhao C., Xue D. and Chen Y. (2005). A fractional order PID tuning algorithm for a class of fractional order plants. *Mechatronics and Automation, 2005 IEEE International Conference* , IEEE, 1, 216-221.

23. Yan Z., He J., Li Y., Li K. and Song C. (2013). Realization of Fractional Order Controllers by Using Multiple Tuning-Rules. *International Journal of Signal Processing, Image Processing and Pattern Recognition*, 6(6), 119-128.

24. Padula F. and Visioli A. (2013). Set-point weight tuning rules for fractional-order PID controllers. *Asian Journal of Control*, 15(3), 678-690.

25. Hui-fang W., Qiu-sheng H., Zhi-cheng Z. and Jing-gang Z. (2015). A design method of fractional order PI λ D μ controller for higher order systems. Control Conference (CCC), 2015 34th Chinese, IEEE, 272-277.

26. Lachhab N., Svaricek F., Wobbe F. and Rabba H. (2013). Fractional order PID controller (FOPID)-toolbox. In *Control Conference (ECC), 2013 European*, IEEE, 3694-3699.