Research Journal of Engineering Sciences ______ ISSN 2278 – 9472
Vol. **6(6),** 19-23, July **(2017)** *Res. J. Engineering Sci.*

Performance measurement of different M-Ary phase signalling schemes in AWGN channel

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Available online at: www.isca.in, www.isca.me
Received 8th April 2017, revised 8th July 2017, accepted 21th July 2017

Abstract

It is most important in design of digital communication systems to receive correct and errorless data at the receiver end with least SNR and bandwidth. The performance of M-ary PSK communication system is analyzed in terms of probability of error considering AWGN channel. The performance is compared for M =4, 8, 16, 32 in MATLAB Simulink environment. It is observed that SNR requirement for a given BER increases as M increases while bandwidth efficiency increases with M. Further M-PSK systems are bandwidth efficient and have large data carrying capacity.

Keywords: Multiple Phase Shift Keying (MPSK), Probability of Error, Additive White Gaussian Noise (AWGN), Signal to Noise Ratio (SNR).

Introduction

In early days the analog communication systems were used for communication. These systems have lot of disadvantages, like, more expensive, consume more power and require more repeaters. For long distance communication large numbers of amplifiers are required to amplify the signal[1]. These amplifiers also amplify the noise. These disadvantages of analog communication system forced to use digital communication systems, which are more reliable, cost effective, flexible, easily modified and easy to recover digital signal. There are several digital modulation schemes like binary and M-ary Amplitude Shift Keying (ASK), Phase Shift Keying (PSK), Frequency Shift Keying (FSK) and Quardrature Amplitude Modulation (QAM).

Initially binary PSK signaling scheme was used in communication systems because it is the best in binary schemes. Later on with increasing demand M-ary PSK signaling schemes were developed and used in communication systems[1]. The modulators and demodulators for higher value of M are discussed in this paper and performance is measured in terms of probability of error for M = 4, 8, 16, 32.

Additive White Gaussian Noise (AWGN)

The unwanted electrical signals considered as noise signal are always present in all electrical systems. The noise sources may be external and internal to the system. The external noise sources are manmade and natural. The manmade noise can be reduced by careful observations and natural noise is not in our control. For performance analysis only the internal noise is considered. The internal noise is defined as thermal noise and shot noise. These noises are modeled as a zero-mean white Gaussian noise with probability density function defined as,

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n}{\sigma}\right)^2\right] \quad (1)$$

Where: σ^2 is variance of Gaussian noise, the normalized Gaussian density function (zero mean and σ^2=1) is plotted in Figure-1.

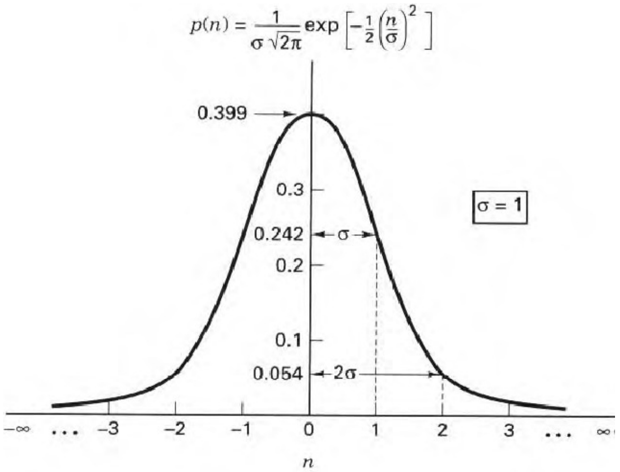

Figure-1: Normalized Gaussian PDF[2].

M-ARY PSK Signalling schemes

M-ary PSK is a very efficient signaling scheme. With increase in M, data rate and band width efficiency increases. In M-Ary PSK, the PSK modulated signal is written as

$$S_m(t) = \left(\sqrt{\frac{2E_S}{T_S}}\right) \cos(2\pi f_c t + \phi_m) \quad (2)$$

$0 < t < T_s$

Where: E_s is signal energy, f_c is carrier frequency and phase ϕ_m will depends on the transmitted symbol. Phase ϕ_m is given by

$$\phi_m = \frac{2\pi(i-1)}{M} + \lambda \qquad (3)$$

Where: i = 1, 2, --------- M and $\lambda = \frac{\pi}{M}$ is fixed phase offset. For example,
For M = 4, phases are 45°, 135°, 225° and 315°.

For M = 8, phases are 22.5°, 67.5°, 112.5°, 157.5°, 202.5°, 247.5°, 292.5° and 337.5°.

For M = 16, phases are 11.25°, 33.75°, 56.25°, 78.75°, 101.25°, 123.75°, 146.25°, 168.75°, 191.25°, 213.75°, 236.25°, 258.75°, 281.25°, 303.75°, 326.25° and 348.75°.

For M = 32, phases are 5.625°, 16.875°, 28.125°, 39.375°, 50.625°, 61.875°, 73.125°, 84.375°, 95.625°, 106.875°, 118.125°, 129.375°, 140.625°, 151.875°, 163.125°, 174.375°, 185.625°, 196.875°, 208.125°, 219.375°, 230.625°, 241.875°, 253.125°, 264.375°, 275.625°, 286.875°, 298.125°, 309.375°, 320.625°, 331.875°, 343.125° and 54.375°.

These phases are shown in constellation diagrams of MPSK for M=4, 8, 16, 32 in Figures-2, 3, 4, 5.

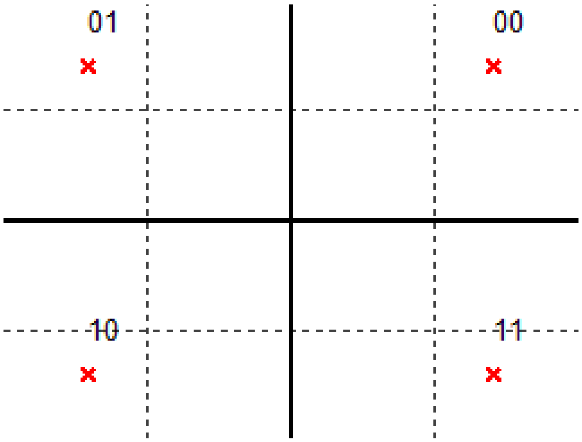

Figure-2: Constellation diagram of QPSK (M=4).

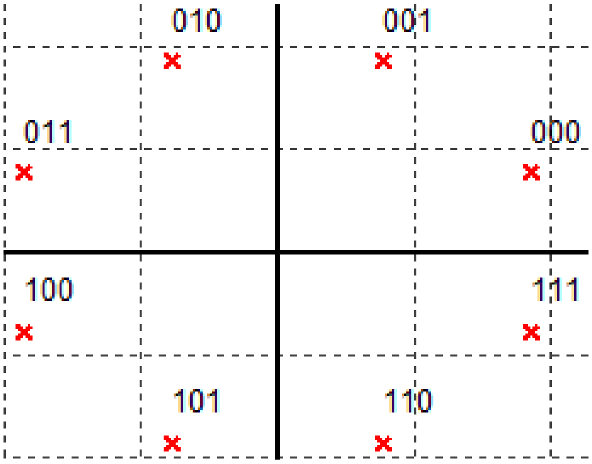

Figure-3: Constellation diagram of 8PSK.

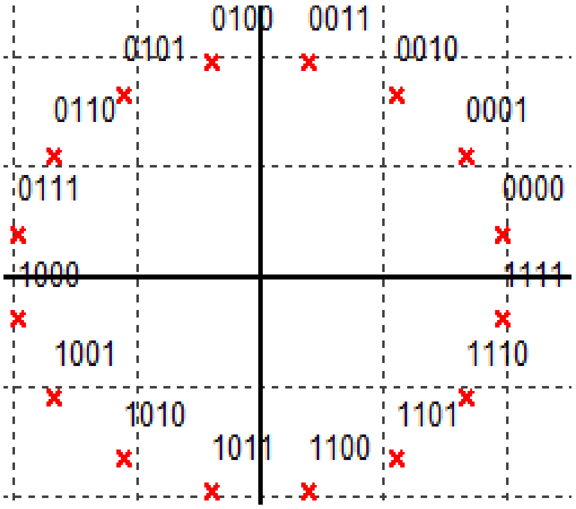

Figure-4: Constellation diagram of 16PSK.

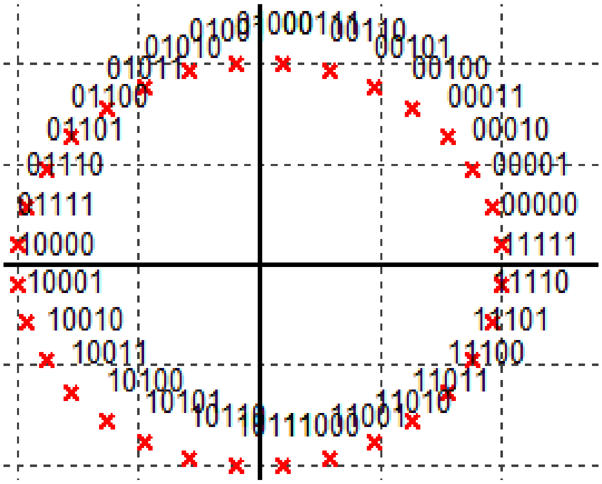

Figure-5: Constellation diagram of 32PSK.

Substituting value of phase in equation (2) and expanding the cosine term, S_m (t) is written as

$$S_m(t) = \sqrt{2P_s}\cos\phi_m \cos 2\pi f_c t - \sqrt{2P_s}\sin\phi_m \sin 2\pi f_c t \sin$$

$$0 < t < T_s \qquad (4)$$

Where: P_s is power of the transmitted signal

This is further written as

$$S_m(t) = A_{cm}\cos 2\pi f_c t - A_{sm}\sin 2\pi f_c t \qquad (5)$$

Where $A_{cm} = \sqrt{2P_s}\cos\phi_m$ 6(a)
And $A_{sm} = \sqrt{2P_s}\sin\phi_m$ 6(b)

It appears that MPSK is equivalent to modulation of two sinusoidal carriers with A_{cm} and A_{sm}. The value of A_{cm} and A_{sm} depends on the symbol.

Modulator

The modulator structure is straight forward and is given in Figure-6. From the data, A_{cm} and A_{sm} are obtained and fed into two balanced modulators. The outputs of both modulators are added to form a MPSK modulated signal.

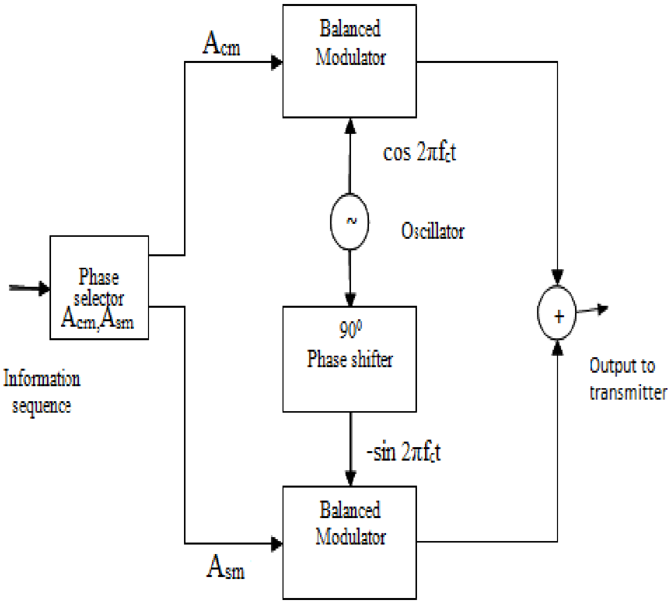

Figure-6: Block diagram of M-PSK modulator[3].

Receiver

M-PSK demodulator is shown in Figure-7 for recovering the noise corrupted signal components A_{cm} and A_{sm}, from which the vector V is formed. Phase detector computes the phase of vector V, and selects the signal from the set $\{S_m(t)\}$ having a phase closest to θ.

The approximate phase of V is given by [3]

$$p(\theta) \approx \sqrt{\frac{\gamma}{\pi}}\cos\theta\ e^{-\gamma\sin^2\theta} \tag{7}$$

Where: $\gamma = \alpha^2 E_s/N_o$ is SNR per symbol, α is channel attenuation and $N_o/2$ is power spectral density of white noise.

If the phase falls outside of the range of $-\pi/M \le \theta \le \pi/M$, a decision error is made. For correct decision the phase of received signal should lie in the range.

Thus the probability of symbol error is given by

$$P_M = 1 - \int_{-\pi/M}^{\pi/M} p(\theta)d\theta \tag{8}$$

Substituting $p(\theta)$ in equation (8) and changing the variable from θ to $u = \sqrt{\gamma}\sin\theta$, it is find that

$$P_M \approx 1 - \int_{-\pi/M}^{\pi/M}\sqrt{\frac{\gamma}{\pi}}\cos\theta\ e^{-\gamma\sin^2\theta}d\theta \qquad 9(a)$$

$$P_M \approx 1 - \left(1 - \frac{2}{\sqrt{\pi}}\int_{\sqrt{\gamma}\sin\pi/M}^{\infty} e^{-u^2}\,du\right) \qquad 9(b)$$

$$P_M \approx \operatorname{erfc}\left(\sqrt{\gamma}\sin\frac{\pi}{M}\right) = \operatorname{erfc}\left(\sqrt{n\gamma_b}\ \sin\frac{\pi}{M}\right) \qquad 9(c)$$

Where: $n = log_2$M. This expression is valid for all values of M>4.

Complimentary error function is defined as

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-t^2}dt$$

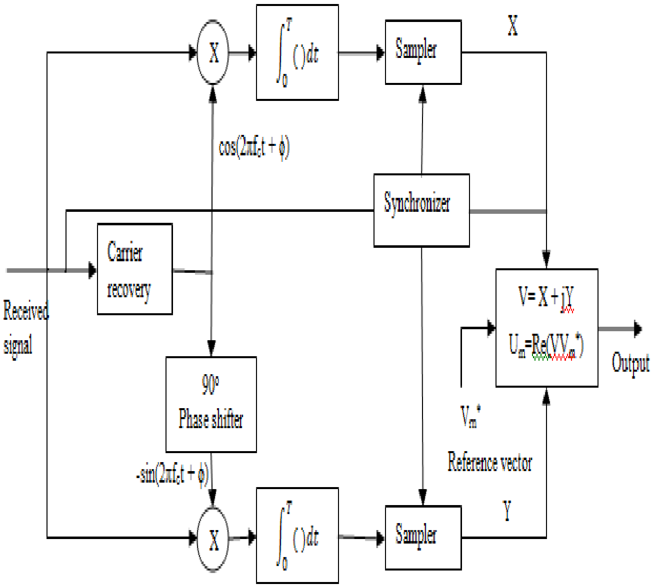

Figure-7: Block diagram of M-PSK demodulator[3].

Bandwidth efficiency

The bandwidth efficiency is defined as R/W, ratio of data rate (R) and bandwidth (W).

$$\text{Bandwidth efficiency} = \frac{R}{W} = \frac{\frac{\log_2 M}{T_s}}{{}^1/_{T_s}} = \log_2 M$$

$$T_s = \log_2 {}^M/_R,\quad W = {}^1/_{T_s}$$

Where: T_s is symbol duration and $W = {}^R/_{\log_2} M$

Therefore as M increases, the bandwidth efficiency increases[3].

Matlab simulation model of M-PSK

The simulation model of M-PSK is shown in Figure-8, connected and placed with suitable parameters of each block for different value of M=4, 8, 16, 32.

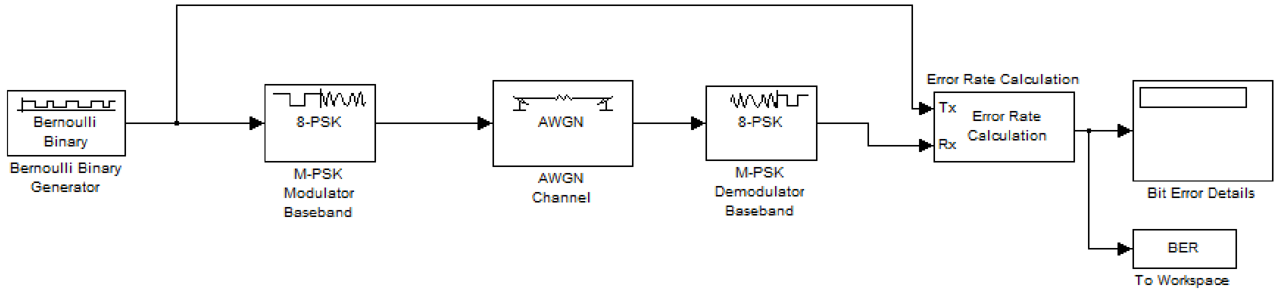

Figure-8: Simulation model of M-PSK.

The system model consists of a random bit generator and the generated bit stream is fed to the M-PSK modulator. This modulated signal passes through the AWGN channel block to add white Gaussian noise with required SNR[4]. This AWGN channel output is fed to the receiver side which consist M-PSK demodulator to get an approximated original signal. There may be some errors due to presence of noise. This error is calculated with the use of error rate calculator compared with original signal[5]. It can be displayed in the display block and is obtained plot between Eb/N_o and bit error rate (BER plot) for different value of M = 4, 8, 16, 32 (Figure-9).

Simulation Results

In Figure-9 both simulated and theoretical results are shown. It is observed that simulated results almost match with theoretical results for small BER. The simulated results are also shown in tabular form in Table-1. Further it is observed that for a given BER, the requirement of SNR increases as M increases, for example, BER at 1E-9 the required SNR for M=4 SNR is 13dB, for M=8 SNR is 16dB, for M=16 SNR is 21dB and for M=32 SNR is 26dB. This is a disadvantage of increasing M.

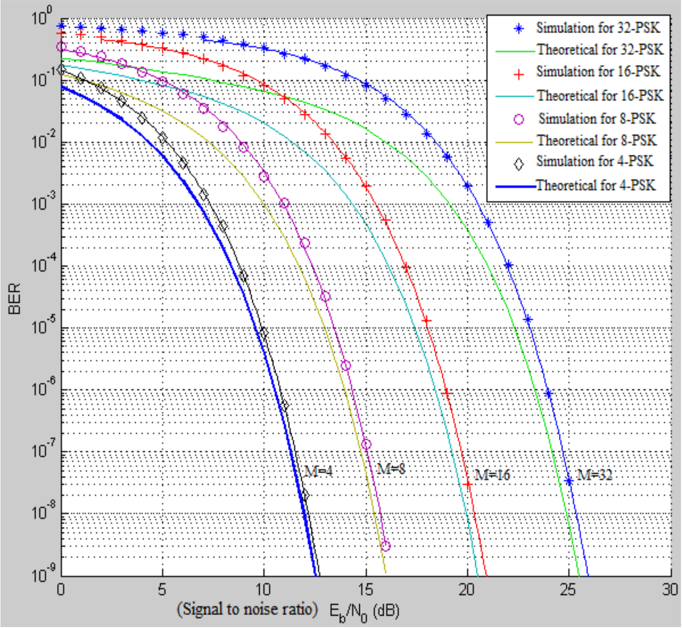

Figure-9: Eb/No Vs BER plots of M-PSK.

Table-1: E_b/N_o and BER table for 4-PSK, 8-PSK, 16-PSK and 32-PSK.

E_b/ N_o	%BER for 4-PSK	%BER for 8-PSK	%BER for 16-PSK	%BER for 32-PSK
0	0.1506	0.347	0.5802	0.7551
1	0.1092	0.2920	0.5344	0.7271
2	0.0737	0.2371	0.4864	0.6961
3	0.0452	0.1854	0.4364	0.6614
4	0.0243	0.1365	0.382	0.6235
5	0.0118	0.0949	0.3265	0.5818
6	0.0046	0.0608	0.2711	0.5360
7	0.0014	0.0352	0.2161	0.4879
8	4.4E-4	0.0177	0.1651	0.4361
9	7.0 E -5	0.0082	0.1190	0.3812
10	8.19E-6	0.0028	0.0807	0.3255
11	5.61E-7	0.0010	0.0501	0.2703
12	1.98E-8	2.3E-4	0.0278	0.2164
13	3.00E-10	3.21E-5	0.0133	0.1664
14		2.45E-6	0.0056	0.1202
15		1.31E-7	0.0019	0.0811
16		3.00E-9	5.45E-4	0.0506
17			9.45E-5	0.0281
18			1.31E-5	0.0135
19			8.69E-7	0.0057
20			2.97E-8	0.0019
21			8.00E-10	5.15E-4
22				1.03E-4
23				1.40E-5
24				8.77E-7
25				3.37E-8
26				7.0E-10

Conclusion

The Bit Error Rate performance has been analyzed in this paper by simulation of different M-ary PSK signalling schemes over AWGN channel in MATLAB Simulink environment. It is found that simulated results are almost identical with theoretical results. For a fixed value of BER the bandwidth efficiency increases with increase in M while SNR requirement increases with M. Due to closeness of the symbols the bit error rate increases therefore more signal to noise ratio is required to achieve same BER. Higher order PSK systems are useful where sufficient power is available and bandwidth is limited.

References

1. Taub Herbert, Schilling Donald and Saha Goutam (2008). Principles of Communication Systems. Tata McGraw-HILL Publishing Company Limited, India, 321-341. ISBN-13:978-0-07-064811-1
2. Sklar Bernard and Ray Pabitra Kumar (2009). Digital Communications. Fundamentals and Applications, Pearson Education, India 30-33. ISBN:978-81-317-2092-9
3. Proakis John G. (1985). Digital Communications. McGraw Hill Book Co. Singapore, 156-171. ISBN:0-07-118183-0
4. Malleswari Naga P. (2015). Simulink Based Comparative Analysis of M-ary Phase Shift Keying Modulation Schemes. *IJIREC*, 2(3), 9-18.
5. Kaur Harjot, Jain Bindiya and Verma Amit (2011). Comparative Performance Analysis of M-ary PSK Modulation Schemes using Simulink. *IJECT*, 2, 204-209.