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Vibration Analysis of Railway Concrete Sleeper for Different Contact Condition using MATLAB

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Abstract

In this work Eigen frequencies of a railway sleeper with ballast beneath having voids at different positions have been evaluated. Before this evaluation a previous work in this field has been validated by an MATLAB program. Then, by considering five different types of void positions (five different cases of supporting), which commonly happen in sleeper and ballast interaction, the results for these situations are discussed. In order to clearly understand and interpret the behavior of the flexural modes, the shapes of them, for a simple theoretical case, are presented. To find out the frequencies of the sleeper and to find out the variation of 1st mode and 2nd mode rigid-body frequencies different MATLAB programs have been developed and through these programs frequencies for different void configurations with different void length have been calculated. The graphs generated show the variation of the frequencies for different void length.

Keywords: Eigen frequencies, Railway sleeper, Ballast, MATLAB program.

Introduction

Railway tracks get loaded, when trains pass over it. Due to the weight of the train, static load is applied, and due to the irregularities of the vehicle and the track, dynamic load is applied. These loads may harm track and can be a reason of everlasting settlement, and they can also cause ground vibration that can exasperate residents of nearby buildings. So, in order to deal with these types of complications, adequate knowledge about track dynamics and interface between vehicle and track is very important.

The dynamic wheel/rail force of interaction depends governed by dynamic properties of the train and deflection of the track due to the load of the train. Also, the irregular contact force then creates an increased loading and deterioration of the ballast bed below the sleeper. The sleeper conveys mainly three types of forces to the ballast bed i.e. vertical, lateral, and longitudinal forces from the rail down. The outcome of unsupported sleeper will be rise of the variations of the dynamic interaction forces between train and track.

Producing a prestressed concrete railway sleeper is a complex design task that requires in-depth knowledge in several areas. The load generated from the passing trains is strongly influenced by possible irregularities of the wheels or the rail and variations of the stiffness of the underlying ballast supporting the entire track structure. The contents of dynamic loads can be substantial and must be considered or at least noticed during the design process of a sleeper. Static design system for concrete sleeper can be used to design the sleepers for the maximum load reached during a train passage, but the effects of rapid shifts in the motion of the sleeper can only be treated by a dynamic design system.

The railway sleepers, uniformly transfer and distribute loads from the rail foot to underlying ballast bed, supporting and holding the rails at the proper gauge by keeping anchorage for the rail fastening system by stabilizing rail inclination; and providing supports for rails; restraining longitudinal, lateral and vertical rail movements by embedding itself onto substructures. The main responsibility of a railway concrete sleeper is to allocate the loads of rolling stocks axle to the supporting formation and then, lastly to the foundation. When the train speeds are low to moderate, the axle load could be considered static or quasistatic.

The performance of railway concrete sleepers is a fundamental concern when studying railway track dynamics[1,2]. The sleeper can be demonstrated in a variety of methods, based on the frequency range of interest i. as a rigid body, ii. as a Euler-Bernoulli, iii. as a Rayleigh-Timoshenko beam, or iv. as a three-dimensional elastic body (at very high frequencies). Frequency, generated by vibrations of sleeper is mostly affected by bending stiffness, mass, and distribution of mass of the sleeper itself and the impact of the surroundings. Maximum sleepers are implanted in ballast, and in most of the cases small voids appear between the ballast and sleepers. The effect of the ballast on the sleeper vibrations is in many cases modelled by a distributed spring stiffness acting along the sleeper. The impact of the part of the track assembly which is located over head the sleeper can be concise by distinct springs acting at the rail locations. These springs are responsible for the rail pad stiffness and stiffness generated due to the rails and the remaining track assembly.

Here, the railway concrete sleeper is supposed to be maintained by an elastic basis (a massless Winkler bed) acting along the complete span of the sleeper, or, in case of a gap between the sleeper and ballast, along part (or parts) of the sleeper. Thus, the sleeper is in contact with the ballast only along one or several sections of its full length. Two distinct springs are also there at the locations of the rails. Vibrations in the sleeper of the in situ sleeper are explored in this report work. Eigen frequencies and Eigen modes are also designed and deliberated.

If the analysis of dynamic performance of a railway track is to be carried out at frequencies much lower than the first bending-mode, Eigen frequency of a free-free sleeper, then in that case sleeper can, be assumed as a rigid body. The effect of the bending stiffness of the sleeper on the Eigen frequencies is then ignored. Then the sleeper vibrations will be affected only by the sleeper mass, its distribution, the foundation stiffness, and the rail pad and the stiffness of rail spring. Euler-Bernoulli beam theory of a beam on elastic foundation can be used at frequencies in the vicinity of the lowest two to three bending-mode Eigen frequencies of the sleeper. Rayleigh-Timoshenko beam theory should be used at the third or upwards Eigen frequency. Then deformation due to shear and inertia in rotation of a beam lamina are considered. Lastly, at very high frequencies where the cross section of the sleeper does not remain plane during the vibration, then in that case a three-dimensional model and the finite element method should be applied. (The Timoshenko beam theory assumes that a beam cross section remains plane during vibration, but a beam lamina will be sheared[3]. The cross section of the beam will not stay planeat very high frequencies and the Timoshenko theory cannot be used.)

When the author, T. Dahlberg worked with topic "Modelling of the dynamic behavior of in situ concrete railway sleepers", he came across with the work done by Kaewunruen S. and Remennikov a M in the topic "Investigation of free vibrations of voided concrete sleepers in railway track system"[3,4]. Evidently he has found that, there are some suspicious results stated in the work of Kaewunruen S and Remennikov a M[4]. Kaewunruen S and Remennikov a M deals with the vibrations of a concrete sleeper either fully supported by the ballast, partly supported or not supported at all (in this last case the sleeper is hanging in the rails). Therefore, the re-analysis is carried out of Eigen frequencies of a free-free sleeper and a partially balanced in situ sleeper, interconnected to the rails through separate springs. Modes of vibrations are also discussed. Frequencies in the lower range only are observed, so the sleeper may be rigid or it may distort according to the Euler-Bernoulli beam theory. Solutions are obtained and compared analytically and using finite element methods.

To validate the dynamic models of the sleeper, comparison is done between calculated values of Eigen frequencies of a free-free sleeper with measured values. Measured values of Eigen frequencies of a free-free sleeper can be obtained as Ågård L, has explained in his work[5]. Measurements were done at the Technical Research Institute of Sweden, SP. These measurements are sometimes measured to when Eigen frequencies of concrete railway sleepers are deliberated[4,6,7]. Comparison is done between the calculated values of Eigen frequencies of a free concrete railway sleeper which is used in Sweden and the measured values[6,8]. Calculations were completed using both E-B (Euler- Bernoulli) and R-T (Rayleigh-Timoshenko) beam theory, and it is seen that the E-B theory gives suitable outcomes only for the lowest two or three Eigen frequencies of the sleeper. Eigen frequencies of an in-situ sleeper, considered as a beam on an elastic basis and elastically connected to the rails, were presented.

In the work reported here, solutions will be done analytically and with the help of finite element methods for a vibrating beam on an elastic foundation, and also elastically connected to the rails, will be presented. The beam is divided into two or more areas that have consistent properties and the distinctive parts could possibly be bolstered by the elastic foundation. (i.e. the ballast). The elastic links to the rails are situated at the rail locations.

The sleeper considered by Dahlberg and Nielsen is a standard gauge sleeper generally used by Swedish railways[6]. Its natural frequencies were established by applying several theories such as Euler-Bernoulli, Timoshenko and Rayleigh-Timoshenko. Also comparisons between the calculated values and the measured ones were performed by Dahlberg and Nielsen[6].

Mathematical Modeling

In order to formulate a mathematical model for the evaluation of Eigen frequencies of a railway sleeper following mentioned beam theories can be adopted with different boundary conditions satisfying various contact types between railway sleeper and ballast. Only Euler–Bernoulli (E–B) beam theory can be used to calculate the lowest few (two or three) Eigen frequencies, because a concrete sleeper is relatively small and stubby. If track dynamics is examined at higher frequencies, it becomes essential to use the Rayleigh–Timoshenko (R–T) beam theory.

The theory of Timoshenko's theory beams institutes an improvement of the Euler-Bernoulli theory; in which it integrates shear deformation effects[9]. In this section a sleeper is considered to be a Rayleigh-Timoshenko beam, in that the two effects, i.e. shear of a beam lamina due to shear force and impact of rotator inertia, are included in the Rayleigh-Timoshenko theory, but these effects are omitted in the Euler-Bernoulli beam theory. In particular, the first effect which is related to the shear force was recommended by the Ukrainian/Russian-born scientist Stephen Timoshenko in the beginning of the 20th century and the second effect, the impact of rotary inertia, was first studied by Rayleigh.

In Rayleigh-Timoshenko beam theory the total deflection of the beam is divided into two parts: one depends on the bending of the beam and the other part depends on shear deformation of the beam. A beam lamina, with shear force and bending moment plus the corresponding deformations, is demonstrated in Figure-1[2].

The relevant deformation to the bending moment M (x, t) is here indicated by wM (x, t) and the other part which is related to the shear force T (x, t) is denoted wS (x, t) [2].

The following formula shows relation between these terms

$$w(x,t) = w_{m(}x,t) + w_{s(}x,t) \quad (1)$$

The deflection due to the bending moment can be given by use of the rotation angle. In this case the relation between deflection and rotation angle is

$$\Psi(x,t) = \frac{\partial w_m(x,t)}{\partial x} \quad (2)$$

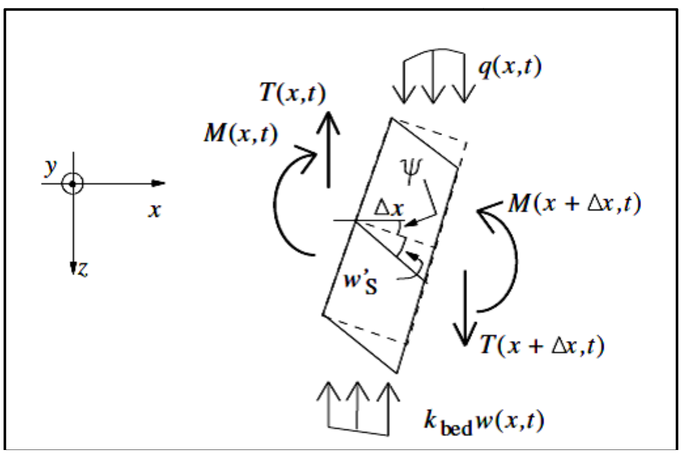

Figure-1
A beam lamina: load *q (x, t)*, deflection *w (x, t)*, shear force *T (x, t)* and shear angle *w´S (x, t)*, bending moment *M (x, t)* and rotation angle *ψ (x, t)*. The elastic foundation influences the beam with a force $k_{bed}w(x,t)$ per unit length.

The relationships between the bending moment M (x, t) and second derivative of the deflection wM (x, t) or the first derivative of the angle Ψ (x, t), and, furthermore, the relation between equivalent deformation and the shear force T (x, t) can be expressed as follows:

$$M(x,t) = \frac{-EI\partial^2 W_m(x,t)}{\partial x^2} = \frac{-EI\partial\Psi(x,t)}{\partial x} \quad (3)$$

And

$$T(x,t) = kGA\frac{\partial W_s(x,t)}{\partial x} = kGA\left[\frac{\partial W_s(x,t)}{\partial x} - \Psi(x,t)\right] \quad (4)$$

Here EI is bending stiffness of the beam, E is Young's modulus, I is second moment of the cross-sectional area A, G is shear modulus which can be calculated from the modulus of elasticity E and the Poisson ratio υ, the factor κ is a constant, the Timoshenko shear factor, that depends on the form of the cross section and the Poisson ratio υ. The value of factor κ is taken from the work of author Ågård L[5].By considering the forces in vertical direction, the equilibrium equation of the given lamina becomes:

$$m\frac{\partial^2 w(x,t)}{\partial t^2} - \frac{\partial T(x,t)}{\partial x} = q(x,t) - k_{bed}x(x,t) \quad (5a)$$

Here m (kg/m) is mass per unit length of the beam and q (x, t) is distributed load (N/m). The elastic foundation is assumed to give a counteracting force (per unit length) that is proportional to the deflection w(tot); the proportionality constant is $k_{\mathbf{bed}}$.

Also, the rotational equilibrium equation of the beam lamina (per unit length) can be written:

$$\frac{\partial M(x,t)}{\partial x} + T(x,t) - \rho I\frac{\partial^2\Psi(x,t)}{\partial t^2} = 0 \quad (5b)$$

where, ρ(kg/m3) is density, giving mass distribution m = ρA. Therefore, ρI is inertia of mass (per unit length) of the beam. The second moment of area I can be expressed $I= Ar^2$ where A (m^2) is area of cross-sectional and r is the radius of gyration with respect to the y-direction. Thus, ρI can also be written $\rho I=mr^2$.

First, the bending moment M (x, t) and shear force T (x, t) are eliminated from the above equations (5.a, b). For this purpose, the T(tot) value from (4) should be inserted into (5a). It gives

$$m\frac{\partial^2 w(x,t)}{\partial t^2} - kGA\left[\frac{\partial^2 w(x,t)}{\partial x^2} - \frac{\partial\Psi(x,t)}{\partial x}\right] = q(x,t) - k_{bed}x(x,t) \quad (6a)$$

$$EI\frac{\partial^2\Psi(x,t)}{\partial x^2} + kGA\left[\frac{\partial w(x,t)}{\partial x} - \Psi(x,t)\right] - \rho I\frac{\partial^2\Psi(x,t)}{\partial t^2} = 0 \quad (6b)$$

Differential equations (6a, 6b) together with the boundary and initial conditions present the solution of the vibration problem of a beam described by Rayleigh-Timoshenko beam theory when the beam is loaded with a distributed load q(tot).

Now, eliminate ψ from (6.a,b). Solve for $\frac{\partial\psi}{\partial x}$ from (6.a) and insert into (6.b). It gives (with w(tot) written in short term as w as well as q (x, t) is written q only)

$$EI\frac{\partial^4 w}{\partial x^4} - \frac{EI\,k_{bed}}{kGA}\frac{\partial^2 w}{\partial x^2} - \frac{mEI}{kGA}\frac{\partial^4 w}{\partial x^2\partial t^2} - \rho I\frac{\partial^4 w}{\partial x^2\partial t^2} + \frac{m\rho I}{kGA}\frac{\partial^4 w}{\partial t^4} + m\frac{\partial^2 w}{\partial t^2} + \frac{\rho I\,k_{bed}}{kGA}\frac{\partial^2 w}{\partial t^2} + k_{bed}w = q + \frac{\rho I}{kGA}\frac{\partial^2 q}{\partial t^2} - \frac{EI}{kGA}\frac{\partial^2 q}{\partial x^2} \quad (7)$$

This equation is the differential equation which, together with boundary conditions and initial conditions, determines the beam deflection w(tot) according to the Rayleigh-Timoshenko beam theory. Here w(tot) includes both the effects of bending deformation and shear deformation. By eliminating w(tot) from (6a, 6b) and obtains a differential equation.

$$EI\frac{\partial^4\Psi}{\partial x^4}-\frac{EI\,k_{bed}}{kGA}\frac{\partial^2\Psi}{\partial x^2}-\frac{mEI}{kGA}\frac{\partial^4\Psi}{\partial x^2\partial t^2}-\rho I\frac{\partial^4\Psi}{\partial x^2\partial t^2}+\frac{m\rho I}{kGA}\frac{\partial^4\Psi}{\partial t^4}+ m\frac{\partial^2\Psi}{\partial t^2}+\frac{\rho I\,k_{bed}}{kGA}\frac{\partial^2\Psi}{\partial t^2}+k_{bed}\Psi=+\frac{\partial q}{\partial x} \quad (8)$$

It is worth noting that the form of the left hand side in (7) and in (8) are equal, only w and ψ are switched. Because of this the homogeneous solutions of w(tot) and ψ(tot) are equal. However, the particular solutions are difference in these equations.

Equations (7 and 8) have homogenous solutions w (x, t) and ψ (x, t) which contains four integration constants each. These eight constants are not independent from each other; consequently, a total of four independent constants only are obtained. These are determined by boundary conditions. Boundary conditions are different from those one gets at the technical beam theory. Some of the boundary conditions that can be stated for a beam of length L (for the coordinate x for 0 ≤x ≤L) are as follows.

Boundary Conditions: Sleeper or Beam with both end restricted. For this type of boundary condition, deflection and the rotation are zero at both ends, giving

$$\mathrm{w}(0,\mathrm{t})=0 \text{ and } \Psi(0,\mathrm{t})=0\ ;\ \mathrm{w}(\mathrm{L},\mathrm{t})=0 \text{and } \Psi(\mathrm{x},\mathrm{t})=0 \quad (9)$$

Note that w' (0, t) and w' (L, t) are not zero. This is because w(tot) contains the shear angle and this angle needs not be zero at the beam end even if the beam is clamped.

Sleeper or Beam with one end restricted and other end simply supported.

$$\mathrm{w}(0,\mathrm{t})=0 \text{ and } \Psi(0,\mathrm{t})=0 \quad (10)$$

and

$$w(L,t)=0\ \ and\ M(L,t)=\ -EI\frac{\partial\Psi(L,t)}{\partial x}\ =\ 0 \quad (11)$$

These conditions are because of the simply supported end of the beam

Both end simply supported: As mentioned at the preceding boundary condition, when the beam is simply supported at one end, this end has zero deflection and zero moment at that end. It gives

$$w(0,t)=0\ \ and\ M(0,t)=\ -EI\frac{\partial\Psi(x,t)}{\partial x}\ =\ 0 \quad (12)$$

$$w(L,t)=0\ \ and\ M(x,t)=\ -EI\frac{\partial\Psi(x,t)}{\partial x}\ =\ 0$$

One end restricted and other end free: From the previous boundary conditions, it is clear that at the clamped end deflection and angle of rotation are zero. Thus at x=0

$$w(0,t)=0\quad and\quad \Psi(0,t)=0 \quad (13)$$

The other end is in this case assumed to be free. When a beam is free at one end the moments and shear forces should be zero on that end. Therefore, the following relation holds at x=L

$$M(L,t)=\ -EI\frac{\partial\Psi(L,t)}{\partial x}\ =\ 0 \quad (14)$$

And

$$T(L,t)=\ kGA\left[\frac{\partial w(L,t)}{\partial x}-\ \Psi(L,t)\right]$$

By applying the condition in Equation (3.6b) above, the following equation can be obtained:

$$T(L,t)=\ -EI\frac{\partial^2\Psi(x,t)}{\partial x^2}+\ \rho I\frac{\partial^2\Psi(x,t)}{\partial t^2}\ =\ 0 \quad (15)$$

Both ends free: Moments and shear forces are zero in the case that the beam ends are free. This give

$$M(0,t)=\ -EI\frac{\partial\Psi(0,t)}{\partial x}=0\ \ and\ T(0,t)=kGA\left[\frac{\partial w(0,t)}{\partial x}-\ \Psi(0,t)\right] \quad (16)$$

Same condition is true for the second end of the beam, at x=L,

$$M(L,t)=\ -EI\frac{\partial\Psi(L,t)}{\partial x}=0\ \ and\ T(L,t)=kGA\left[\frac{\partial w(L,t)}{\partial x}-\ \Psi(L,t)\right] \quad (17)$$

Which is the same condition as used before.

In a last step of a Free-Free beam, instead of obtaining expression (15), the Equations (3) and (4) can be combined to give

$$T(L,t)=\ \frac{\partial M(x,t)}{\partial x}+\ \rho I\ \frac{\partial^2\Psi(x,t)}{\partial t^2}=\ -EI\ \frac{\partial^2\Psi(x,t)}{\partial x^2}+\ \rho I\frac{\partial^2\Psi(x,t)}{\partial t^2} \quad (18)$$

This expression can be convenient in the case that E-B beam theory is preferred as a limiting case of the R-T theory, i.e., in condition κGA→∞ (18) should be considered in the calculation.

Prescribed quantities of beam length: If a prescribed displacement, for instance -, is given for the beam at any beam end, then

$$w(*,t)=\ \delta \quad (19)$$

Where the * is the coordinate of the beam end, i.e. 0 or L.

A prescribed rotation angle, for instance *, is given at the beam end. Then

$$\Psi(*,t)=\ \theta \quad (20)$$

Where the * can be 0 or L.

In this case there is a prescribed moment such as M0 (positive) which can be applied at the beam end ('*' represents 0 or L)

$$M(*,t) = -EI\,\frac{\partial\Psi(*,t)}{\partial x} = M_0 \quad (21)$$

A prescribed shear force such as P0 (positive) can be applied at a beam end (* is 0 or L).

$$T(*,t) = kGA\left[\frac{\partial w(*,t)}{\partial x} - \Psi(*,t)\right] = P_0 \quad (22)$$

In the case studied here the sleeper can be considered as a Free-Free beam and the boundary conditions should be considered as discussed above.

Results and Discussion

An analytical solution of a vibrating beam on an elastic foundation, and elastically connected to the rails, has been presented in the work of T. Dahlberg [2]. Eigen frequencies are calculated by using R–T beam theory for a beam on an elastic foundation. The beam is separated into three sections that have piecewise constant properties, with the central section of the beam being slightly thinner as compared to that of outer parts. Each one of the three parts may or may not be supported by the elastic foundation. The elastic connections to the rails are positioned at the two connections of the three sleeper sections. In his work T. Dahlberg has examined free vibration of an R–T beam on elastic foundation. Therefore, the value of q (x, t) is put as 0 into equations (7 and 8)

In the present work a MATLAB program has been generated to calculate the Eigen Frequencies of a sleeper as per the mathematical analysis done by T. Dahlberg in his work. In his work T. Dahlberg only considered a free-free boundary condition to calculate the Eigen Frequencies. But in the present work more boundary conditions have been studied mathematically which has been discussed later.

After execution of the MATLAB program we get the following Eigen frequencies for Mode-1, Mode-2 and Mode-3 vibration of a railway sleeper with free-free boundary condition. The units of vibrations are in Cps or Hz.
eigenfreq1 = 132.4221
eigenfreq2 = 338.5644
eigenfreq3 = 644.3510

These Eigen frequencies have been compared with the Eigen frequencies calculated by T. Dahlberg in his work experimentally[7].

So it is concluded that the above MATLAB program generates the satisfactory result in calculating Eigen Frequencies of a railway sleeper with free-free boundary conditions.

Next five more boundary conditions have been considered along with voids in the ballast to calculate Eigen Frequencies of the railway sleeper. This consideration creates a very nearly real situation of a railway sleeper.

Table-1
Comparison of frequencies

Mode of Vibration	Eigen frequency calculated experimentally [2]	Eigen frequency calculated in this work.	% age error
Mode 1	131 Hz	132.4221 Hz	1.05%
Mode 2	333 Hz	338.5644 Hz	1.67%
Mode 3	627 Hz	644.3510 Hz	2.76 %

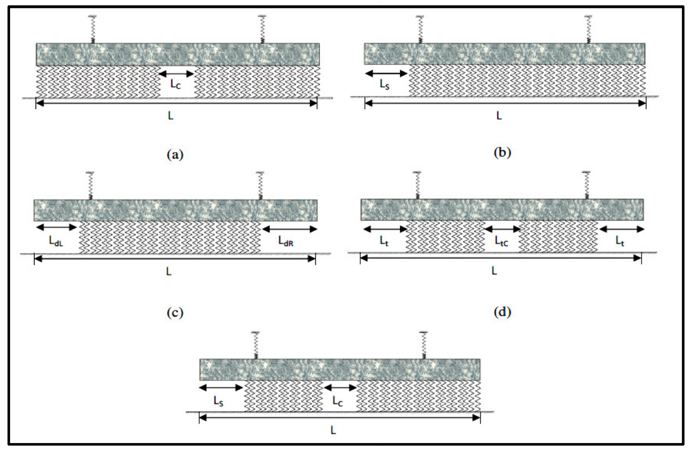

Figure-2
Sleeper/ballast contact patterns, (a) central void, (b) single hanging, (c) double hanging, (d) triple hanging, and (e) side-central voids

Single Hanging: In this section, vibration frequencies of a sleeper under conditions called 'single hanging' are evaluated. By 'single hanging' is here meant that one part at the end of the sleeper is hanging. To start with the sleeper is fully supported by the ballast along its full length. After that the contact length between sleeper and ballast diminishes by steps of five percent of the sleeper length until all support from the ballast is removed and the sleeper is free (then only hanging in the rails).

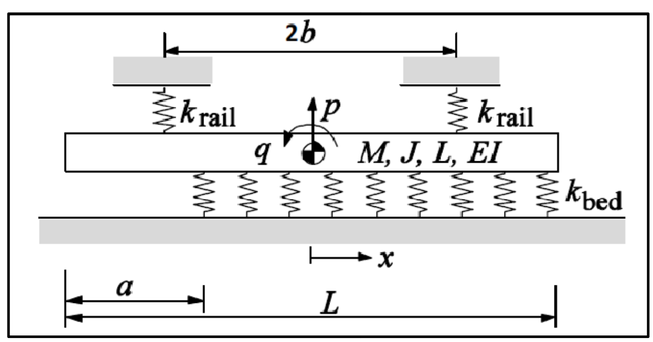

Figure-3
Rigid-body modelling of semi supported railway sleeper

The center of mass of the sleeper translates a distance p (m) upwards and rotates an angle q (rad) counter clockwise. The equations of motion become (coordinate x = 0 at sleeper mass center)

$$M\ddot{p} + \int_{\frac{L}{2}+a}^{\frac{L}{2}} k_w(p + xq)dx + 2k_r p = 0 \quad (23)$$

$$J\ddot{q} + k_w(p + xq)xdx + 2k_r p = 0$$

Here 'b' is the distance (in the x direction) between the center of the sleeper and the rail position (thus 2b between the rails). In this study the center to center distance between the rails is assumed to be 1500mm and the length of the sleeper is 2500mm so that b is 750mm. Also 'J' is mass moment of inertia (in rotation)

$$J = J_q = \frac{ML^2}{12} \quad (24)$$

Other notations are the same as before. After integration and some simplification one obtains

$$M\ddot{p} + k_w(L - a)p + k_w\frac{1}{2}a(L - a)q + 2k_r p = 0$$

$$J\ddot{q} + k_w\frac{1}{2}a(L - a)p + k_w\frac{1}{12}\{L^3 - 3L^2a + 6La^2 - 4a^3\}q + 2k_r b^2 q = 0 \quad (25)$$

For further simplification the following definitions are introduced

$$L_1 = L - a \quad L_2 = \frac{1}{2}a(L - a) L_3 = \frac{1}{12}\{L^3 - 3L^2a + 6La^2 - 4a^3\} \quad (26)$$

The following equation gives the final relation for the Eigen Frequencies:

$$\omega_{1,2}^2 = s\left\{1 \mp \sqrt{1 - \frac{(k_w^2(L_1L_3 - L_2^2) + 2k_rk_w(L_3 + L_1b^2) + ak_r^2b^2)4MJ}{(Mk_wL_3 + Jk_wL_1 + 2Mk_rb^2 + 2Jk_r)^2}}\right\} \quad (27)$$

Study the cases a= 0, a= 0.1L, a= 0.2L and so on.

In the case that a=0 one obtains

$$L_1 = L \qquad L_2 = 0 \qquad L_3 = \frac{1}{12}\{L^3\} \quad (28)$$

Following are the 1st and 2nd mode of rigid body frequencies for different position of voids generated from a MATLAB program.

Figure-4 depicts the behavior of the modes during various amount of the voided part for the 'Single hanging' condition. It is seen that the rigid-body frequencies decreased by increasing the unsupported part. The final values of these frequencies are around 70 percent of the initial values. Therefore, these changes are important. The situations for the first and last points are explained above (the modes are uncoupled).

Central Void: Now changing the expression of L1, L2 and L3 as mentioned in Equation 26 to the following we get Eigen Frequencies by substituting new values of L1, L2 and L3 in Equation 27.

Table-2
Rigid Body Frequencies for Single Hanging configuration

a/L	Mode 1 rigid body frequency (Hz)	Mode 2 rigid body frequency (Hz)
0	81.9208e+000	83.5794e+000
0.05	79.2099e+000	82.5464e+000
0.1	75.9169e+000	82.4917e+000
0.15	72.9573e+000	82.4886e+000
0.2	70.3656e+000	82.4837e+000
0.25	68.1334e+000	82.4591e+000
0.3	66.2409e+000	82.4000e+000
0.35	64.6622e+000	82.2907e+000
0.4	63.3669e+000	82.1124e+000
0.45	62.3221e+000	81.8431e+000
0.5	61.4942e+000	81.4566e+000
0.55	60.8496e+000	80.9222e+000
0.6	60.3566e+000	80.2040e+000
0.65	59.9862e+000	79.2598e+000
0.7	59.7121e+000	78.0401e+000
0.75	59.5115e+000	76.4863e+000
0.8	59.3642e+000	74.5281e+000
0.85	59.2522e+000	72.0795e+000
0.9	59.1563e+000	69.0339e+000
0.95	59.0391e+000	65.2662e+000
1	58.5764e+000	60.8744e+000

New expressions for L1, L2 and L3 are

$$L_1 = L - \beta(l - a)$$

$$L_2 = 0 \quad (29)$$

$$L_3 = \frac{1}{12} \times (L^3 - a^3)$$

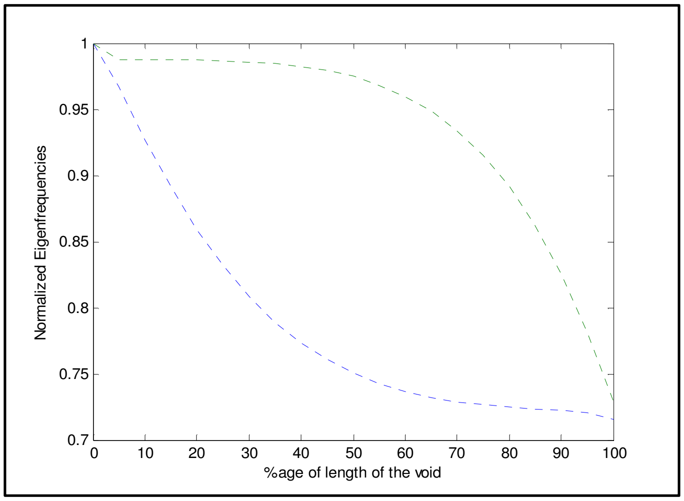

Figure-4
Normalized Eigen Frequencies of single hanging sleeper

Table-3
Rigid Body Frequencies for Central Void configuration

a/L	Mode 1 rigid body frequency (Hz)	Mode 2 rigid body frequency (Hz)
0	81.9208e+000	83.5794e+000
0.05	80.9137e+000	83.5769e+000
0.1	79.8939e+000	83.5597e+000
0.15	78.8609e+000	83.5131e+000
0.2	77.8142e+000	83.4222e+000
0.25	76.7532e+000	83.2722e+000
0.3	75.6774e+000	83.0479e+000
0.35	74.5860e+000	82.7338e+000
0.4	73.4785e+000	82.3140e+000
0.45	72.3540e+000	81.7719e+000
0.5	71.2117e+000	81.0897e+000
0.55	70.0508e+000	80.2486e+000
0.6	68.8703e+000	79.2280e+000
0.65	67.6693e+000	78.0051e+000
0.7	66.4465e+000	76.5541e+000
0.75	65.2008e+000	74.8454e+000
0.8	63.9309e+000	72.8440e+000
0.85	62.6352e+000	70.5074e+000
0.9	61.3121e+000	67.7829e+000
0.95	59.9599e+000	64.6024e+000
1	58.5764e+000	60.8744e+000

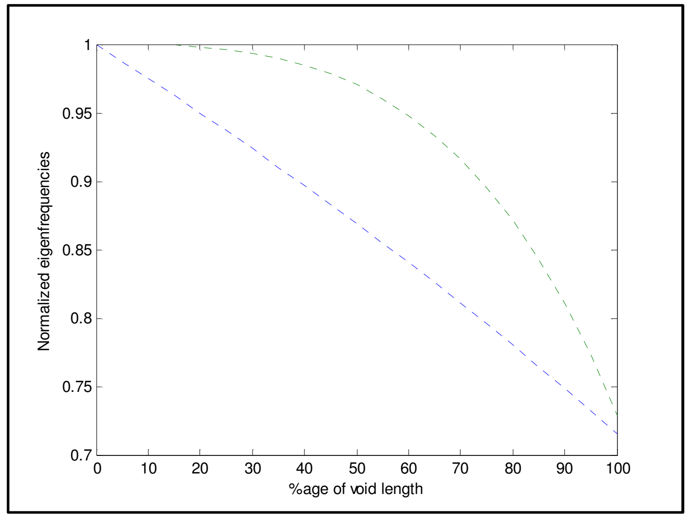

Figure-5
Normalized Eigen Frequencies of sleeper having central void

Double Hanging: Now expression of L1, L2 and L3 as mentioned in Equation 26 will be changed as per bellow for the 'Double Hanging' condition of a railway sleeper and a MATLAB program has been written below to evaluate the frequencies of the sleeper under this condition and also a graph has been generated to depict the change in frequencies with void positions.

New expressions for L1, L2 and L3 for this condition are

$$L_1 = L - \beta(l - a)$$

$$L_2 = \frac{(\beta^2 \times L^2 - L^2 \times \beta - a^2 + L \times a)}{2}$$

$$L_3 = \frac{\left(\left(\frac{L}{2} - \beta \times L\right)^3 - \left(-\frac{L}{2} + a\right)^3\right)}{3}$$

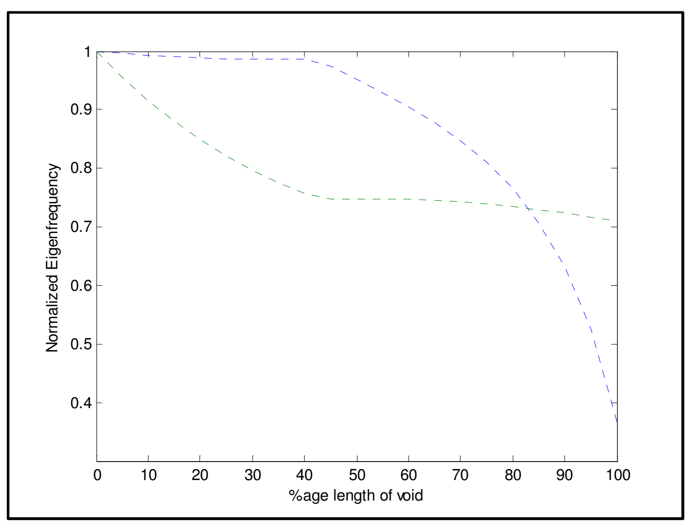

Figure-6
Normalized Eigen Frequencies of a double-hanging sleeper

Table-4
Rigid Body Frequencies for Double Hanging configuration

a/L	Mode 1 rigid body frequency (Hz)	Mode 2 rigid body frequency (Hz)
0	61.4942e+000	81.4566e+000
0.05	61.2681e+000	77.8108e+000
0.1	61.0731e+000	74.5555e+000
0.15	60.9147e+000	71.6709e+000
0.2	60.7970e+000	69.1287e+000
0.25	60.7220e+000	66.8930e+000
0.3	60.6870e+000	64.9222e+000
0.35	60.6809e+000	63.1741e+000
0.4	60.6523e+000	61.6383e+000
0.45	59.9451e+000	60.9024e+000
0.5	58.5764e+000	60.8744e+000
0.55	57.1554e+000	60.8647e+000
0.6	55.6578e+000	60.8112e+000
0.65	54.0117e+000	60.6908e+000
0.7	52.1231e+000	60.4909e+000
0.75	49.8688e+000	60.2084e+000
0.8	47.0836e+000	59.8486e+000
0.85	43.5340e+000	59.4243e+000
0.9	38.8590e+000	58.9517e+000
0.95	32.3942e+000	58.4473e+000
1	22.3990e+000	57.9254e+000

Side Central: For this condition expression of L1, L2 and L3 will be changed as per bellow in Equation 24. For the ‘Double Hanging’ condition of a railway sleeper a MATLAB program has been written below to evaluate the frequencies of the sleeper and also a graph has been generated to depict the change in frequencies with void positions.

New expressions for L1, L2 and L3 for this condition are

$$L_1 = L - \beta(l - a)$$

$$L_2 = \frac{a}{2} \times (L - a)$$

$$L_3 = \frac{\left(\left(-\beta \times \frac{L}{2}\right)^3 - \left(-\frac{L}{2} + a\right)^3 - \left(\beta \times \frac{L}{2}\right)^3\right)}{3}$$

Table-5
Rigid Body Frequencies for Side Central configuration

a/L	Mode 1 rigid body frequency(Hz)	Mode 2 rigid body frequency(Hz)
0	71.2117e+000	81.0897e+000
0.05	69.6439e+000	78.6636e+000
0.1	67.3256e+000	77.3669e+000
0.15	64.6596e+000	76.7879e+000
0.2	61.9985e+000	76.5583e+000
0.25	59.5115e+000	76.4863e+000
0.3	57.2641e+000	76.4759e+000
0.35	55.2749e+000	76.4706e+000
0.4	53.5395e+000	76.4293e+000
0.45	52.0413e+000	76.3161e+000
0.5	50.7573e+000	76.0955e+000
0.55	49.6604e+000	75.7300e+000
0.6	48.7223e+000	75.1788e+000
0.65	47.9138e+000	74.3965e+000
0.7	47.2053e+000	73.3318e+000
0.75	46.5665e+000	71.9266e+000
0.8	45.9631e+000	70.1145e+000
0.85	45.3517e+000	67.8205e+000
0.9	44.6638e+000	64.9646e+000
0.95	43.7650e+000	61.4820e+000
1	42.3236e+000	57.4083e+000

Figure-7 shows the 1st mode and 2nd mode rigid-body frequencies of a railway sleeper with for different positions of void and for different length of void.

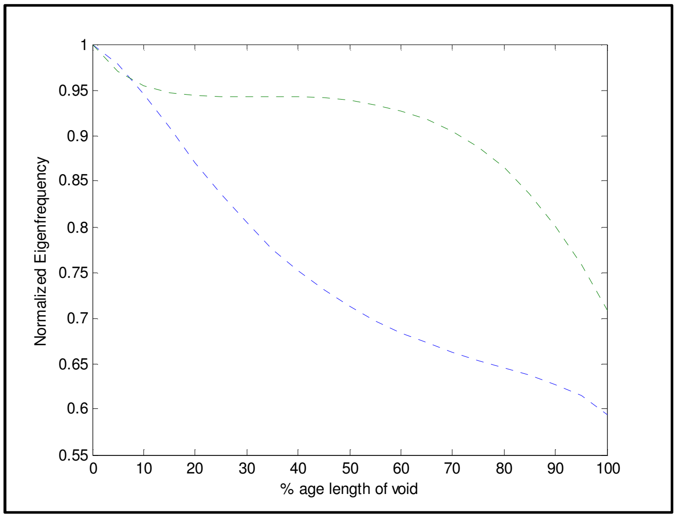

Figure-7
Normalized Eigen Frequencies of a sleeper with side central void

Conclusion

In MATLAB and by mathematical analysis, detailed study of Eigen frequencies, Eigen modes and vibration, of an in-situ concrete railway sleeper have been carried out. The sleeper can be modelled in either of the ways i.e. as an elastic body or as a rigid body. Two discrete springs are used to link sleeper with the rail and it is supported by a constantly distributed spring modelling the ballast. In between sleeper and the ballast, pockets in the form of voids may appear Five patterns of irregularities have been taken into account and estimated. Also a comparison between in situ sleeper with rail stiffness and in situ sleeper without rail stiffness has been made.

In the symmetric cases the rigid-body modes are pure translation or rotation. Once the model is not symmetric the first and second rigid-body modes are combination of rotation and translation. When the mode is pure translation the center of oscillation is at infinity and for pure rotation the center of oscillation coincides with the mass center of the beam.

The following conclusions from this report can be drawn: The foundation stiffness effects the two rigid-body Eigen frequencies the maximum (decreasing up to 70 percent of its initial value when the foundation stiffness is removed). The higher Eigen frequencies are more or less unaffected by the foundation stiffness. The rate of Eigen frequency reducing for bending modes is directly related to the place of the pocketing (the voiding). If the voiding part is placed under the high amplitude section of the bending mode, it decreases the Eigen frequency a lot, and vice versa. The fifth bending mode stays the same during the voiding of ballast. The impact of rail pad (and rail) stiffness on the sleeper rigid-body Eigen frequencies is considerable, but for the bending-mode Eigen Frequencies this influence is negligible.

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