



# Analysis and Mitigation of Stress Concentration factor in Rectangular plate with Central Circular hole under Transverse Loading

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## Abstract

Many techniques are available for the study of stress concentration around holes in rectangular plates. The distribution of stress and deflection in isotropic and orthotropic rectangular plate with central circular hole under transverse static loading has been done by applying finite element method. The analysis has been done for different size of hole for two different boundary conditions. The models have been modified by applying two methods for mitigation of SCF. The finite element formulation have carried out in software ANSYS.

**Keywords:** Finite element method, SCF, Transverse loading, Boundary conditions.

## Introduction

Rectangular plates with central circular hole under transverse loading have found widespread applications in various fields of engineering. Knowledge of stresses, deflection and study of stress concentration which arises from any abrupt changes in geometry of plate are required. Analytical solutions are available for SCF in literature. The work carried out by various researchers for analysis of SCF is compiled and presented by Peterson<sup>1</sup>. Formulation and graphs have given for different discontinuities under different loading conditions.

Fedorov<sup>2</sup> studied the effect of D/A ratio and different loading on stress concentration in a glass reinforcement plastic specimen and extended the work on anisotropy of material. Fedorov has studied 12 different models for different loads and sizes. Paul and Rao<sup>3,4</sup> evaluated stress and stress concentration in fibre reinforced composite fibrous plate containing central circular hole and two coaxial holes subjected to transverse load by using finite element method and Lo Christensen Wu higher order bending theory.

Mittal and Jain<sup>5</sup> analyzed the effect of fibre orientation on stress concentration factor in fibrous plate with central circular hole under transverse static loading by using two dimensional finite element methods. Mittal and Jain<sup>6</sup> analyzed the design of simply supported isotropic square plate with central circular hole subjected to transverse static loading by Finite Element Method. They have reported around 30% reduction in SCF. They proposed four auxiliary holes around circular hole in square geometry. Rao et. al.<sup>7</sup> evaluated the stress around square and rectangular cutouts in symmetric laminates. It has been analyzed that the maximum stress and its location is mainly influenced by the type of loading<sup>8-12</sup>. Worked on composite plates and laminated plates under different loading conditions.

## Problem Description

Rectangular plate of 400 mm\*100 mm having central circular hole of diameter D under uniformly distributed load in transverse direction has been considered (Figure-1). Isotropic material and four different composite materials with respective material properties are selected for the analysis. Plate has been modified for mitigation of SCF by introducing auxiliary holes around the central circular hole.

To study the variation in SCF model has been generated using a 3-D solid element. An eight noded structural 3D Shell element specified as Shell 93 in ANSYS of 1 mm length is selected for modeling. Two boundary conditions considered for analysis are simply supported and fixed, Figure-2. The loading is kept constant in solid plate and in plate with hole i.e. 0.02N/m<sup>2</sup>.

All the models of different plate material have been analyzed for both boundary conditions. In plate A all the edges are fixed and in plate B all the edges are simply supported. Stress concentration factor has been determined for stresses in  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_{xy}$  and also  $\sigma_{von}$ . Deflection in Z direction has also been analyzed.

Two models are considered for analysis Model 1 is plate with central circular hole. Model 1 has been modified by introducing two auxiliary holes around main hole as Model 2 has been generated. For further analysis, Model 2 has been modified by introducing four auxiliary holes around central main hole, as Model 3. The size and location of auxiliary holes are optimized. The SCF for all the considered stresses has been studied. The deflection in Z direction is compared with the deflection of solid plate in Z direction for similar geometric and loading conditions.

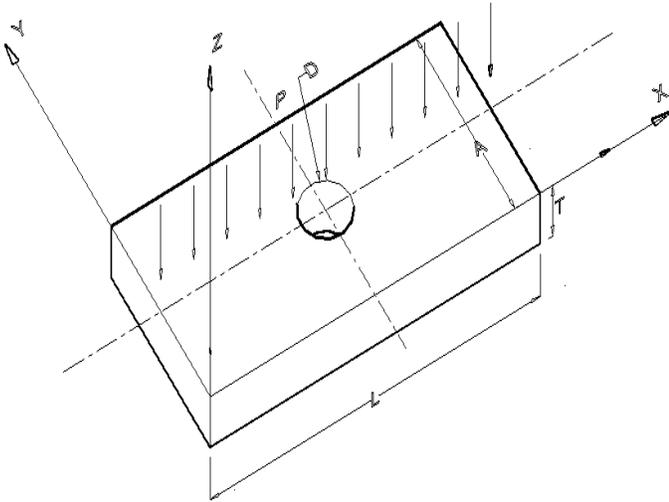


Figure-1

Plate model with hole under transverse loading, Model 1

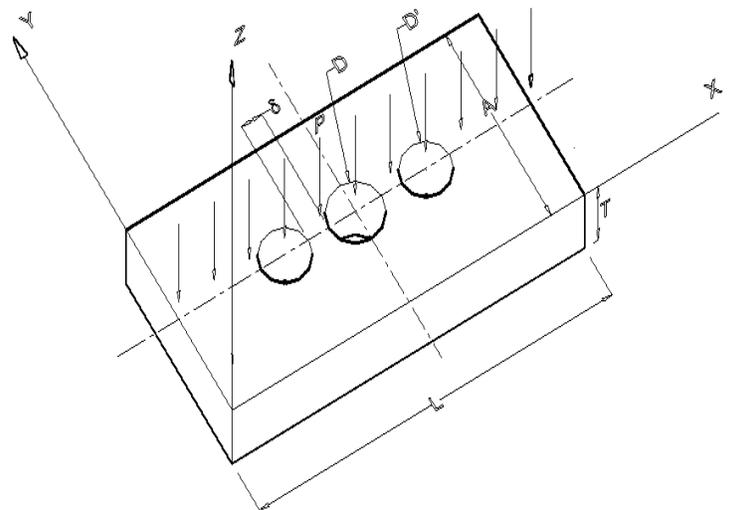
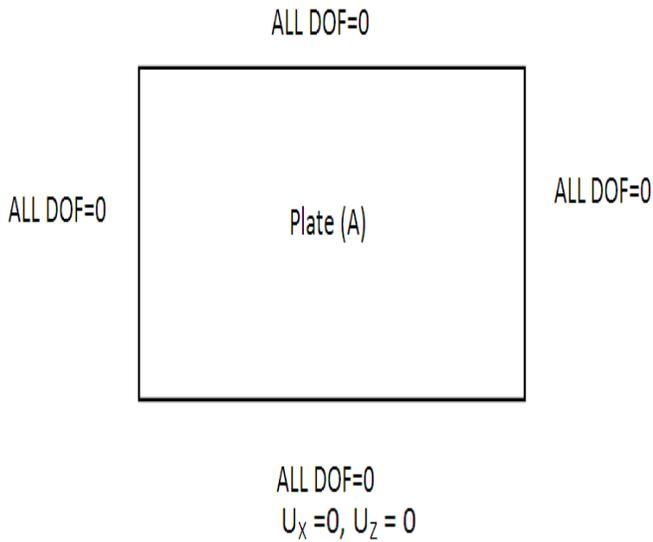


Figure-3

Plate with one set of auxiliary holes under transverse loading, Model 2



ALL DOF=0

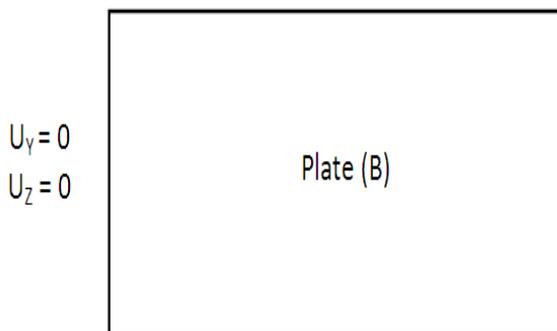
ALL DOF=0

Plate (A)

ALL DOF=0

ALL DOF=0

$U_x=0, U_z=0$



$U_y=0$

$U_z=0$

Plate (B)

$U_y=0$

$U_z=0$

$U_x=0, U_z=0$

Figure-2

Boundary conditions at all edges of Plate A fixed, Plate B simply supported

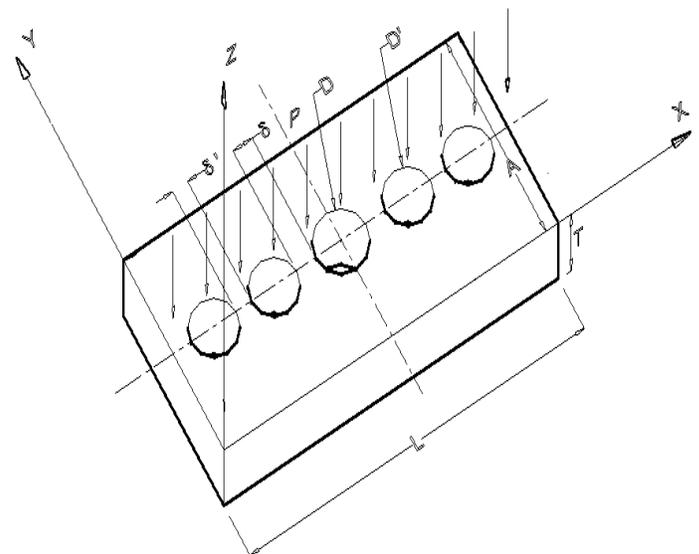


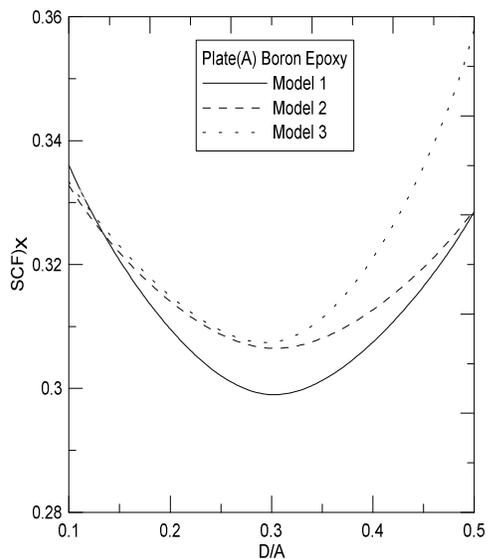
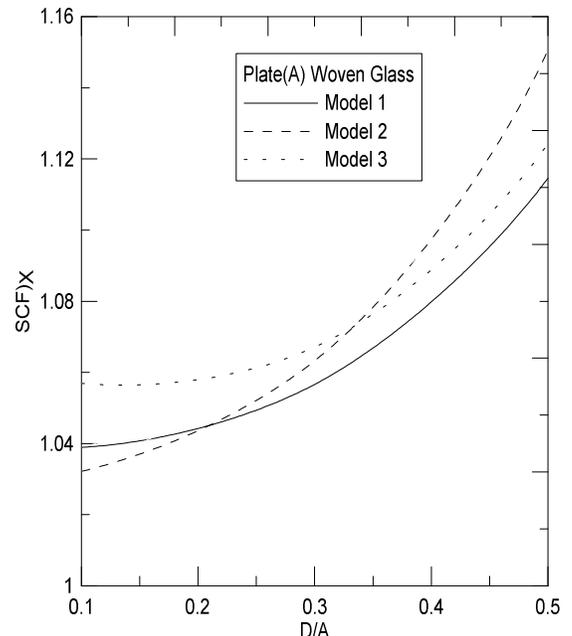
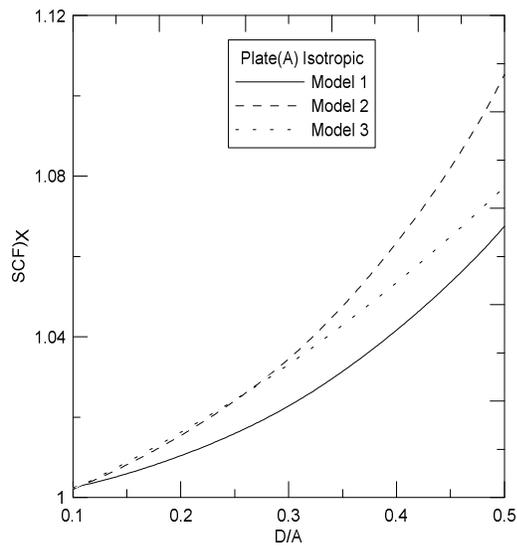
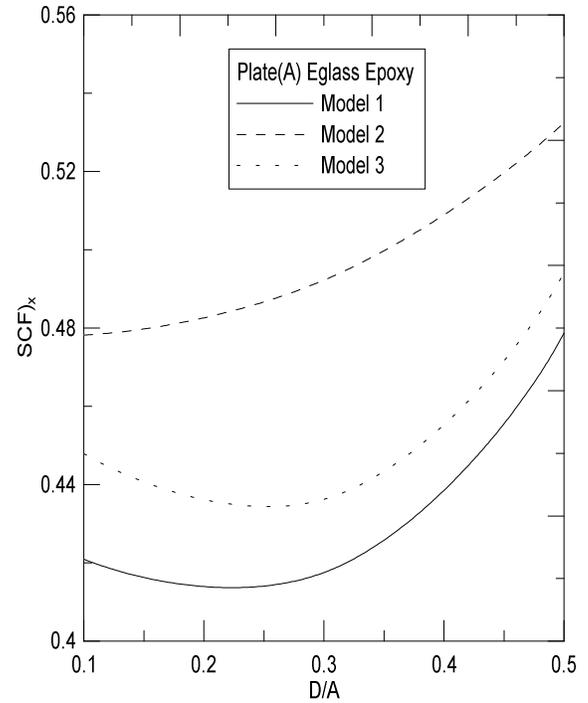
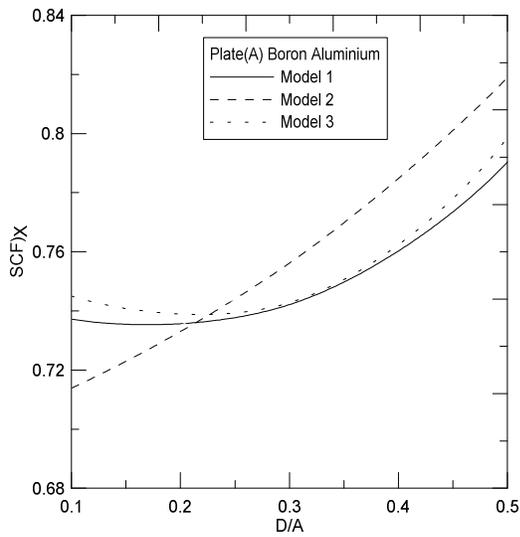
Figure-4

Plate with two sets of auxiliary holes under transverse loading, Model 3

## Results and Discussion

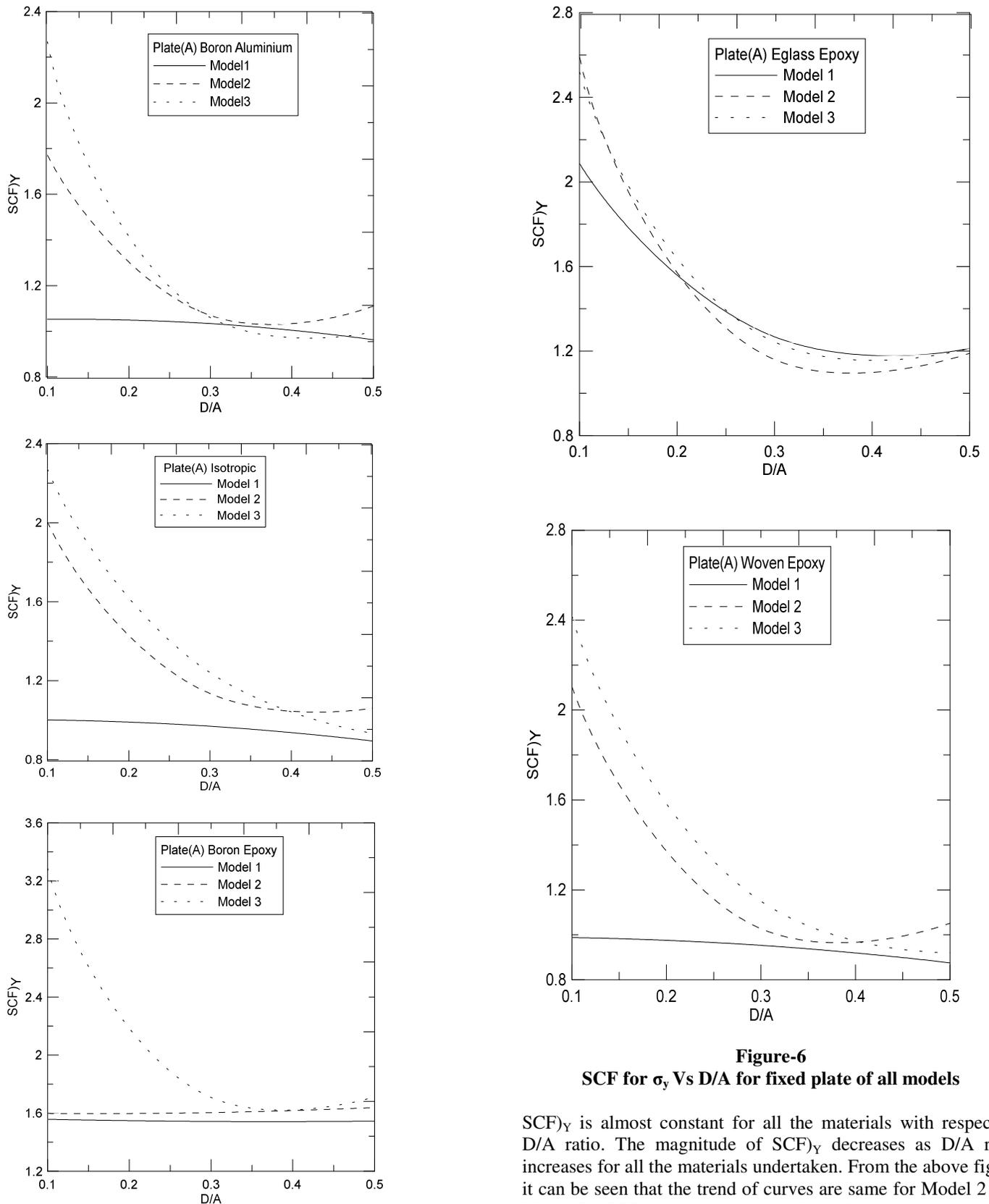
Results of SCF for different models thus obtained for various materials are presented in graphical forms.

**Fixed Plate under transverse loading Plate A: Variations of SCF for  $\sigma_x$ ,  $\sigma_y$ ,  $\tau_{xy}$ ,  $\sigma_{von}$  and  $U_z/U_{zs}$  versus D/A:** The SCF's for all the stresses have been plotted against D/A ratio in case of all the models. The ratio of deflection in Z direction has also been plotted against D/A ratio for all the models. The trend of plots is different for special materials and for different models.



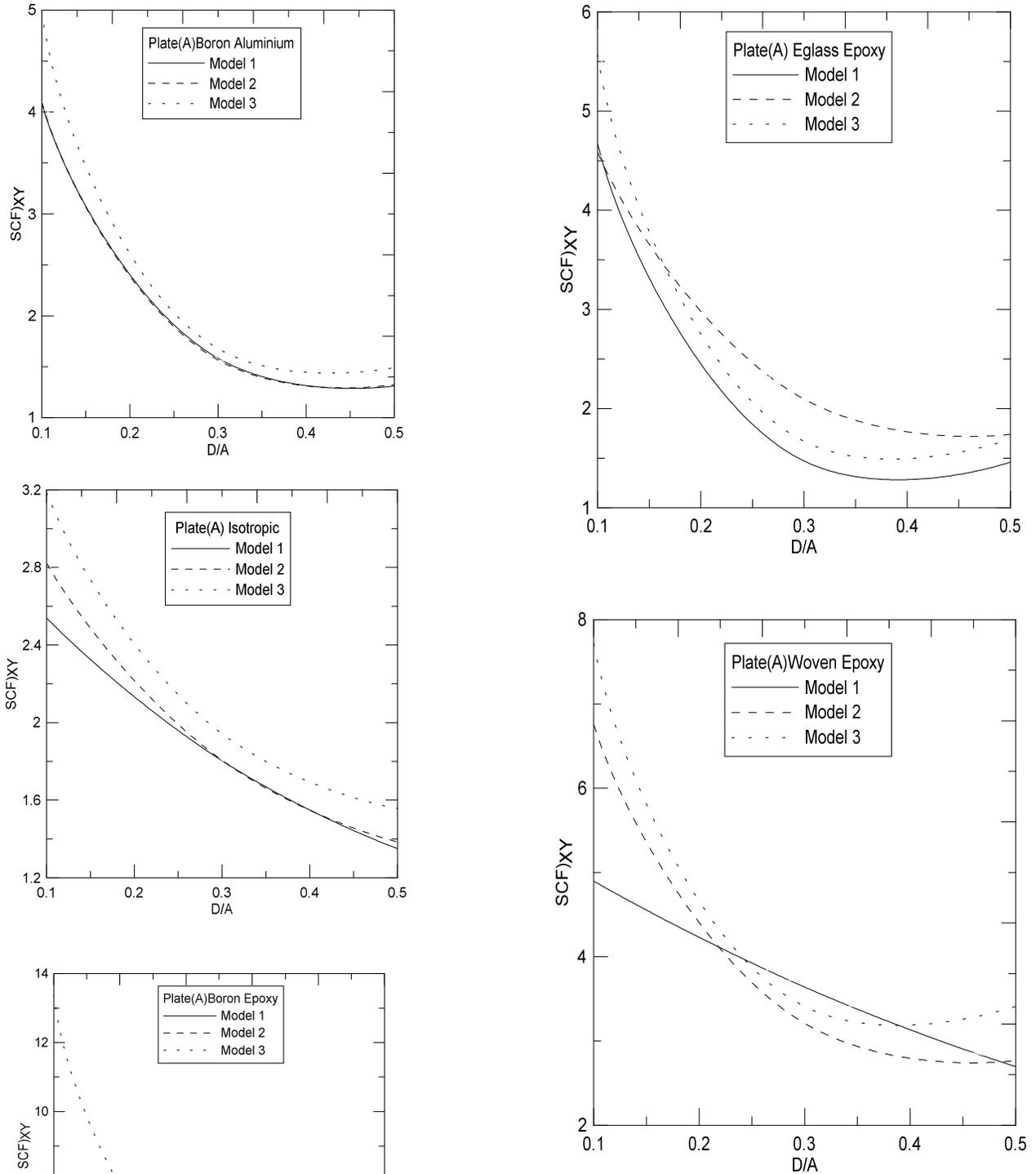
**Figure-5**  
 $SCF_x$  Vs  $D/A$  for fixed plate of all models

It has been noted that the effect of  $D/A$  ratio is prominent for  $SCF$  of  $\sigma_x$  for all cases considered. It is observed from Figure-5, that there is no reduction in  $SCF$  by providing auxiliary holes along with the main hole. The trend of curves for Model 1 and Model 3 are same. The  $SCF_x$  is more in case of Model 3 than that for Model 1. The  $SCF_x$  is maximum in case of Model 2 for higher  $D/A$  ratio. The  $SCF_x$  increased with  $D/A$  ratio for all the materials.



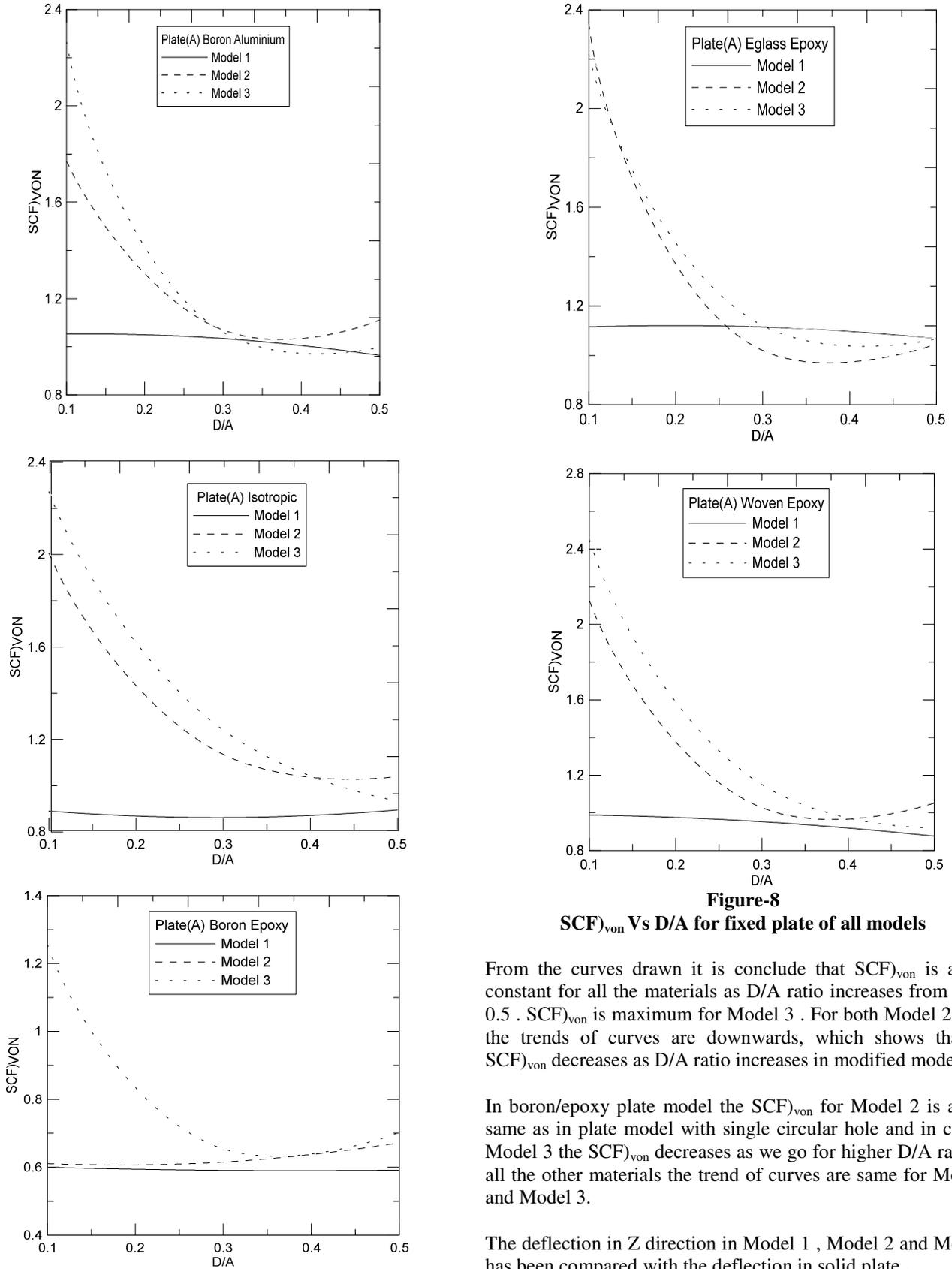
**Figure-6**  
 SCF for  $\sigma_y$  Vs D/A for fixed plate of all models

(SCF)<sub>Y</sub> is almost constant for all the materials with respect to D/A ratio. The magnitude of (SCF)<sub>Y</sub> decreases as D/A ratio increases for all the materials undertaken. From the above figure it can be seen that the trend of curves are same for Model 2 and Model 3. (SCF)<sub>Y</sub> is maximum for Model 3.



**Figure-7**  
**SCF<sub>xy</sub> Vs D/A for fixed plate of all models**

The curves show downwards trend for all the materials as D/A ratio increases. The curve is a straight line for all the materials expect for e- glass/epoxy considering Model 1. The (SCF)<sub>XY</sub> is maximum for Model 3 and minimum for Model 1. SCF for  $\sigma_{xy}$  shows less influence of D/A ratios for all materials considered.

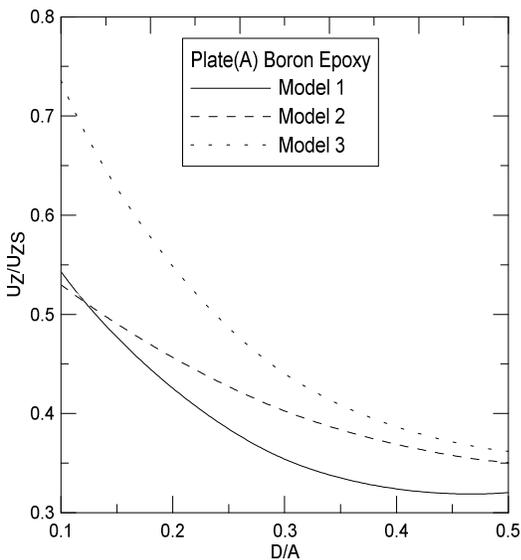
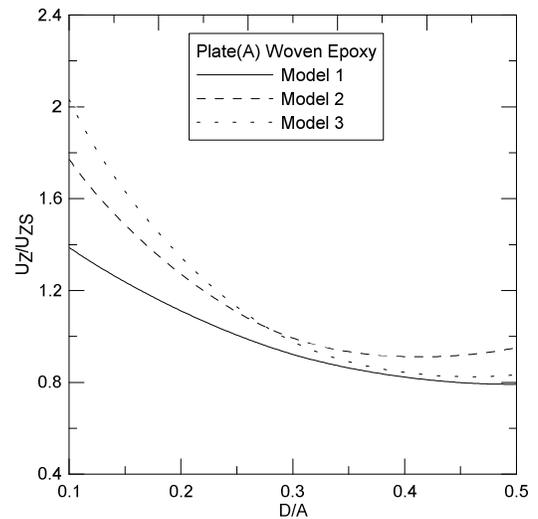
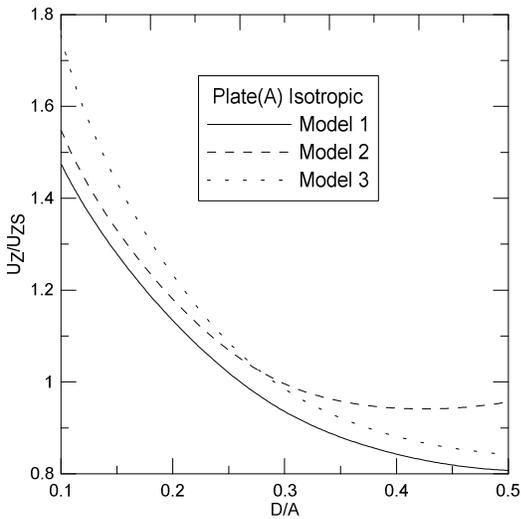
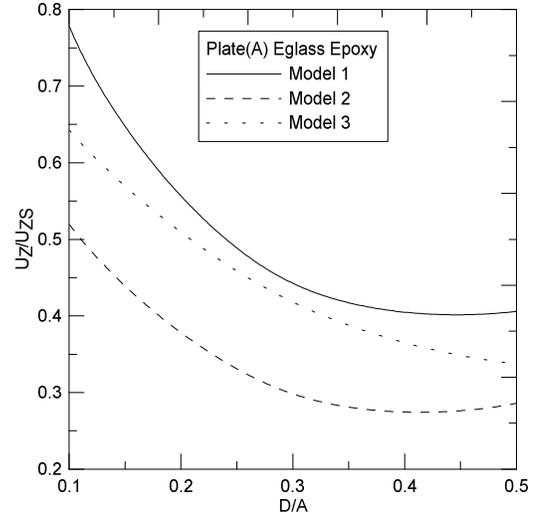
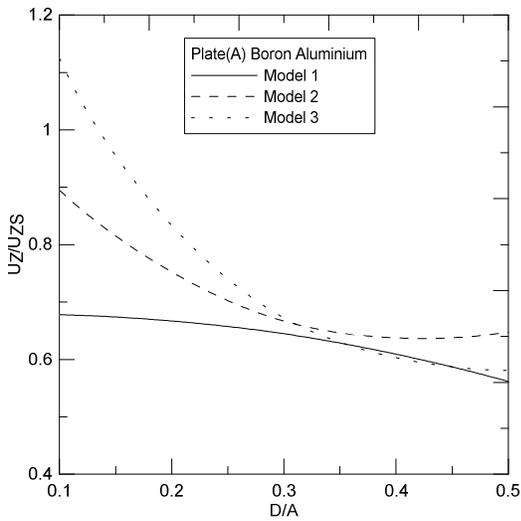


**Figure-8**  
**SCF)<sub>von</sub> Vs D/A for fixed plate of all models**

From the curves drawn it is conclude that  $SCF)_{von}$  is almost constant for all the materials as  $D/A$  ratio increases from 0.1 to 0.5 .  $SCF)_{von}$  is maximum for Model 3 . For both Model 2 and 3 the trends of curves are downwards, which shows that the  $SCF)_{von}$  decreases as  $D/A$  ratio increases in modified models.

In boron/epoxy plate model the  $SCF)_{von}$  for Model 2 is almost same as in plate model with single circular hole and in case of Model 3 the  $SCF)_{von}$  decreases as we go for higher  $D/A$  ratio. In all the other materials the trend of curves are same for Model 2 and Model 3.

The deflection in Z direction in Model 1 , Model 2 and Model 3 has been compared with the deflection in solid plate .



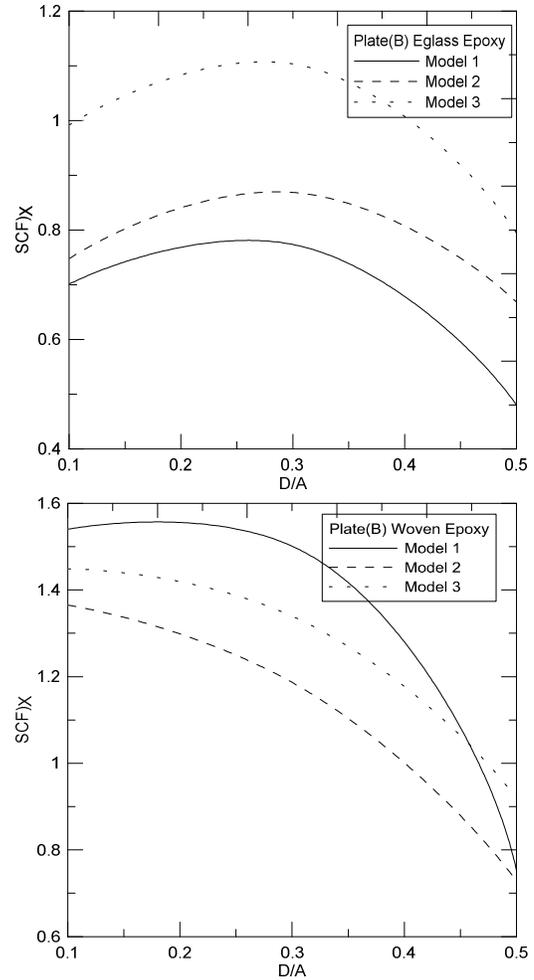
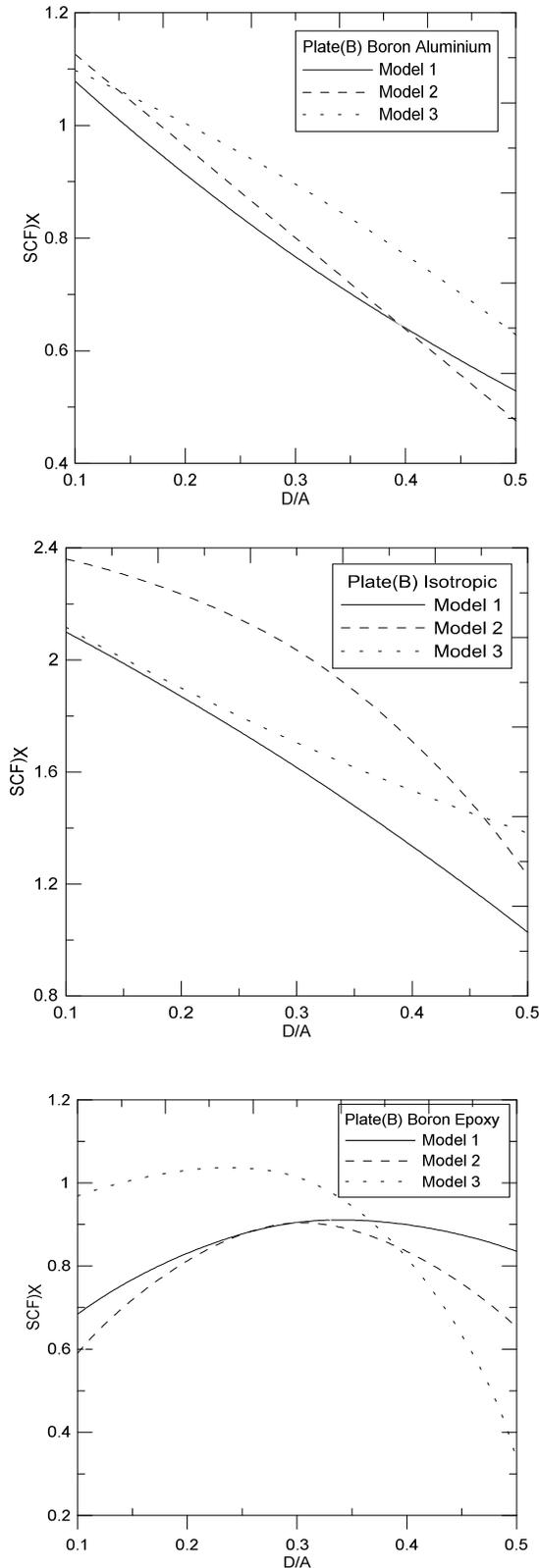
**Figure-9**  
 $U_z/U_{zs}$  Vs  $D/A$  for fixed plate of all models

Deformation in plate in Z direction in solid plate and deformation in modified model are compared. The curves are plotted between ratio of these deflection and  $D/A$  ratio for all the models. The influence of  $D/A$  ratio is substantial for ratios of  $U_z/U_{zs}$  for all cases of materials.

The curves show that this deformation ratio decreases for isotropic material and E - glass/ epoxy after introducing the auxiliary holes around main hole. This reduction is more for lower values of  $D/A$  ratio. The curves represents that this ratio is almost constant for Model 2 and Model 3 for all  $D/A$  ratios.

For Boron/ Aluminium, Boron/Epoxy and Woven/Epoxy this ratio increases as auxiliary holes are introduced. This shows that by introducing auxiliary holes in a fixed plate of these materials the deformation increases as compared to the solid plate of same material and under same loading conditions for all  $D/A$  ratio.

**Simply supported plate under transverse loading, Plate B**  
**Variations of SCF for  $\sigma_x$ ,  $\sigma_y$ ,  $\tau_{xy}$ ,  $\sigma_{von}$  and  $U_z/U_{zs}$  versus D/A**

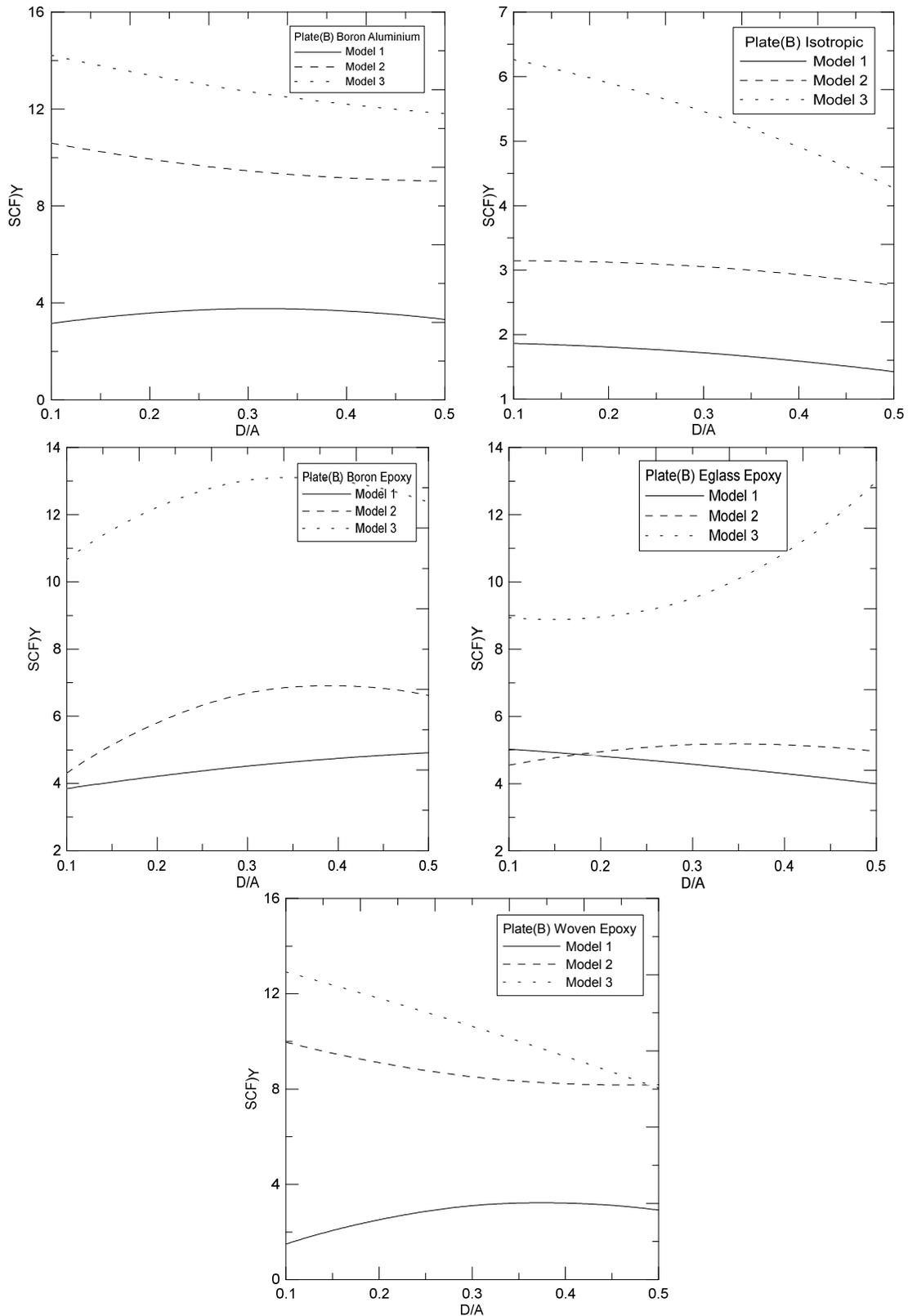


**Figure-10**  
**SCF)<sub>x</sub> Vs D/A for simply supported plate of all models**

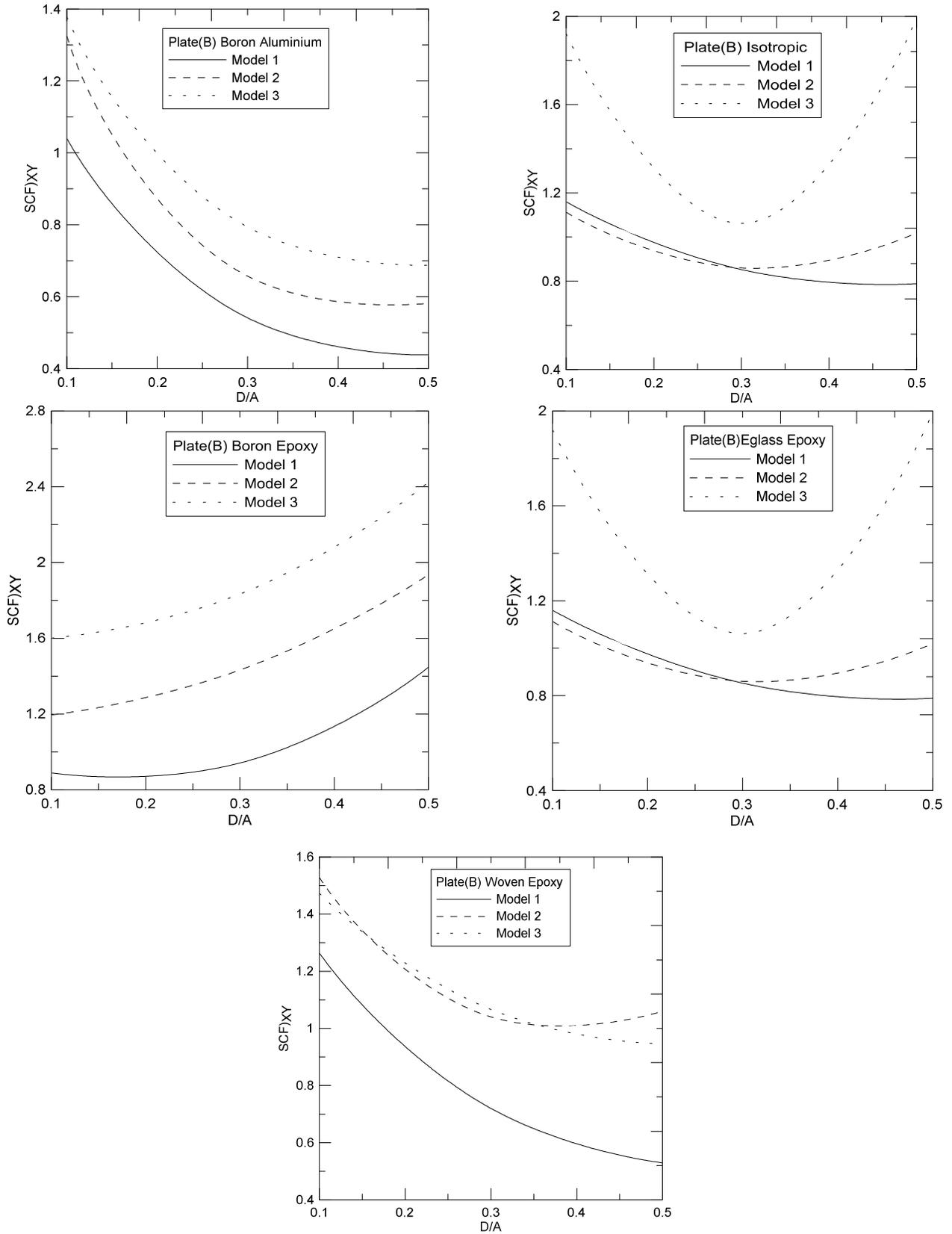
SCF for  $\sigma_x$  varies from 0.8 to 1.4 for all the cases. SCF decreases as D/A ratio increases for all the materials except for boron/epoxy. In boron/epoxy plate its magnitude increases somewhat with D/A ratio.

For boron/aluminium the SCF is 0.8 for D/A ratio =0.1 which decreases and is minimum, equal to 0.53 for D/A =0.5 for Model 1. For Model 2 and 3, the (SCF)<sub>x</sub> is 1.12 for D/A=0.1 and decreases as D/A ratio increases. The trend of curve for Model 2 and Model 3 is almost similar.

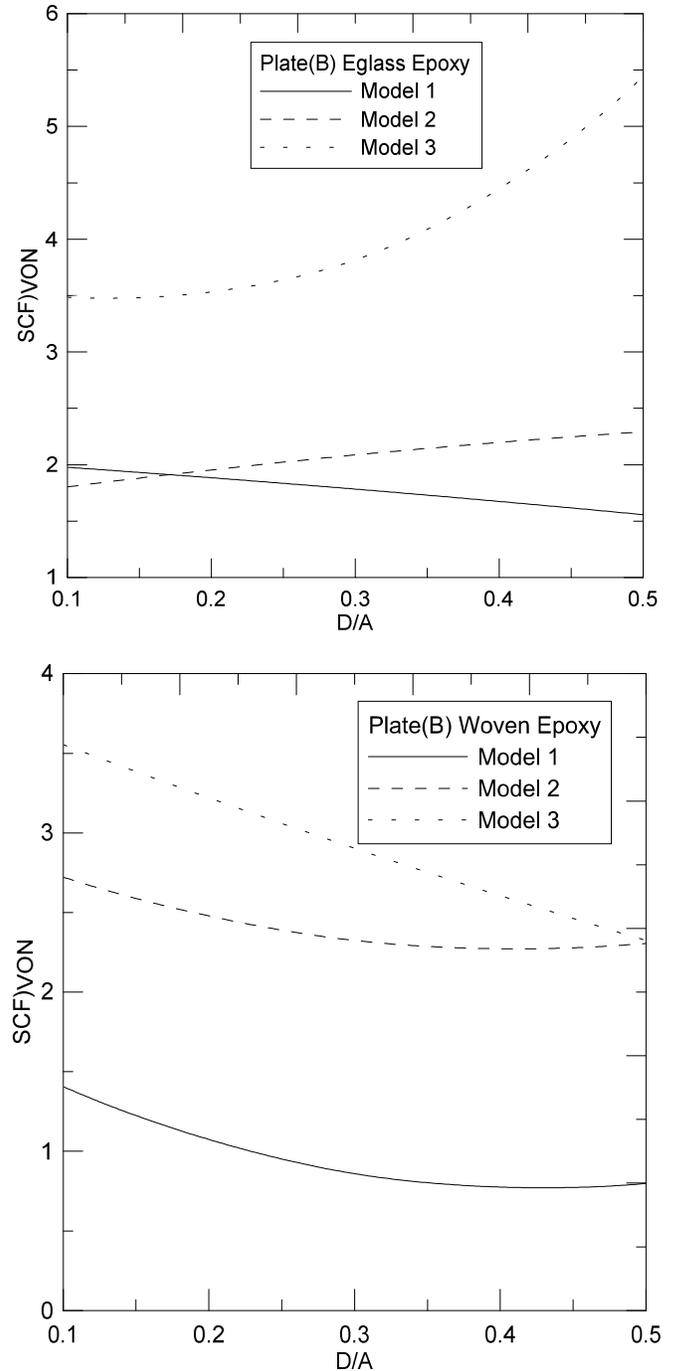
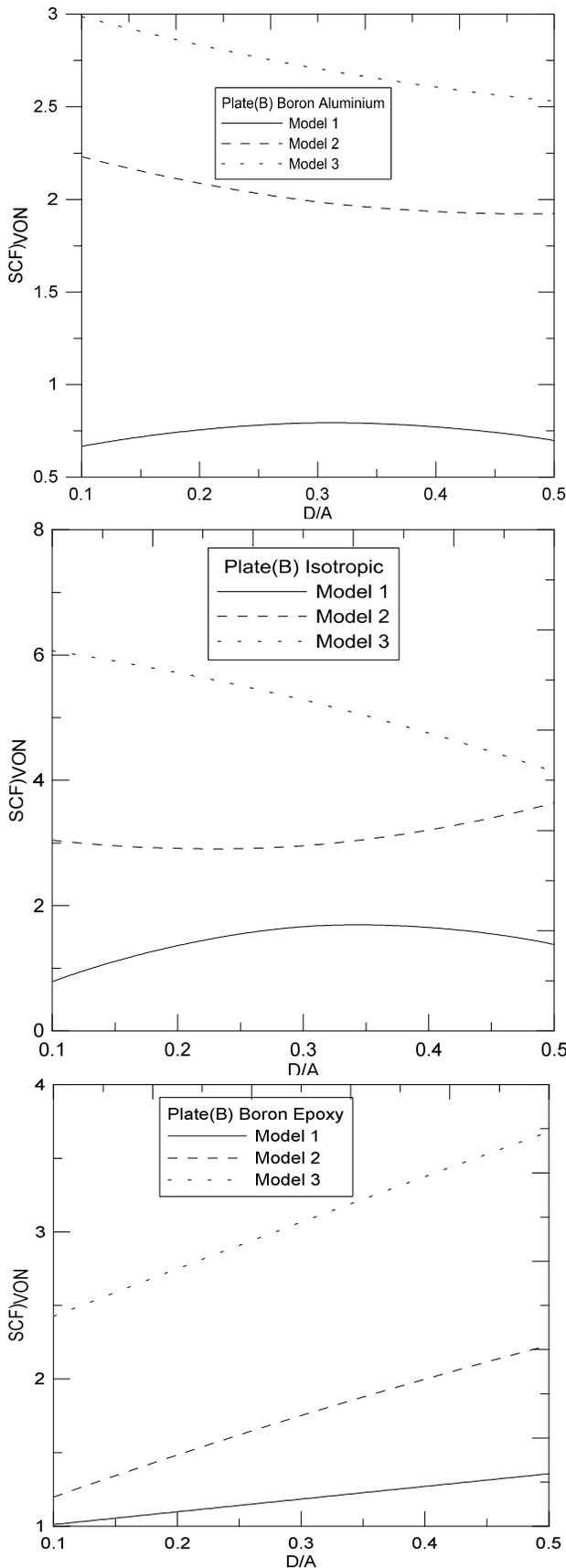
SCF)<sub>y</sub> decreases with higher values of D/A ratio for all the materials considered. For boron/ aluminium the (SCF)<sub>y</sub> is almost constant for all D/A ratio. SCF)<sub>y</sub> is 3 for all D/A ratio for Model 1. It can be seen from the Figure-11 that the SCF)<sub>y</sub> is 10.5 for D/A = 0.1 for Model 2 and reduces to 9.0 for D/A = 0.5. By studying the plot for Model 3 it can be deduced that the value of SCF)<sub>y</sub> decreases as D/A ratio increases. SCF)<sub>y</sub> is maximum for Model 3 and it is varying from 14.0 to 12.0 for all D/A ratios.



**Figure-11**  
 SCF<sub>Y</sub> Vs D/A for simply supported plate of all models



**Figure-12**  
**SCF)<sub>XY</sub> Vs D/A for simply supported plate of all models**



**Figure-13**

**SCF)<sub>VON</sub> Vs D/A for simply supported plate of all models**

SCF)<sub>VON</sub> is between 0.5 to 2.0 for D/A=0.1 for all materials, as we analyze the curves. SCF)<sub>VON</sub> is 2, which is maximum for e-glass/ epoxy and increases for Model 2 and Model 3. For Model 3 SCF)<sub>VON</sub> increases from 3.5 to 5.5. For all the material considered SCF)<sub>VON</sub> is minimum for Model 1, increases for Model 2 and is maximum for Model 3. Curves are not following any particular trend, as the trend is different for all the models.

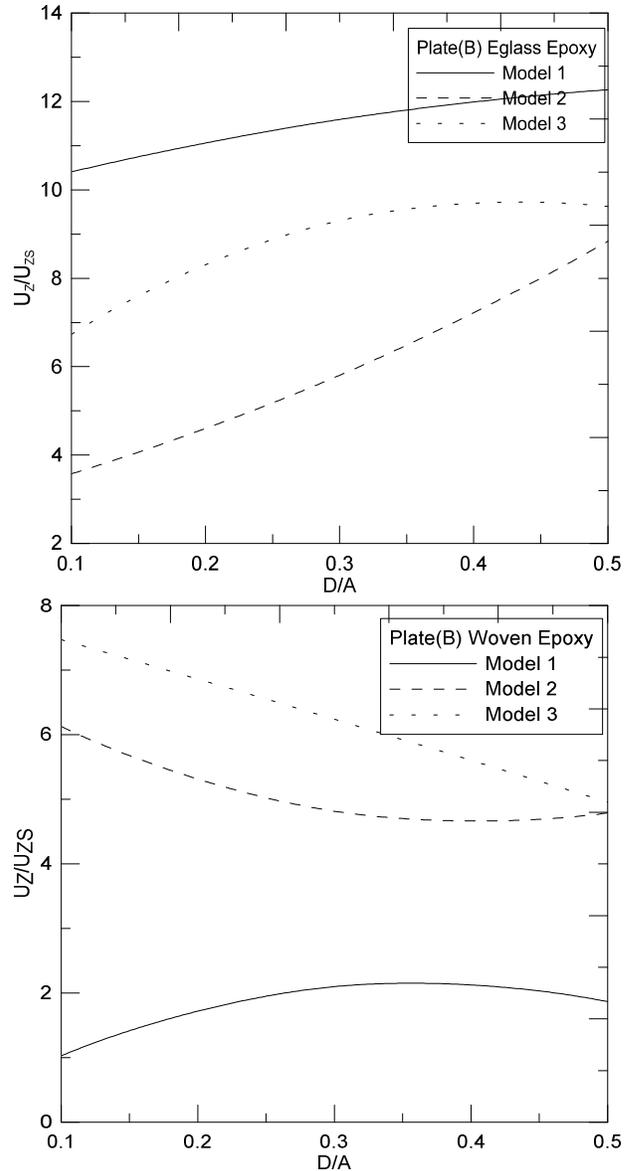
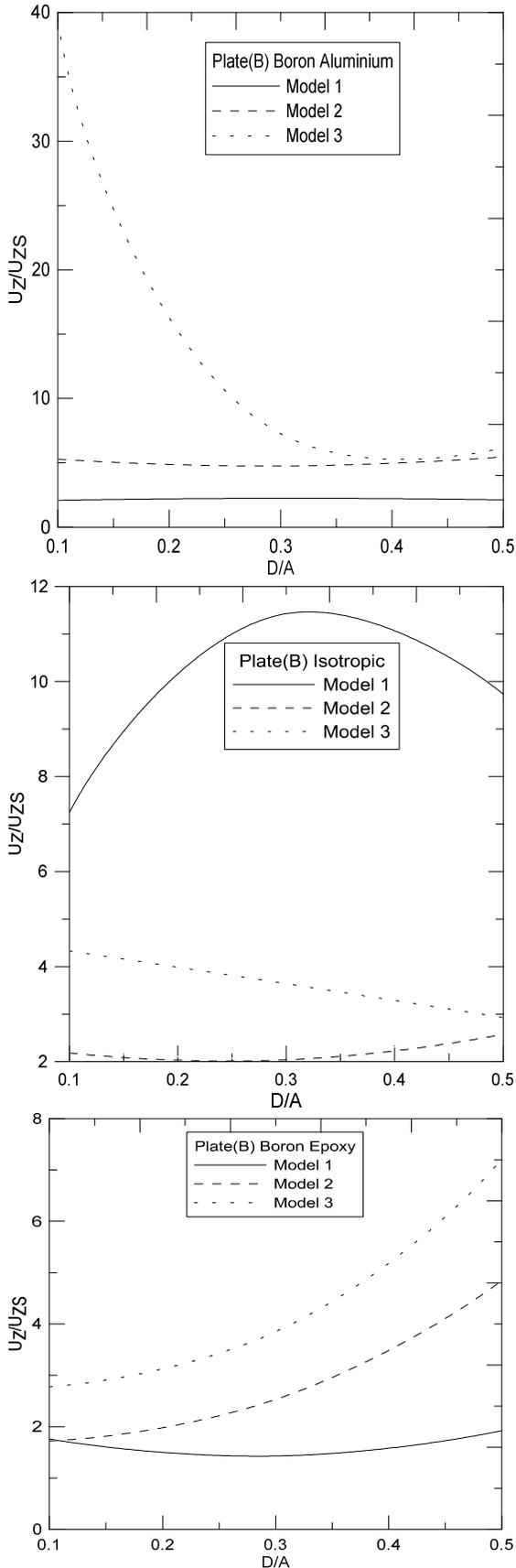


Figure-14

**$U_z/U_{zs}$  VS  $D/A$  for simply supported plate of all models**

Deformation in plate in Z direction in solid plate and deformation in modified model are compared. The curves are plotted between ratio of these deflection and  $D/A$  ratio for all the models.

For isotropic material  $U_z/U_{zs}$  increases from 7.0 to 11 from  $D/A=0.1$  to 0.3 then decreases to 9.7 for  $D/A =0.5$ .  $U_z/U_{zs}$  is almost constant for boron/epoxy.

For boron/aluminium, boron/epoxy and woven/epoxy this ratio increases as auxiliary holes are introduced. This shows that by introducing auxiliary holes in a fixed plate of these materials the deformation increases as compared to the solid plate of same material and under same loading conditions for all  $D/A$  ratios.

## Conclusion

$SCF_Y$  is almost constant for all the cases. By introducing auxiliary holes in a fixed plate of these materials the deformation increases as compared to the solid plate of same material and under same loading conditions for all D/A ratios.

It has been observed that there is no reduction in any SCF by providing auxiliary holes along with the main hole.

Results obtained of SCF for  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_{xy}$ ,  $\sigma_{eqv}$  and  $U_z$  in simply supported case for all materials is symmetric. Higher  $E_x/E_y$  and  $E_x/G_{xy}$  ratios increases the values of SCF for  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_{xy}$ ,  $\sigma_{eqv}$  and  $U_z$  of e-glass/epoxy and boron/epoxy and have the highest values compared to woven glass/epoxy & boron/aluminium. SCF for  $\sigma_x$  is maximum on the hole boundary along the width with its direction on the lower face of the plate.  $\sigma_Y$  is maximum towards x direction.  $\sigma_{XY}$  and  $\sigma_{eqv}$  are observed to be maximum at extreme edge corners of the plate in diagonal direction.  $U_z$  is maximum at extreme edge on all sides of the plate covering the complete edge thickness.

D/A ratio play a vital role in SCF for  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_{xy}$ ,  $\sigma_{eqv}$  and  $U_z$  showing ample variation in all D/A ratios. Introduction of auxiliary holes around main hole increases SCF for  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_{xy}$ ,  $\sigma_{eqv}$  and deflection  $U_z$  for all the cases undertaken.

$SCF_X$  decreases with increase in D/A ratio for all the materials.  $SCF_Y$  is very high as compared to SCF for all the other stresses in all the cases considered.  $SCF_Y$  values are showing very less variation for all the cases.  $SCF_{XY}$  shows different trend for dissimilar material. The variation in  $SCF_{XY}$  is very less for all materials. The magnitude of SCF increases to as we introduce auxiliary holes.

It can be concluded from the study that for transverse loading the introduction of auxiliary holes around the main hole is not the solution for minimization of SCF around the main hole. The reduction in different stresses has been achieved by introducing auxiliary holes as the point of maximum stress is not on the hole. In transverse loading we can extend the work to optimize the shape of the holes to cavities for reduction in stresses in Y direction which is important in case of transverse loading.

## Acknowledgment

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