



# Mathematical Modelling for Reverse Supply Chain in Manufacturing System

Shiena Shekhar<sup>1\*</sup> and Ankur Malviya<sup>2</sup>

<sup>1</sup>Department of Mechanical Engineering, Bhilai Institute of Technology, Durg, Chhattisgarh, India

<sup>2</sup>Department of Mechanical Engineering, Christian College Of Engineering and Technology, Bhilai, Chhattisgarh, India  
ankur.malviya8@gmail.com

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## Abstract

This paper describe about how reverse supply chain is important in manufacturing system similar to the forward supply chain and suggested an feasible mathematical model for the reverse supply chain considering remanufacturing . We know that because of emerging innovations and technologies, products are changing faster than ever due to this now consumer are interested to adopt new product rather than the older products. This results environment gets deteriorates due to steady growth of wastage in the form of not in used products. Product gets obsolete due to many reasons like there not functioning, old technology and many more. So our papers discussed about mathematical model which incorporate between returned products and the out coming parts in reverse supply chain in manufacturing system considering remanufacturing.

**Keywords:** Remanufactured Product (REMP), Supply Chain Management (SCM), Reverse Supply Chain Management (RSCM).

## Introduction

From many years, more efforts are done on supply chain management systems for getting maximum benefits in the market. This supply chain management system involves suppliers, manufacturer, distributor, retails and consumers as shows in the Figure-1. And the objective is to fulfilling the demand of consumer in supply chain and gets benefit from producing new products.

They are focusing on forward supply chain system but the other side there is a weak condition is that continuously there is increase of landfill of waste, because after end of life of the product or outdated technology etc, consumers rejected the product. So to minimize the wastage either in terms of materials or energy we can make the product in reuse in terms of repair,

recycle, cannibalize and remanufacturing. Now contribution is doing in reverse logistics effort, the focus is to minimize the cost and also ensures the satisfaction of costumers. Now companies have to put effort on various types of recovery programs and involve in the planning during direct supply chain.

Product become obsolete if it is not returned back by the consumers in proper way when it's not function properly, therefore it is required to have proper supply chain stages since form the beginning of manufacturing, distribution and finally products reaching to the consumers. So the returns are divided into three parts consumer return, distribution return and manufacturing returns. The following Figure-2 shows how reverse supply chain works.

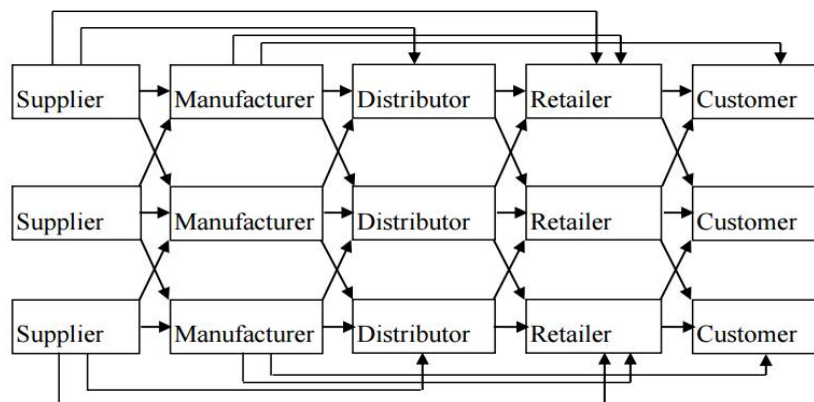
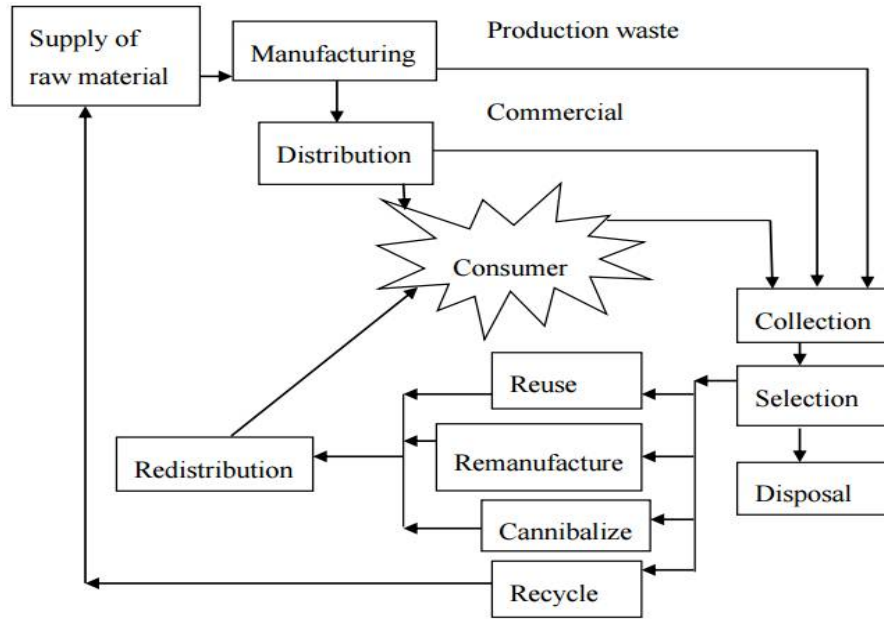


Figure-1  
Phases in supply chain management system<sup>1</sup>



**Figure-2**  
 Frame work explains the reverse order managements works in SCM<sup>2</sup>

Recovering the parts to reuse, to repair and to cannibalize in reverse logistics which emphasising sustainability and attractive business. Now the many companies are coming forward for this approach such like companies for electronic products, beverages, etc the automotive companies are demanding the change in virtual and physical supply<sup>1</sup>. There is a trend of recycling that is stretching throughout worldwide which involves all the phases of supply chains.

For the Reverse supply chain there are various challenges such as data management, it is difficult to obtain and manage relevant information to analyse it. Volume management, most of the returns are time based and it is required to manage it properly to avoid loss. To meet consumers requirement, it is always difficult to maintain frame work of reverse supply chain, because consumers always requires best prize with return policies<sup>2</sup>.

**Aim of research:** To compare the sensitivity of price differences with amount of product for both the industries and to estimate the return price and unit buying cost of end of use products from the consumers and also making the model to balance the supply chain using remanufacturing in manufacturing system.

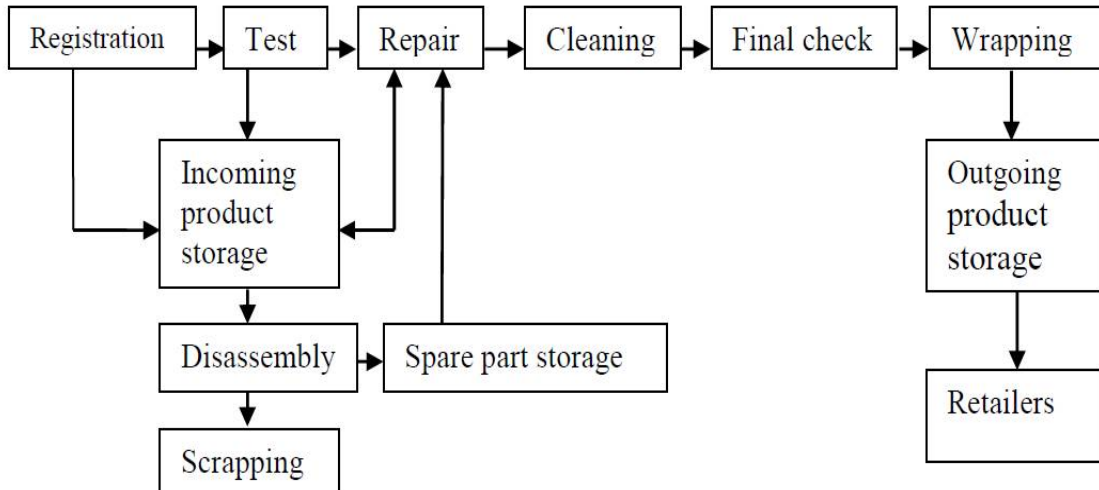
**Literature Survey:** Till date many researchers have proposed many models for reverse supply chain by considering different conditions such as manufacturing, time, business. Our work will be about to develop generalized optimized mathematical model for the RSC which can be utilised for product manufacturing and may be use to return the products which might have potential of creating unnecessary waste inventory<sup>3,4,5</sup>. For deciding this model consumer satisfaction, the life expectancy

and cost of product manufactured are in the route. Tang and Grubbstrom, studied this system with two parameters a constant demand and other is stochastic lead time<sup>6</sup>. Franke, explained how a mobile phone company remanufacturer at service centers are working, if product not possible to repair on site, then it send to industries through the transports<sup>7</sup>.

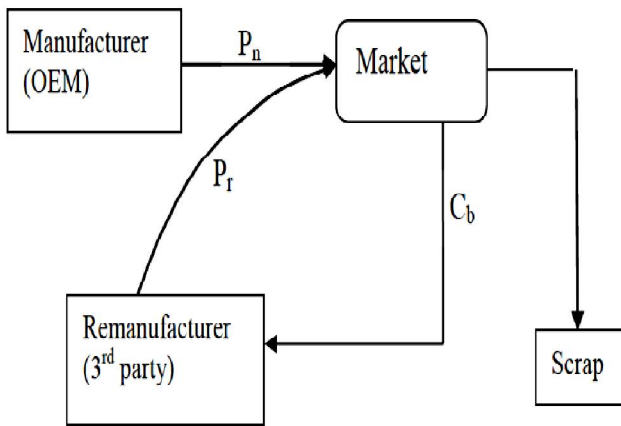
As product arrived they registered in a software system, and decide what decision has to take either the product used as spare or go for remanufacturing (Figure-3) and if product having some error then it get repaired on the basis of type of problems and errors it required time (30-60 minutes), if the spare parts are available from the storage, otherwise again it takes more time, since spare part has to ordered from the facility. Once the product repaired, after cleaning wrapped and transport to retails for resale<sup>8</sup>. As per the service centers people their found and suggested that at cleaning stage more care has to taken because it the root cause of the bottleneck in remanufacturing system. And also he found if control is there for two things firstly administration of the entire process and secondly product storage, so company can hold the largest cost shares<sup>9</sup>.

Remanufacturing model was presented by Jung, as per this model under this system the brand new products (OEM) are sold at  $A_{OEM}$  and company OEM take the responsibility of paying corresponding penalties for the returned products<sup>10</sup>.

The OEM through the remanufactures collected the product from the consumers when its life time ends at the cost of  $B_c$ . For the returned products the price/unit of  $R_p$  shows in Figure-4. On the basis of competition and demands the policies for pricing depends for the producers and reproducers. For getting good profit for coordination of producer and reproducer is required.



**Figure-3**  
 The main material flow at mobile phone Company at their remanufacturing system<sup>7</sup>



**Figure-4**  
 Model shows the remanufacturing system<sup>10</sup>

**Methodology**

**Modelling rscm with remanufacturing:** For policy of the take back offer all new product sells for the price  $S_{np}$  and product are collected by remanufacturer when the lifetime of the product ends, at the back price of  $B_c$ .

The model is shown in the Figure-5, the area where we focus are the demand rate of new product, remanufactures product, the amount of product supply to the market  $Q_s$ , the amount returned product  $Q_r$  and the amount of waste product  $Q_w$ . There might be two situations which may arise during the process. The situations are discussed under the following two cases.

Case 1: When the demand rate of new product is equal to the greater than the REMP.

$$D_{np} \geq D_{remp} \text{ i.e : } D_{np} \alpha B_c.$$

Case 2: When the demand rate is more for the REMP compare to the return rate of the product, after end of use.  $D_{remp} > D_1$  i.e:  $D_{np} \alpha B_c$

After manufacturing the new product, it goes to holding inventory, remanufacturing holding inventory of used product.

**Notations:**  $D_{np}, D_{remp}$  = Product demand of the product for new, remanufactured, ( $\frac{Units}{Time}$ ).

$C_{np}, C_{remp}$  = Cost of new, remanufactured product, ( $\frac{Rupees}{Units}$ )  
 $\alpha (B_c)$  = Coefficient for the supply for the returned products  
 $= \frac{Back\ Cost}{Scaling\ factor}$

$s = C_{np} - C_{remp}$ , scaling factor, ( $\frac{Rupees}{Units}$ )

$S_{np}, S_{rp}$  = Product Selling price for new, remanufactured, ( $\frac{Rupees}{Units}$ ).

$Q_{np}, Q_{remp}$  = Quantity of the product to the market of for new, remanufactured (Units).

$Q_r, Q_w$  = Quantity of returned, waste product (Units).

$a_{np}, a_{remp}$  = the maximum units of sales of new, remanufactured product, (Units).

$b_{np}, b_{remp}$  = the price sensitive to the customer for new, remanufactured product, ( $\frac{Rupees}{Units}$ ).

$\gamma$  = Demand coefficient for the difference in per price, ( $\frac{Units}{Rupees}$ ).

To avoid the complicacies in the model, the assumptions are being considered because above model is based on various conditions during manufacturing and subsequent use of the product by consumer and the buy and back policy use of the product. The assumptions are shortage not allowed, demand rates is depends on the price of the product and buy and back cost includes remanufacturing only.

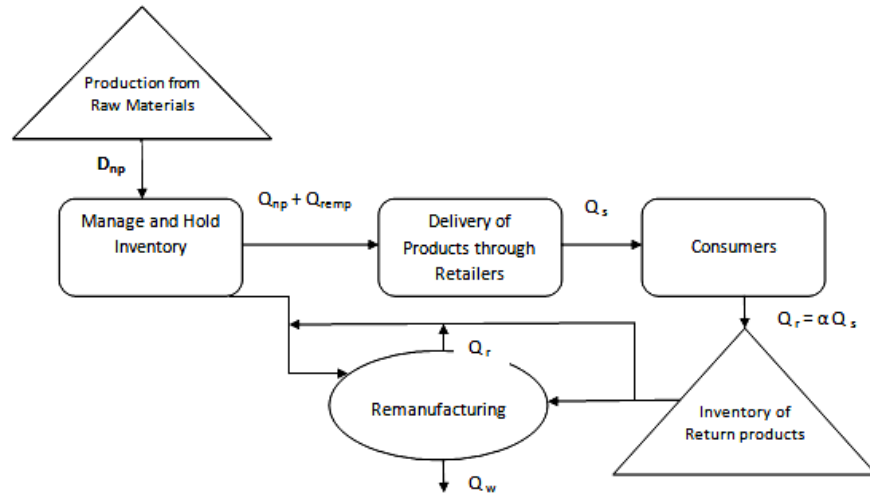


Figure-5  
Supply chain model with remanufacturing.

**Mathematical Modelling:** For the demand changes to REMP instead of brand new products, lower price of the this demand of remanufactured product are assumed in our study and we know that people always migrate for price difference it is also state by Bell modelled the demand leakage for difference between the prices as a linear function<sup>8</sup>.

Let us consider the demand function =  $f(D_x, e)$

Where:  $D_x$  is the demand for some quantity and “e” is an additive error sensitive to  $S_{np}$  and  $S_{rmp}$ . Considering a linear demand curve with additive error (e):

$$Q_s = D_x + e \quad (1)$$

From the actual linear demand curve between variable “a” and “b” and the price per unit “p”

$$Q_s = a - bp \quad (2)$$

Therefore from the equation (1) and (2):

$$\begin{aligned} Q_s &= D_x + e \\ D_x &= Q_s - e \\ D_x &= a - bp - e \end{aligned} \quad (3)$$

We are using Homoscedasticity theory since the variance of errors is the same across all levels, so from this model the demand can obtain.

For the demand rate of the new product from OEM,  $p_{np} > p_{rmp}$

$$D_x = a_x - b_x p_x - e, \quad x = \text{New Product}(np), \text{ Remanufactured Product}(rmp) \quad (4)$$

With,  $e = Y(p_{np} - p_{rmp})$

For the new product,

$$D_{np} = a_{np} - b_{np} p_{np} - Y(p_{np} - p_{rmp}) \quad (5)$$

And for the case when  $p_{rmp} > p_{np}$ ,

$$D_{rmp} = a_{rmp} - b_{rmp} p_{rmp} - Y(p_{rmp} - p_{np}) \quad (6)$$

As per the Figure-5, the number of units of new product supply

to the market can be taken as the sum of number of new and remanufactured products with unit time:

$$Q_{np} = D_{np} + D_{rmp} \quad (7)$$

Now equating the demand rate of remanufactured product to the amount of remanufacturing with the unit time,

$$Q_{rmp} = D_{rmp} \quad (8)$$

Considering the landfill product

$$Q_r = Q_{rmp} + Q_w \quad (9)$$

$$D_{np} = a_{np} - b_{np} p_{np} - Y(p_{np} - p_{rmp})$$

$$D_{rmp} = a_{rmp} - b_{rmp} p_{rmp} - Y(p_{rmp} - p_{np})$$

As now considering 35% on the new product and 70% on the remanufactured product referred in the paper<sup>6</sup>.

$$P_{np} = 1.35 C_{np} \quad (10)$$

$$P_{rmp} = 1.7 C_{rmp} \quad (11)$$

The minimum and maximum values of  $B_c$  max and  $B_c$  min can be calculated with optimization of profit functions for the manufacturer. The average value of  $B_c$  taken care of to find and compare the results for the wide range of variables for above two cases.

Case-1: Examining for the industry with the change in Y value: Considering the parameter from Equation (12).

Case-2: Examining for the industry with the change in Y value: From Equation-13

Case 1: When the demand rate of new product is equal to the greater than the REMP,  $D_{np} \geq D_{rmp}$ . i.e:  $D_{np} \alpha B_c$ .

$$B_c = \frac{s(a_2 - b_2 p_{rmp} - Y(p_{rmp} - p_{np}))}{a_1 - b_1 p_{np} - Y(p_{np} - p_{rmp})} \quad (12)$$

$$B_c = C_{np} - C_{rmp}$$

Case 2: When the demand rate of REMP is more compare to the return rate,

$$B_c = \frac{D_{remp} > D_{np} i.e: D_{np} \alpha B_c}{2} \quad (13)$$

$$a_{np}=6000\text{Units}, \quad a_{remp}=3000\text{Units}, \quad b_{np}=1.10\text{Unit/Rs},$$

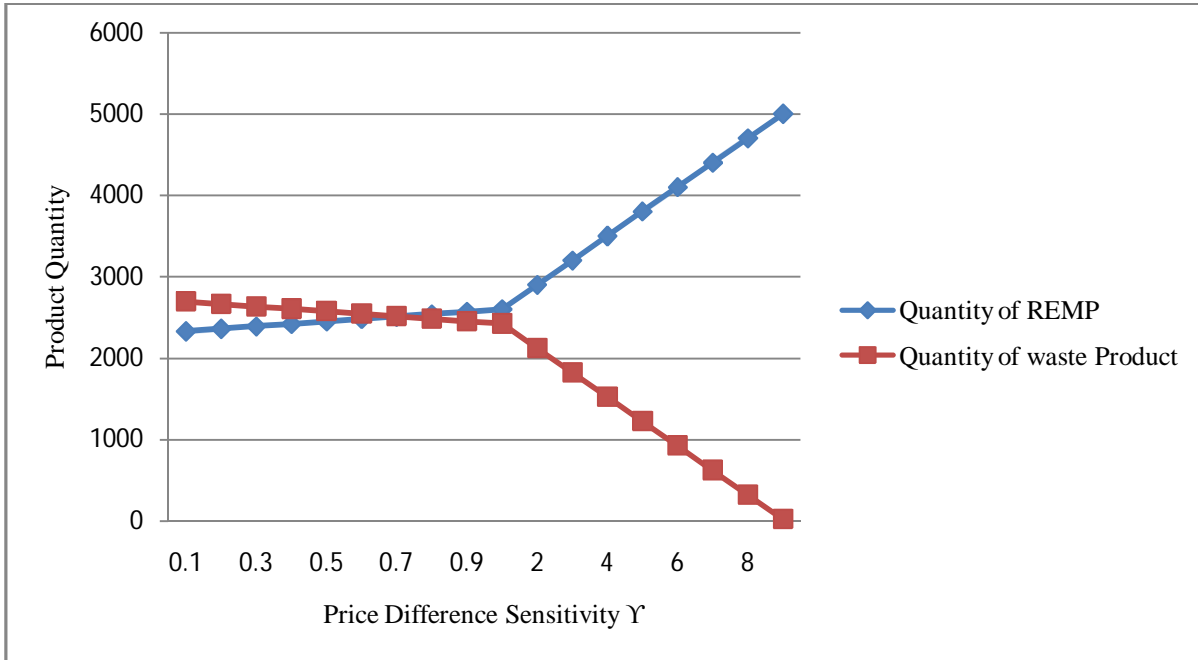
$$b_{remp}=\text{Rs}1\text{Units/Rs}, \quad P_{np}=\text{Rs}1000/\text{Units}, \quad P_{remp}=\text{Rs}700/\text{Units},$$

$$Y=0.2 \text{ Units/Rs}$$

**Result and Discussion**

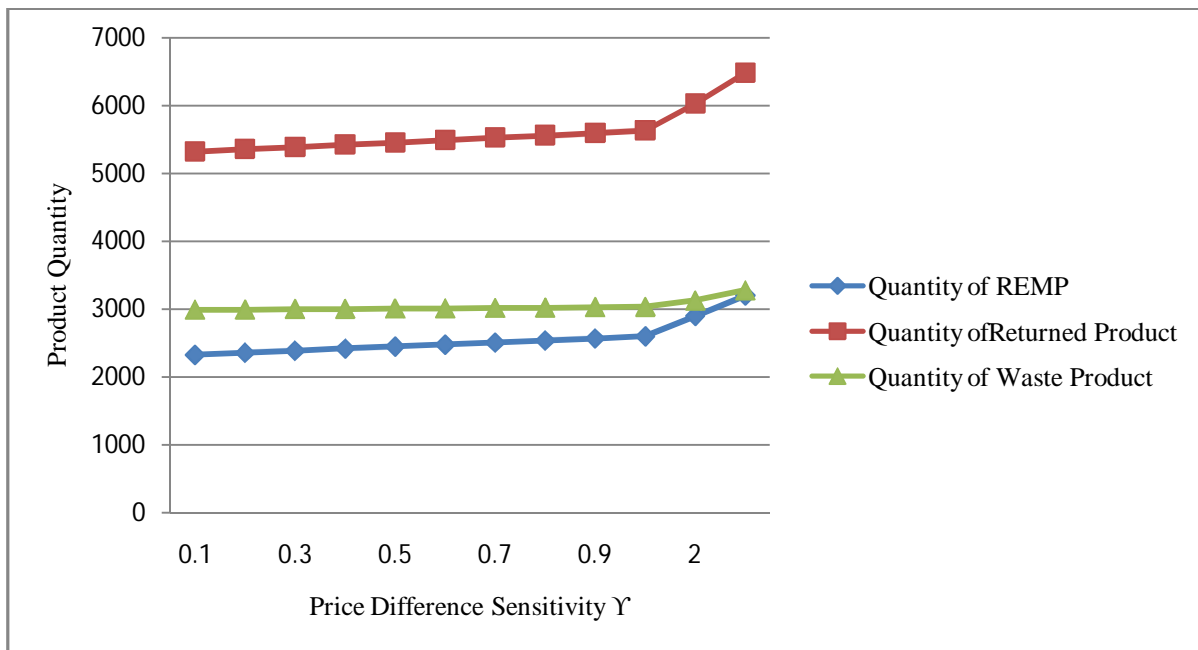
To test the above model we are using the data's from the cartridge industry and put these values in the respective equation from Equation (4-13). The data's are:

The results which we calculated are showing in the following Figure 6-13.



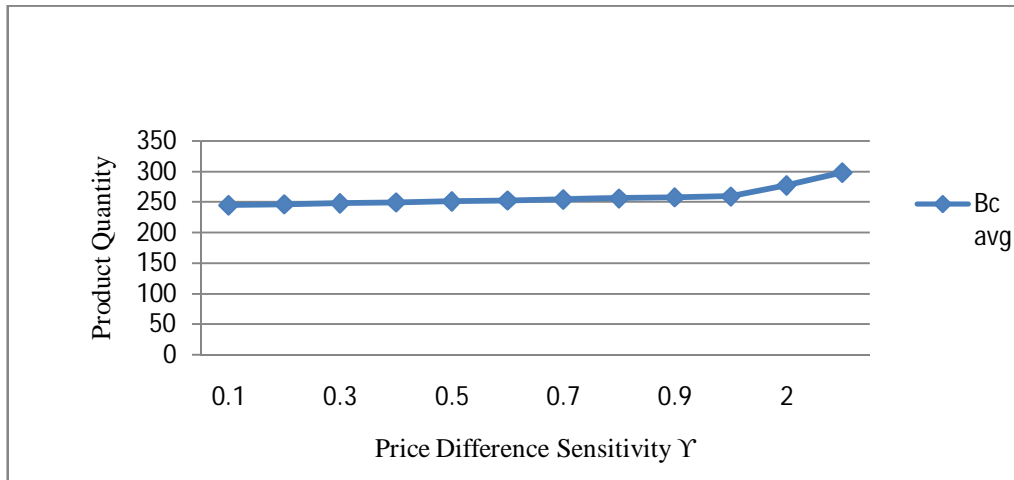
**Figure-6**

Changes in the quantity of remanufactured and waste product with the corresponding change in Y value

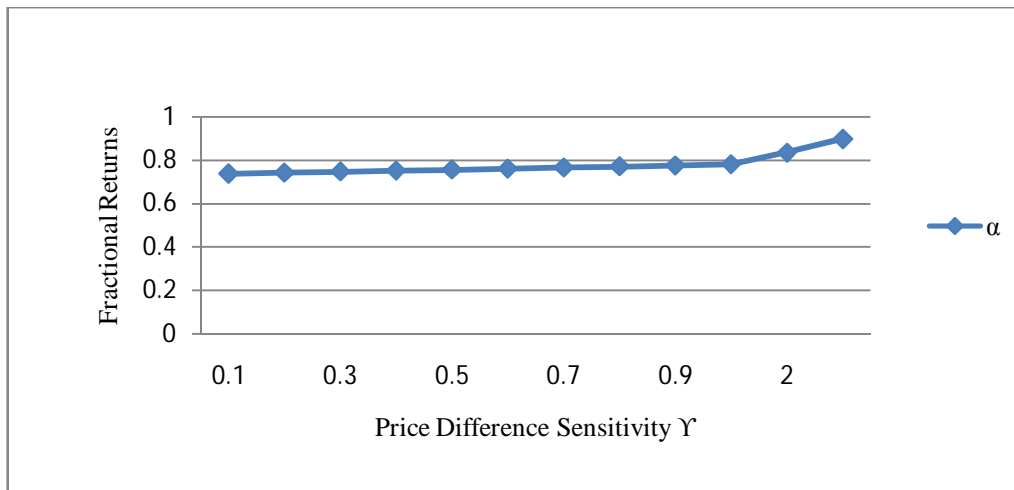


**Figure-7**

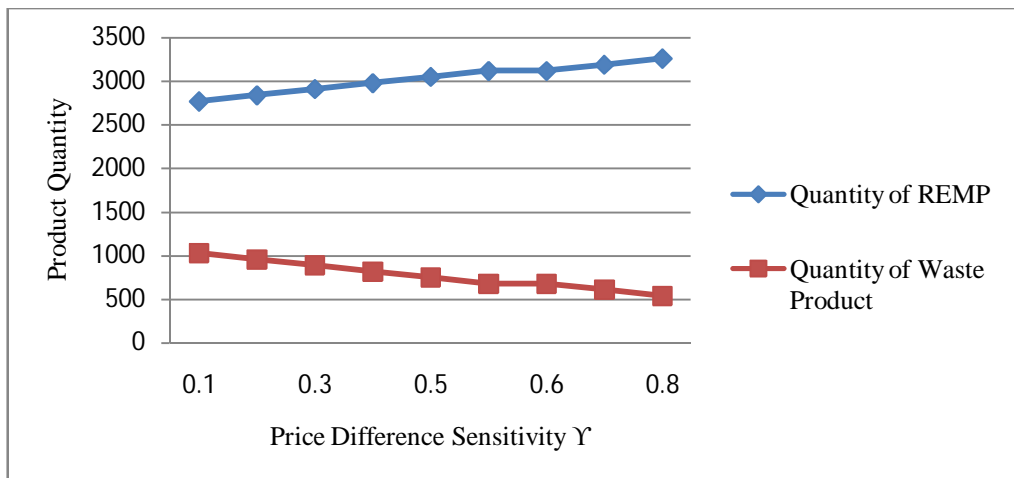
Changes in the quantity of remanufactured, returned and waste product with the corresponding change in Y value



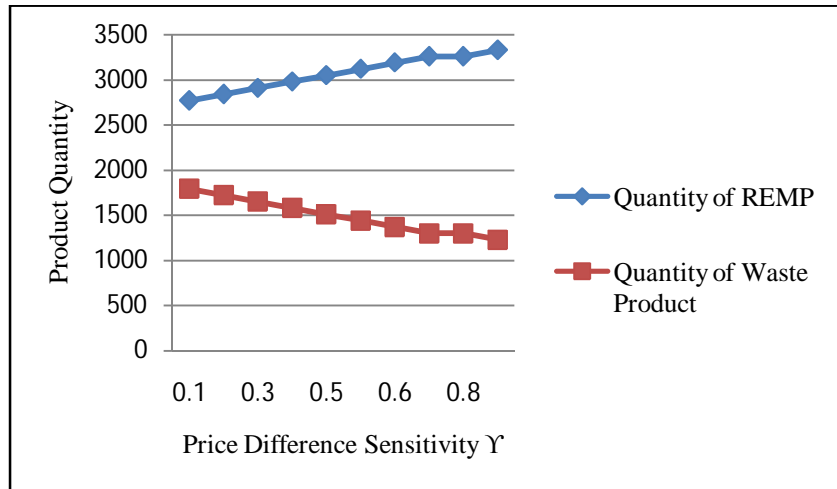
**Figure-8**  
 Changes in the value of  $B_c$  avg with corresponding change in  $\Upsilon$  value



**Figure-9**  
 Changes in the value of  $\alpha$  with corresponding change in  $\Upsilon$  value

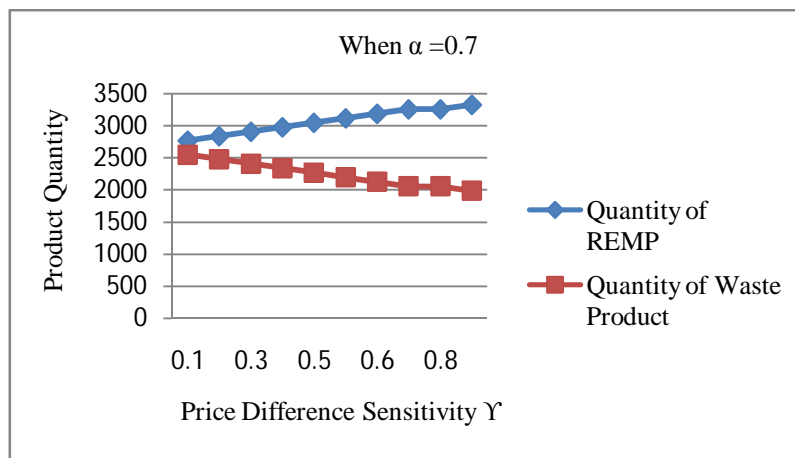


**Figure-10**  
 Changes in the quantity of remanufactured, returned and waste product with the corresponding change in  $\alpha = 0.5$



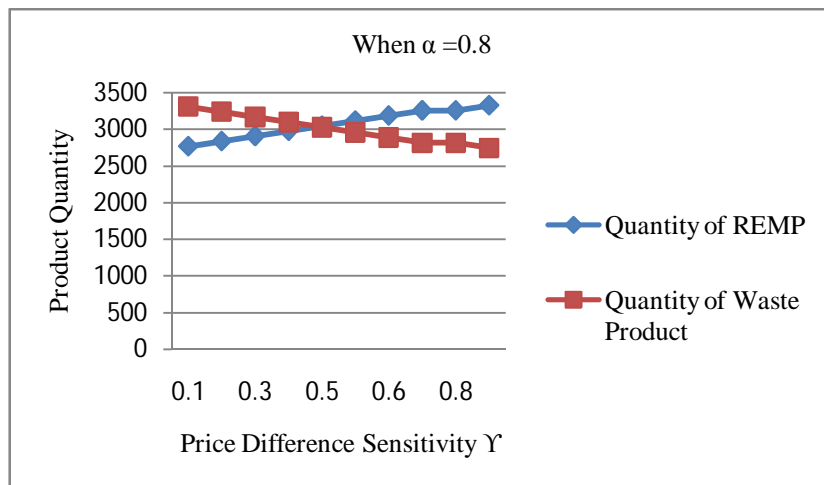
**Figure-11**

Variation in the quantity of remanufactured, returned and waste product with the corresponding change in  $\alpha = 0.6$



**Figure-12**

Variation in the quantity of remanufactured, returned and waste product with the corresponding change in  $\alpha = 0.7$



**Figure-13**

Variation in the quantity of remanufactured, returned and waste product with the corresponding change in  $\alpha = 0.8$

## Conclusion

This model of reverse supply chain with remanufacturing was tested with practical data and during the analysis, the sensitive parameters of the model were identified and their effect on the behaviour of the model under consideration was observed. For various values of sensitivity  $\gamma$  are presented for the product cartridge and discussed the model of supply chain management with the remanufacturing, the amount of waste product is calculated.

**Future Scope:** Here in this paper we considered only one case for reverse supply chain that is remanufacturing, with some assumptions of constant behaviour of demands and supply. So there are various possibility for further work by considering dynamic behavior of market. By taking the other parameters in RSC such as recycling, cannibalization, repairing, transport, technology, time, etc can be taken for further study. This work will be beneficial for not only to the industry and consumer but also it get benefite for the healthy enviornment and wealthy nation.

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