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A Computational Model on: Vibration of Square Plate with Varying Thickness and Thermal effect in Two Directions

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Abstract

A computational model presented here is to study the vibration of visco-elastic isotropic square plate with thermal effect on two direction varying thickness parabolically. Equation of frequency is derived by using Rayleigh-Ritz technique with a two-term deflection function. All the calculation made for the first two modes of vibration, for various values of thermal gradients and taper constant.

Keywords: Square plate, frequency, thickness, thermal effect, taper constant.

Introduction

Square plates have wide applications in ships, aircrafts, bridges, and so on. A thorough dynamic study of their behavior and characteristics is essential to assess and use the full potentials of plates. In the aeronautical field, analysis of plates with variable thickness has been of great interest due to their utility in aircraft wings. There are different kinds of visco-elastic plates of variable thickness such as rectangular plates, square plates, circular plates, parallelogramic plates.

Modern engineering structures are based on different design types, which involve various types of anisotropic and non-homogeneous materials in the form of their structural components. Depending upon the requirement, durability and reliability, materials are being developed so that they can be used to provide better strength and efficiency. The equipment used in air jets, communications, and in other similar technological industries take into consideration such materials, which not only reduce the weight and size but also are reliable in terms of efficiency, strength, and economy.

Further, the study of vibration behaviour in the presence of thermal gradient of visco-elastic plates is required due to its practical importance in the field of engineering because Machines very repeatedly operate under diverse temperature conditions. In majority of cases the impact of temperature are ignored yet they need to be taken in to consideration. Most of engineering materials are found to have linear relationship between modulus of elasticity and temperature. Applications of such materials are due to lessening of weight and size, low operating cost and enhancement in efficiency and strength.

The objective of the present study is the thermal effect on vibration of square plate of varying thickness parabolically. It is clamped supported on all the four edges.

Methodology

Differential equation of transverse motion of a visco-elastic plate in Cartesian co-ordinates[1]:

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{yx}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \rho h \frac{\partial^2 w}{\partial t^2} \quad (1)$$

The expression for M_x, M_y, M_{yx} are given by

$$\left.\begin{aligned} M_x &= -\widetilde{D} D_1 \left(\frac{\partial^2 w}{\partial x^2} + \vartheta \frac{\partial^2 w}{\partial y^2}\right) \\ M_y &= -\widetilde{D} D_1 \left(\frac{\partial^2 w}{\partial y^2} + \vartheta \frac{\partial^2 w}{\partial x^2}\right) \\ M_{yx} &= -\widetilde{D} D_1 (1-\vartheta) \frac{\partial^2 w}{\partial y \partial x} \end{aligned}\right\} \quad (2)$$

where $\widetilde{D}$ is visco-elastic operator.
On substitution the values M_x, M_y and M_{yx} from equation (2) in (1) and taking w, as a product of two function, equal to w(x,y,t)=W(x,y)T(t), equation (1) become:

$$\left[D_1 \left(\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + 2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3} + \frac{\partial^3 W}{\partial x \partial y^2}\right) + 2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3} + \frac{\partial^3 W}{\partial x^2 \partial y}\right) + \frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2} + \vartheta\frac{\partial^2 W}{\partial y^2}\right) + \frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2} + \vartheta\frac{\partial^2 W}{\partial x^2}\right) + 2(1-\vartheta)\frac{\partial^2 D_1}{\partial x \partial y}\frac{\partial^2 W}{\partial x \partial y} \right] \Big/ \rho h W = -\frac{\ddot{T}}{\widetilde{D} T} \quad (3)$$

Here dot denote differentiation with respect to t, taking both sides of equation (3) are equal to a constant p^2 (square of frequency), we have

$$[D_1(W_{,xxx}+2W_{,xxyy})-2D_{1,x}(W_{,xxx}+W_{,xyy})+2D_{1,y}(W_{,yyy}+W_{,yxx})$$

$$+D_{1,xx}(W_{,xx}+W_{,yy})+D_{1,yy}(W_{,yy}+W_{,xx})+2(1-)D1_{,xy}W_{,xy}]-php^2W=0 \quad (4)$$

is a differential equation of transverse motion for non-homogeneous plate of variable thickness. Here, D_1 is the flexural rigidity of plate i.e.

$$D_1 = \frac{Eh^3}{12(1-\vartheta^2)} \quad (5)$$

and corresponding two-term deflection function is taken as[2]

$$W = \left[\left(\frac{x}{a}\right)\left(\frac{y}{a}\right)\left(1-\frac{x}{a}\right)(1-y/a)\right]^2 \left[A_1 + A_2\left(\frac{x}{a}\right)\left(\frac{y}{a}\right)\left(1-\frac{x}{a}\right)(1-y/a)\right] \quad (6)$$

In the above equation A_1 and A_2 are constants satisfy boundary conditions. Also, it is assumed that temperature varies parabolically in two directions i.e.

$$\tau = \tau_0(1 - x^2/a^2)(1 - y^2/a^2) \quad (7)$$

where τ denotes the temperature excess above the reference temperature at any point on the plate and τ_0 denotes the temperature at any point on the boundary of plate and "a" is the length of a side of square plate. The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed in this form[3]:

$$E = E_0(1 - \gamma\tau) \quad (8)$$

where, E_0 is the value of the Young's modulus at reference temperature i.e. $\tau = 0$ and γ is the slope of the variation of E with τ. The modulus variation (5) become

$$E = E_0[1 - \alpha(1 - x^2/a^2)(1 - y^2/a^2)] \quad (9)$$

where $\alpha = \gamma\tau_0 (0 \le \alpha < 1)$, thermal gradient.

It is assumed that thickness also varies parabolic in x and y directions as shown below:

$$h = h_0(1 + \beta_1 x^2/a^2)(1 + \beta_2 y^2/a^2) \quad (10)$$

where β_1 is taper parameters in x- directions respectively and $h=h_0$ at x=y=0.

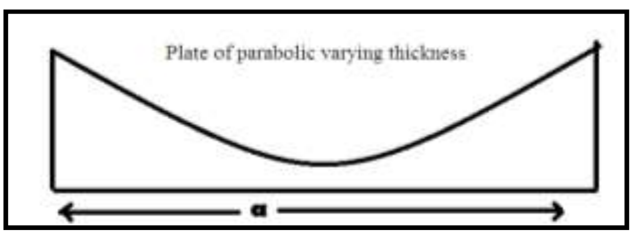

Figure-1
Plate with parabolic varying thickness

Put the value of E and h from equation (9) and (10) in the equation (5), one obtain

$$D_1 = \frac{[E_0[1-\alpha(1-x^2/a^2)(1-y^2/a^2)]h_0(1+\beta_1 x^2/a^2)(1+\beta_2 y^2/a^2)]}{12(1-\vartheta^2)} \quad (11)$$

Rayleigh-Ritz technique is applied to solve the frequency equation. In this method, one requires maximum strain energy must be equal to the maximum kinetic energy[4]. So it is necessary for the problem under consideration that

$$\delta(K^* - S^*) = 0 \quad (12)$$

for arbitrary variations of W satisfying relevant geometrical boundary conditions.

Since the plate is assumed as clamped at all the four edges, so the boundary conditions are:

$$\left.\begin{array}{l} W = W_{,x} = 0,\ x = 0, a \\ W = W_{,y} = 0,\ y = 0, a \end{array}\right\} \quad (13)$$

Now assuming the non-dimensional variables as

$$X = \frac{x}{a},\ Y = \frac{y}{a},\quad \overline{W} = \frac{W}{a},\ \ \overline{h} = \frac{h}{a} \quad (14)$$

The kinetic energy K* and strain energy S* are [5]

$$K^* = \left(\frac{1}{2}\right)\rho p^2 \overline{h_0} a^5 \int_0^1\int_0^1 [(1+\beta_1 x^2/a^2)(1+\beta_2 y^2/a^2)\quad \overline{W}^2\ ]\ dYdX \quad (15)$$

and

$$S^* = Q\int_0^1\int_0^1 \Big[[1-\alpha(1-X^2)(1-Y^2)][(1+\beta_1 x^2/a^2)(1+\beta_2 y^2/$$

a2 3 W,XX2+W,YY2+2ϑW,XX W,YY+2(1−ϑ)W,XY2 dYdX(16)

where, $Q = \frac{E_0 {h_0}^3 a^3}{24(1-\vartheta^2)}$

Using equations (15) and (16) in equation (12), one get

$$(S^{**} - \lambda^2 K^{**}) = 0 \quad (17)$$

where, $S^{**} = \int_0^1\int_0^1 \Big[[1-\alpha(1-X^2)(1-Y^2)][(1+\beta_1 x^2/a^2)(1+\beta_2 y^2/a^2)\quad]^3 \left\{(\overline{W}_{,XX})^2 + (\overline{W}_{,YY})^2 + 2\vartheta \overline{W}_{,XX}\overline{W}_{,YY} + 2(1-\vartheta)(\overline{W}_{,XY})^2\right\}\Big]\ dYdX$ (18)

and

$$K^{**} = \int_0^1\int_0^1 [(1+\beta_1 x^2/a^2)(1+\beta_2 y^2/a^2)\quad \overline{W^2}]\ dYdX \quad (19)$$

Here, $\lambda^2 = 12\frac{\rho p^2(1-\vartheta^2)a^2}{E_0 {h_0}^2}$ is a frequency parameter.

Equation (19) consists two unknown constants i.e. A_1 and A_2 arising due to the substitution of W. These two constants are to be determined as follows[6]:

$$\frac{\partial(S^{**} - \lambda^2 K^{**})}{\partial A_n}, = 0,\ \text{n=1, 2} \quad (20)$$

On simplifying (20), we get $b_{n1}A_1 + b_{n2}A_2 = 0$, n=1, 2 (21)

where bn_1, bn_2 (n=1,2) involve parametric constant and the frequency parameter.

For a non-trivial solution, the determinant of the coefficient of equation (21) must be zero. So one gets, the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} \quad (22)$$

With the help of equation (22), one can obtains a quadratic equation in λ^2 from which the two values of λ^2 can found. These two values represent the two modes of vibration of frequency i.e. λ_1 (Mode1) and λ_2 (Mode2) for different values of taper constant and thermal gradient for a clamped plate.

Results and Discussion

Calculation of the frequency parameter are carried out with the help of computer software i.e. MATLAB. Computation has been done to obtain first two modes of frequency of square plate of variable thickness for different values of taper constants (β_1 and β_2), thermal gradient (α) at different points.

Table-1
Frequency vs. Thermal Gradient

α=0	$\beta_1 = \beta_2 = 0$		$\beta_1 = \beta_2 = 0.6$	
0	140.88	35.99	273.42	70.88
0.2	134.28	34.31	264.27	68.46
0.4	127.34	32.52	254.81	65.91
0.6	119.99	30.63	244.98	63.21
0.8	112.18	28.60	234.77	60.31
1	103.78	26.39	224.12	57.17

It is clearly seen that value of frequency decreases as value of thermal gradient α increases from 0.0 to 1.0 for $\beta_1 = \beta_2 = 0.0$ and $\beta_1 = \beta_2 = 0.6$ for both modes of vibrations.

Table-2
Frequency vs. Taper parameter

β_1	$\beta_2=0.2$ and $\alpha=0.3$		$\beta_2=0.2$ and $\alpha=0.6$	
0	147.14	37.78	135.71	34.82
0.2	166.37	42.79	154.40	39.67
0.4	186.58	48.09	174.01	44.78
0.6	207.61	53.63	194.36	50.11
0.8	229.33	59.35	215.37	55.59
1	251.64	65.22	236.93	61.22

Also it is obvious to understand the increment in frequency as value of tapper constant β_1 from 0.0 to 1.0 for i. $\beta_2=0.2$ and $\alpha=0.3$., ii. $\beta_2=0.2$ and $\alpha=0.6$

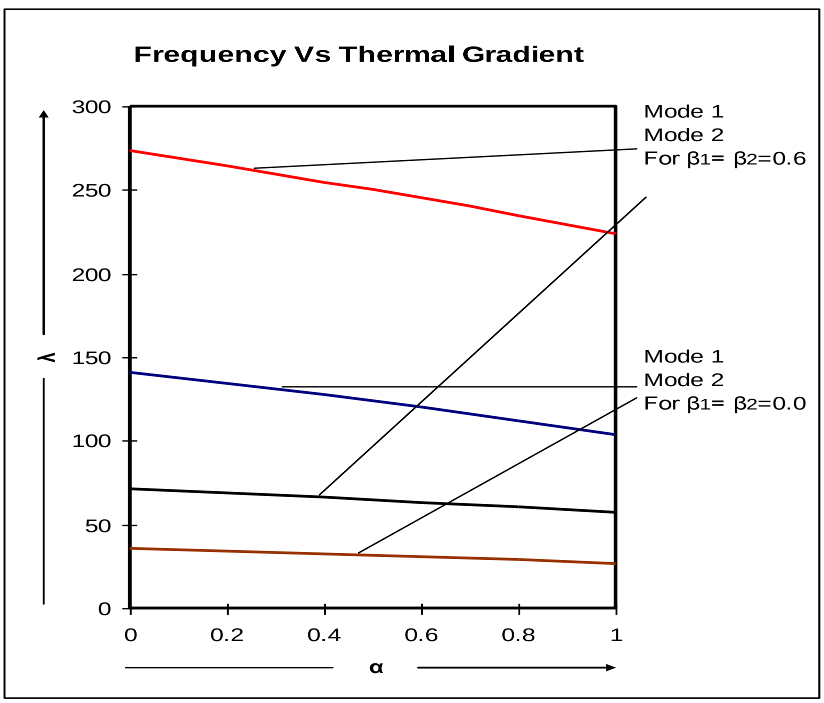

Figure-2
Frequency vs. Thermal Gradient (For $\beta_1 = \beta_2 = 0.0$ and $\beta_1 = \beta_2 = 0.6$.)

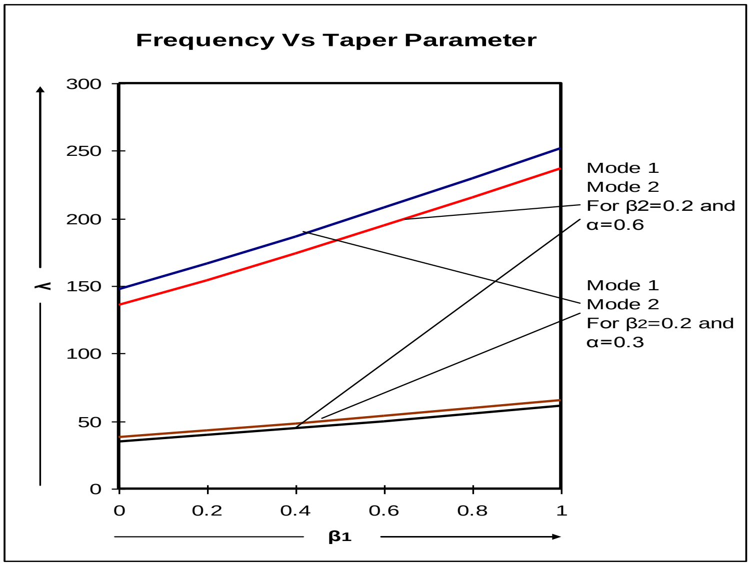

Figure-3
Frequency vs. Taper Parameter (For β_2=0.2 and α =0.3and β_2=0.2 and α =0.6)

Conclusion

Motive is to provide such kind of a mathematical design so that scientist can perceive their potential in mechanical engineering field and increase strength, durability and efficiency of mechanical design and structuring with a practical approach. Actually this is the need of the hour to develop more but authentic mathematical model for the help of mechanical engineers/researchers/practitioners. Therefore mechanical engineers and technocrats are advised to study and get the practical importance of the present paper and to provide much better structure and machines with more safety and economy.

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