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Graph Theoretic approach (GTA) – A Multi-Attribute Decision Making (MADM) Technique

Attri Rajesh[1*], Dev Nikhil[1] and Sharma Vivek[2]
[1]Department of Mechanical Engineering, YMCA University of Science and Technology, Faridabad – 121006, INDIA
[2]Department of Mechanical Engineering, Advanced Institute of Technology and Management, Palwal - 121105, INDIA

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Abstract

This paper presents the basic concepts details of graph theoretic approach as a decision-making method in the manufacturing environment. Graph theory and matrix approach as a decision-making method is relatively new, and offers a generic, simple, easy, and convenient decision-making method that involves less computation. The method lays emphasis on decision-making methodology, gives much attention to the issues of identifying the attributes, and to associating the alternatives with the attributes, etc. The measures of the attributes and their relative importance are used together to rank the alternatives, and hence provides a better evaluation of the alternatives. The permanent concept fully characterizes the considered selection problem, as it contains all possible structural components of the attributes and their relative importance. The graph theoretical methodology consists of three steps – digraph representation, matrix representation and permanent function representation. These are also explained in the paper.

Keywords: GTA, approach, digraph, matrix, permanent function.

Introduction

Decision making is the study of identifying and choosing alternatives based on the values and preferences of the decision maker. Making a decision implies that there are alternative choices to be considered, and in such a case, not only as many of these alternatives as possible are identified but also the best one is chosen to meet the decision maker's goals, objectives, desires, and values. Thus, every decision making process produces a final choice[1]. The selection decisions are complex, as decision making is more challenging now a days.

For obtaining the best decision in conjunction with the real-time requirements, a number of MADM approaches are available. MADM methods are generally discrete, with a limited number of pre-specified alternatives. These methods require both intra and inter-attribute comparisons, and involve explicit tradeoffs that are appropriate for the problem considered. Most commonly used MADM approaches are weighted sum method (WSM), weighted product method (WPM), Analytic hierarchy process (AHP), Technique for order preference by similarity to ideal solution (TOPSIS), and Compromise ranking method (VIKOR), Graph theoretic approach (GTA).

The main objective of this paper is to explore the basic concepts of Graph theoretic approach. From the literature it is clear that graph theory and matrix approach as a decision making method is relatively new, and offers a generic, simple, easy, and convenient decision making method that involves less computation. Graph theory is a useful representation because on the one hand the elements of the graph can be defined so as to have a one-to-one correspondence with the elements of many kinds of engineering systems. On the other hand, the theorems and algorithms of graph theory allow one also to represent behavioral properties of the system, such as deformations and forces, or velocities and movements, as properties of the vertices or edges of the graph.

The advanced theory of graphs and their applications are well documented[2-27].

Graph Theory Approach

Graph/digraph model representations have proved to be useful for modeling and analyzing various kinds of systems and problems in numerous fields of science and technology. GTA is a systematic and logical approach which synthesizes the inter-relationship among different parameters or sub-system parameters and provides a synthetic score for the entire system. It also takes care of directional relationship and inter-dependence among parameters. The graph theoretical methodology consists of three steps namely – digraph representation, matrix representation and permanent function representation. A digraph is used to represent the structure of the system in terms of nodes and edges wherein the nodes represent the measure of characteristics and the edges correspond to dependence of characteristics. Matrix representation is one to one representation of the digraph. Permanent representation is the mathematical expression of characteristics and their interdependence. Digraph representation, matrix representation and permanent function are developed for the quality, cost, reliability and efficiency characteristics.

Digraph representation: A digraph is used to represent the elements and their interdependencies in terms of nodes and edges. In an undirected graph, no direction is assigned to the edges in the graph, whereas directed graphs or digraphs have directional edges. The digraph consists of a set of nodes $N = \{n_i\}$ with $i=1, 2,\ldots, M$ and a set of directed edges $E = \{c_{ij}\}$. A node n_i represents i^{th} parameter and edges represent the interdependence between parameters. The total of nodes, M, is equal to the number of parameters considered for the system. If a node i has relative importance over another node j, then a directed edge or arrow is drawn from node i to node j (c_{ij}). If a node j is having relative importance over i, then a directed edge or arrow is drawn from node j to node i (c_{ji}). Let us consider an example of a system consisting six sub-systems in which all the sub-systems are affecting each other. Then the corresponding digraph will be as shown in figure 1.

Matrix Representation: Digraph presentation is very suitable for visual analysis but is not very suitable for computer processing. Moreover if the system is large, its corresponding graph is complex and this complicates its understanding visually. In view of this, it is necessary to develop a representation of the digraph that can be understood, stored, retrieved and processed by the computer in an efficient manner. Many matrix representations for example adjacency and incidence matrix available in literature[2,3] to model the graph mathematically. The adjacency matrix is a square matrix and is selected for this purpose. It being a matrix and multinomial has been derived subsequently based on connectivity only, neglecting the directional properties. The matrix for the considered example is written as below:

$$A = \begin{bmatrix} S_1 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & S_2 & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & S_3 & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & S_4 & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & S_5 & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & S_6 \end{bmatrix} \quad (1)$$

Permanent Representation: Both digraph and matrix representations are not unique in nature because they are altered by changing the labels of their nodes. Hence, to develop a unique representation that is independent of labeling, a permanent function of the system matrix is proposed here. The permanent is a standard matrix function and is used in combinatorial mathematics. The permanent function is obtained in a similar manner as the determinant but unlike in a determinant where a negative sign appears in the calculation, in a variable permanent function positive signs replace these negative signs.

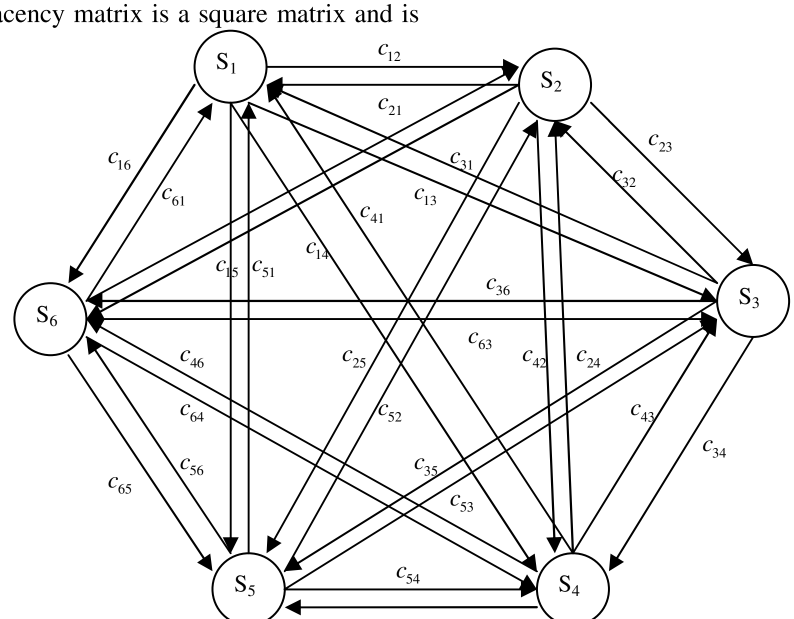

Figure-1
Digraph for a six system attributes

The expression for permanent function corresponding to considered example is written as:

$$\begin{aligned} Per(E_N) = \prod S_i &+ \sum_i \sum_j \sum_k \ldots \sum_m \sum_n c_{ij}^2 S_k S_l S_m S_n \ldots + 2\sum_i \sum_j \sum_k \ldots \sum_m \sum_n (c_{ij} c_{jk} c_{ki}) S_l S_m S_n \ldots \\ &+ 2\sum_i \sum_j \sum_k \ldots \sum_m \sum_n (c_{ij} c_{jk} c_{kl} c_{li}) S_m S_n \ldots + \sum_i \sum_j \sum_k \ldots \sum_m \sum_n (c^2_{ij} c^2_{ij}) S_m S_n \ldots \\ &+ 2\sum_i \sum_j \sum_k \ldots \sum_m \sum_n (c_{ij} c_{jk} c_{kl} c_{lm} c_{mi}) S_n \ldots + 2\sum_i \sum_j \sum_k \ldots \sum_m \sum_n (c_{ij} c_{jk} c_{ki}) c_{lm}^2 S_n \ldots \\ &+ \sum_i \sum_j \sum_k \ldots \sum_m \sum_n (c^2_{ij})(c^2_{kl})(c^2_{mn}) \ldots + 4\sum_i \sum_j \sum_k \ldots \sum_m \sum_n (c_{ij} c_{jk} c_{ki})(c_{lm} c_{mn} c_{ln}) \ldots \\ &+ 2\sum_i \sum_j \sum_k \ldots \sum_m \sum_n (c_{ij} c_{jk} c_{kl} c_{li})(c^2_{mn}) \ldots \end{aligned} \quad (2)$$

The permanent of matrix (i.e. equation 1) is a mathematical expression in symbolic form. Equation 2 contains n! terms. The equation 2 contains terms arranged in N+ 1 groups, where N is number of elements.

The physical significance of various grouping is explained as under: i. The first term (grouping) represents the interaction of six major systems (i.e. $S_1S_2S_3S_4S_5S_6$). ii. The second grouping is absent in absence of self-loops. iii. Each term of the third grouping represents a set of two-system interdependence loop (i.e. c_{ij} c_{ji}) and efficiency measure of the remaining four unconnected systems. iv. Each term of the fourth grouping represents a set of three-system interdependence loop (i.e. c_{ij} c_{jk} c_{ki} or its pair c_{ik} c_{kj} c_{ji}) and efficiency measure of the remaining three unconnected systems. v. The fifth grouping contains two subgroups. The terms of first subgroup consist of two-system interdependence loop (i.e $c_{ij}c_{ji}$ and $c_{kl}c_{lk}$) and measure of two systems efficiency (i.e. S_mS_n). The terms of second subgroup are a product of four-system interdependence (i.e. c_{ij} c_{jk} c_{kl} c_{li}) or its pair (i.e. c_{il} c_{lk} c_{kj} c_{ji}) and measure of two systems efficiency (i.e. S_mS_n). vi. The terms of sixth grouping are also arranged in two subgroups. The terms of first subgroup are a product of a two-system interdependence loop (i.e. c_{ij} c_{ji}) and a three-system interdependence loop (i.e. c_{kl} c_{lm} c_{mk}) or its pair (i.e. c_{km} c_{ml} c_{lk}). The second subgroup consists of a five-system interdependence loop (i.e. c_{ij} c_{jk} c_{kl} c_{lm} c_{mi}) or its pair (c_{im} c_{ml} c_{lk} c_{kj} c_{ji}) and measure of one systems efficiency (i.e. S_n). vii. The terms of seventh grouping are arranged in four subgroups. The first subgroup consists of terms which are a product of two-system interdependence loop (i.e. $c_{ij}c_{ji}$) and four-system interdependence loop (i.e. $c_{kl}c_{lm}c_{mn}c_{nk}$ or $c_{kn}c_{nm}c_{ml}c_{lk}$). The terms of second subgroup are a product of two interdependence loops of three-system each i.e. ($c_{ij}c_{jk}c_{ki}$ and $c_{lm}c_{mn}c_{nl}$). The terms of fourth subgroup are a product of three interdependence loops of two-system each (i.e. $c_{ij}c_{ji}$, $c_{kl}c_{lk}$ and $c_{nm}c_{mn}$). The fourth subgroup consists of a six-system interdependence loop (i.e. c_{ij} c_{jk} c_{kl} c_{lm} c_{mn} c_{ni}).

Quantification of Inheritance and Interdependencies

In order to compute the numerical value of permanent function, the values of S_i and c_{ij} are required. The value of S_i should preferably be obtained from the available or estimated data. Faisal et. al.[26] have explained that if data regarding the variables from some previous research or field study is available it can be used to determine the index. But in case no quantitative values are available and in order to avoid complexity at system or subsystem level, values for inheritance may be taken from table 1. From literature it has been found that Wani and Gandhi[22] have used data from previous research for selecting the values of the variables while Kulkarni[28] had used a questionnaire to measure each attribute in terms of weightage to arrive at the values of the variables. Besides this, Faisal et. al.[26] have also explained that numerical values for the off-diagonal elements (c_{ij}'s) representing relative impacts between the variables at system level or subsystem level cannot be measured directly. Rao and Padmanabhan[20] had used the scale proposed by Saaty in his Analytical Hierarchy Process (AHP) to assign values to the relative importance between attributes. However, values can be assigned through proper interpretation by experts[22, 26]. Table 2 suggests these qualitative values of relative impacts of variables.

Table-1
Quantification of factors affecting the system

S. No.	Qualitative measure of factors	Assigned value of factors
1	Exceptionally low	1
2	Very Low	2
3	Low	3
4	Below average	4
5	Average	5
6	Above Average	6
7	High	7
8	Very High	8
9	Exceptionally High	9

Table-2
Quantification of interdependencies/ off diagonal elements

S. No.	Qualitative measure of interdependencies	c_{ij}
1	Very Strong	5
2	Strong	4
3	Medium	3
4	Weak	2
5	Very weak	1

Methodology

The various steps involved in graph theoretic approach are enlisted in sequential manner as below: i. Identify the various sub-systems affecting the main system. ii. Logically develop a diagraph between the system/sub-system depending upon their interdependencies. iii. Develop a variable permanent function matrix at the sub-system level on the basis of digraph developed in step 2. iii. Using the logical values of the inheritances and interdependencies, obtain the permanent functions at the system/subsystem level. The off- diagonal elements of the matrix representation may be obtained from the graphs, knowledge database interpretation or from the excerpts of the expert's opinion. iv. Evaluate the permanent of the variable permanent function at the system/sub-system level. v. Record the results of this study and document them for future analysis.

Conclusion

A graph theoretic approach helps to analyze and understand the system as a whole by identifying system and subsystem up to the component level. Graph theoretic and matrix model consists of digraph representation, matrix representation and permanent representation. Digraph representation is useful for modelling and visual analysis. Matrix representation is useful in analysing the digraph model mathematically and for computer processing. Permanent multinomial function characterizes the system uniquely and the permanent value of a multinomial represents the system by a single number, which is useful for comparison, ranking, and optimum selection. The following features highlight the uniqueness of this approach over other similar approaches: i. It presents a single numerical index for all the parameters. ii. It is a systematic methodology for conversion of qualitative factors to quantitative values, and mathematic modelling gives an edge to the proposed technique over conventional methods. iii. It permits modelling of interdependence of parameters under consideration. iv. It allows visual analysis and computer processing. v. It leads to self-analysis and comparison of different systems.

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