*Research Journal of Engineering Sciences* ISSN 2278 – 9472
Vol. **1(5),** 17-24, November **(2012)** *Res. J. Engineering Sci.*

# Identification of Rheological Parameters of the linear Viscoelastic Model of two species of tropical woods (*Tectona grandis Lf* and *Diospyros mespiliformis*)

**Houanou K.A.[1*], Tchéhouali A.[1] and Foudjet A.E.[2]**
[1]Laboratoire d'Energétique et de Mécanique Appliquée- Ecole Polytechnique d'Abomey Calavi, Université d'Abomey Calavi, BENIN
[2]CRESA Forêt – Bois, Université de Dschang, CAMEROUN

**Available online at: www.isca.in**
Received 15th October 2012, revised 20nd October 2012, accepted 31st October 2012

## Abstract

*The present work aims to determine the rheological parameters for Tectona grandis L.f and Diospyros mespiliformis considering a Zener model. To achieve that goal, creep tests are carried out on wood samples undergoing constant bending stress along the beam. The moisture content of the samples is 12% obtained through a drying process in a kiln. The level of loading of the samples is fixed at 20% of the yield load. Then, the nonlinear least squares method is applied to the Zener rheological model to identify the optimal values of these parameters. The dynamic elastic modulus "E" and the dynamic constant viscosity "$\eta$" have been computed from the non-linear least square method while the instantaneous modulus of elasticity "$E_0$", has been calculated from the Hooke's law for the two wood species.*

**Keywords:** Wood, creep, dynamic elastic modulus, dynamic viscosity constant, constant bending stress.

## Introduction

Wood is a raw material that meets the current major environmental concerns. It is an abundant, biodegradable, renewable resource whose production is little polluting and little costly energy, and whose sustainable use allows the storage of carbon[1]. Moreover, its increased use significantly reduces the employment of plastics, metal and concrete whose manufacturing releases dioxide of carbon[1]. Today, although a large number of wood properties have been investigated; little important informations are available concerning certain mechanical characteristics of wood. Previous studies have shown that a purely elastic modeling of timber used as a structural material does not account for the delayed behavior events presented by this material, even for low stresses[2]. In this context, several studies have been undertaken and have shown that different behavior of the wood is linear viscoelastic for constraints under 35% of the stress fracture. This behavior may be represented by a model of Kelvin-Voigt, order 2 with dashpot presenting some relative maximum differences on the arrows of 1% while a Zener model could give maximum deviations of 5%[2]. The objective of this work is to identify the rheological parameters of two tropical hard woods available in Benin (Diospyros mespiliformis and Tectona grandis L.f) in the linear viscoelastic field using Zener model (figure 1).

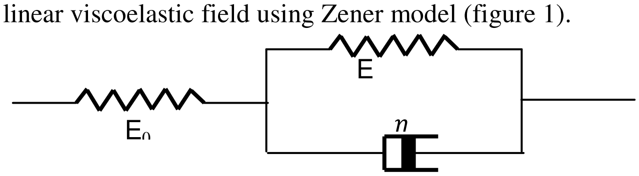


**Figure-1**
**Rheological model of wood**

$E_0$ : Instant elastic modulus, E : Dynamic elastic modulus, $\eta$ : Dynamic constant viscosity.

This model is constituted of a series of Kelvin-Voigt model and spring. The spring characterizes the instantaneous deformation and Kelvin-Voigt model characterizes the own creep.

The corresponding equations are as follows:

$$\begin{cases} \sigma_0 = \varepsilon_0 E_0 \\ \sigma_0 = \varepsilon_1 E + \eta \frac{d\varepsilon_1}{dt} \\ \varepsilon = \varepsilon_0 + \varepsilon_1 \end{cases} \quad (1)$$

Where: $\sigma_0$ = stress, $\varepsilon$ = total strain, $\varepsilon_1$ = delayed strain, $\varepsilon_0$ = instantaneous strain, $\frac{d\varepsilon_1}{dt}$ = creep strain rate, $E$= dynamic elastic modulus, $E_0$ = instantaneous elastic modulus, $\eta$ = dynamic constant viscosity.

The resolution of the differential equation of equation (1) by the Carson-Laplace transformations revealed the following expression:

$$\varepsilon_1(t) = \frac{\sigma_0}{E}\left(1 - e^{-E.t/\eta}\right) \quad (2)$$

with $t \geq 0$.

Thus, the general expression for the total strain is as follows:

$$\varepsilon(t) = \frac{\sigma_0}{E_0} + \frac{\sigma_0}{E}\left(1 - e^{-E.t/\eta}\right) = \sigma_0.\left[\frac{1}{E_0} + \frac{1}{E}.\left(1 - e^{-\frac{E}{\eta}t}\right)\right] \quad (3)$$

## Material and Methods

**Experimental device:** The experimental device is basically the same that Foudjet[2] has used in his work (figure 2 and 3).

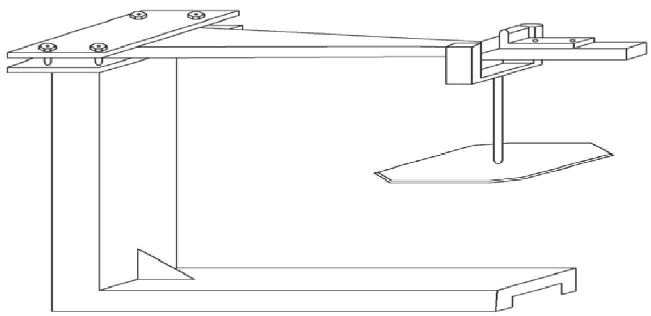

**Figure-2**
**Experimental device for bending test**

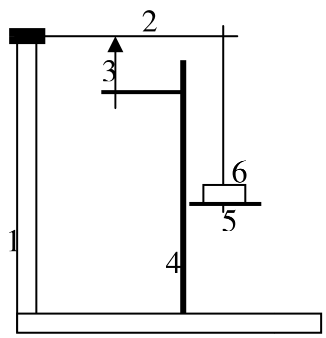


**Figure-3**
**The principle of deflection measurement**
1. Bracket, 2. Sample, 3. Compare, 4. Compare support, 5. Pan suspension, 6. Mass

**Plant material:** Twelve (12) samples were collected in the heart of the wood following the longitudinal direction (figure 4). The samples were conditioned to a moisture content of 12% obtained through a drying process in a kiln. These samples are carefully packaged with foil to prevent the variation of its moisture content during the test. The duration of the experiment is 15 hours. The samples are weighed immediately before and after the test. The test is a two point bending test on beams with uniform extreme fiber stress. During the test, the sample is clamped on one end and a concentrated load is applied on the free end. Then, the extreme fiber stress is maintained to 20% of the stress fracture. The deflection is measured every 30 minutes at a distance of 150 mm from the clamped end by means of a dial gauge whose resolution and race are respectively 0.01mm and 10 mm.

**Method of calculation of longitudinal deformation:** Using the geometric configuration of the distorted beam, the longitudinal deformation Ɛ of beam has been established from below equation[3].

$$\varepsilon = \frac{2fy}{f^2+L^2} \quad (4)$$

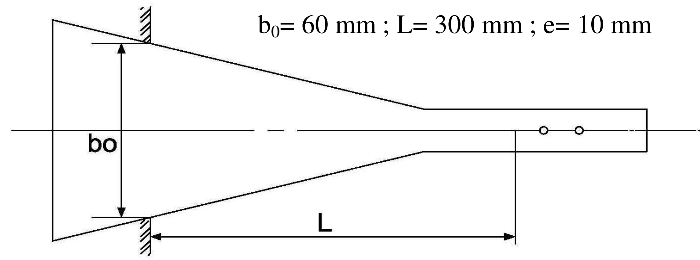


**Figure-4**
**Iso stress sample of creep test**

Where:  : deflection of the beam, $L$ : length of the beam, $y$ : distance from the tenser fiber to the neutral axis.

The neutral axis position relative to the tenser fiber is determined by using the model of the trapezoidal distribution of normal stress on a full surface proposed by PRAGER (figure 5).

Based on Navier-Bernoulli hypothesis and assuming that the quadratic moment of the beam section is constant. The following relationship is established.

$$\frac{y}{h} = \frac{2.\sigma_{uc}.\sigma_{ut}}{(\sigma_{uc}+\sigma_{ut})^2} \quad (5)$$

Where, $\sigma_{ut}$ : tensile stress, $\sigma_{uc}$ : compression stress, $h$ : height of the beam, $y$ : distance from the tenser fiber to the neutral axis with equation (4) and (5), we have:

$$\varepsilon = 4\frac{\sigma_{uc}\sigma_{ut}}{(\sigma_{uc}+\sigma_{ut})^2}\frac{f}{(f^2+L^2)}h \quad (6)$$

Finally, the calculated deformations were used to estimate the rheological parameters of different species considered.

**The parameters identification method:** The identification of the parameters of the law of linear viscoelastic behavior consists in determining the unknown η and E of equation (2) using non-linear least squares method.
On the base of below equation

$$F(t,a,b) = \frac{\sigma_0}{a}\left(1-e^{-\frac{a}{b}t}\right) \quad \text{with } a = E, b = \eta \text{ and } t \geq 0 \quad (7)$$

we adjust the deflection measured ($y_i$) in order to minimize the distance $d_i$ as follow:

$$d_i = \sum_{i=1}^{n}[y_i - F(t,a,b)]^2 \quad (8)$$

By means of equation (8), we have the iterative formulas equation (9) below:

$$\begin{cases} \mathbf{a_{j+1}} = \mathbf{a_j} + \dfrac{(\sum_{i=1}^{n}Ay_i-\sum_{i=1}^{n}AC)\sum_{i=1}^{n}B^2-\sum_{i=1}^{n}AB(\sum_{i=1}^{n}By_i-\sum_{i=1}^{n}BC)}{\sum_{i=1}^{n}A^2\sum_{i=1}^{n}B^2-\left(\sum_{i=1}^{n}AB\right)^2} \\ \mathbf{b_{j+1}} = \mathbf{b_j} + \dfrac{\sum_{i=1}^{n}A^2(\sum_{i=1}^{n}By_i-\sum_{i=1}^{n}BC)-(\sum_{i=1}^{n}Ay_i-\sum_{i=1}^{n}AC)\sum_{i=1}^{n}AB}{\sum_{i=1}^{n}A^2\sum_{i=1}^{n}B^2-\left(\sum_{i=1}^{n}AB\right)^2} \end{cases}$$

with

$$A = \frac{\partial}{\partial a} F(t_i, a_0, b_0) = [-\frac{1}{a_0^2}(1 - e^{-\frac{a_0}{b_0}t_i}) + \frac{t_i}{a_0 b_0} e^{-\frac{a_0 t_i}{b_0}}]\sigma_0$$

$$B = \frac{\partial}{\partial b} F(t_i, a_0, b_0) = [-\frac{t_i}{b_0^2} e^{-\frac{a_0}{b_0}t_i}]\sigma_0$$

$$C = F(t_i, a_0, b_0) = [-\frac{1}{a_0}\left(1 - e^{-\frac{a_0}{b_0}t_i}\right)]\sigma_0$$

With equation (9), the optimal values of $a$ and $b$ have determined. The iteration stopping criterion is set for each parameter as follows:

$$\frac{a_{j+1} - a_j}{a_j} < 10^{-5} \text{ and } \frac{b_{j+1} - b_j}{b_j} < 10^{-5}$$

## Results and Discussion

Table 1 indicates main creep test characteristics such as initial strain $\varepsilon_0$, final strain $\varepsilon_f$, creep coefficient K, creep strain $\Delta\varepsilon$ and instantaneous modulus $E_0$defined as follow:

$$K = \frac{\varepsilon_f}{\varepsilon_0} \quad (10)$$

$$\Delta\varepsilon = \varepsilon_f - \varepsilon_0 \quad (11)$$

$$E_0 = \frac{\sigma_0}{\varepsilon_0} \quad (12)$$

The Different investigations are on one hand used to model the linear viscoelastic behavior of wood and on the over hand to validate this model.

Table 2 presents the optimal value for the parameters of the model obtained by minimizing equation (8).

Figures 6a and 6b show delayed behavior of the two woods species and the experimental and simulated curves.

Using Excel, the parameters E and η have been determined ($10^{-7}$ accuracy). The predictive capability of the model is discussed as follow.

**Test parameters:** The determined parameters (E and η) are tested for each species using the t-Distribution law in considering a risk of 1%. Table 3 shows the different statistic values calculated to carry out test on each parameter. Tables 4 and 5 summarize the different t-Distribution tests executed on parameters which confirm its values identified for each species. The accuracy level of the results doesn't need determination of the confidence interval of the values.

**Predictions fields:** Using statistical law, we have constructed a 99% confidence interval about the regression line (continuous line), and also, 99% prediction interval on future observation (discontinuous line). Figures 7a and 7b illustrate that the prediction limits are always wider than the confidence limits. Table 6 summarizes all rheological parameters that characterize each limit.

**Adequacy of the regression model: residual analysis:** ***Normality of the distribution of residuals:*** For both species, at least 96% of the standardized residuals $d_i = \frac{e_i}{\sqrt{\hat{\sigma}^2}}$ fall in the required field $[-2, +2]$ (Montgomery and Runger 2003). Then, the errors are normally distributed.

**Coefficient of determination $R^2$:** Table 7 presents the coefficient of determination calculated for the studied species. For Tectona grandis, the model accounts for 98% of the variability in the data and for Diospyros mespiliformis, the model accounts for 97% of the variability in the data.

**Validation of the instant modulus of elasticity: Normality of the values distribution:** Figures 8a and 8b show the normal probability plot for instantaneous elastic modulus calculated using HOOKE's law for each of the two species.

**Test on the mean and confidence interval determination**: Test have been carried on the mean value of the instantaneous elastic modulus using the t-Distribution law in considering a 1% risk to check its validity, (tables 8, 9 and 10).

**A futur observation prediction:** The interval in which we could find the (n+1) observation, given the thirteenth one is determined. Table 11 shows the lower and upper bound of this interval.

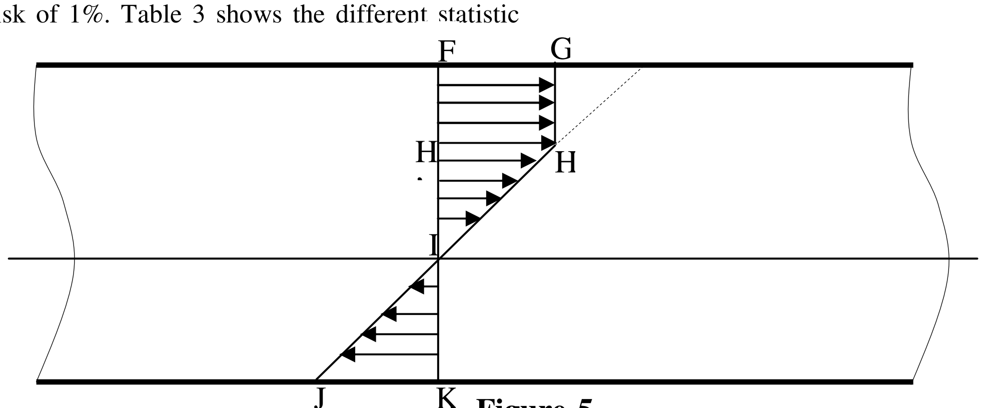


**Figure-5**
**Stress distribution in a straight section of the beam**

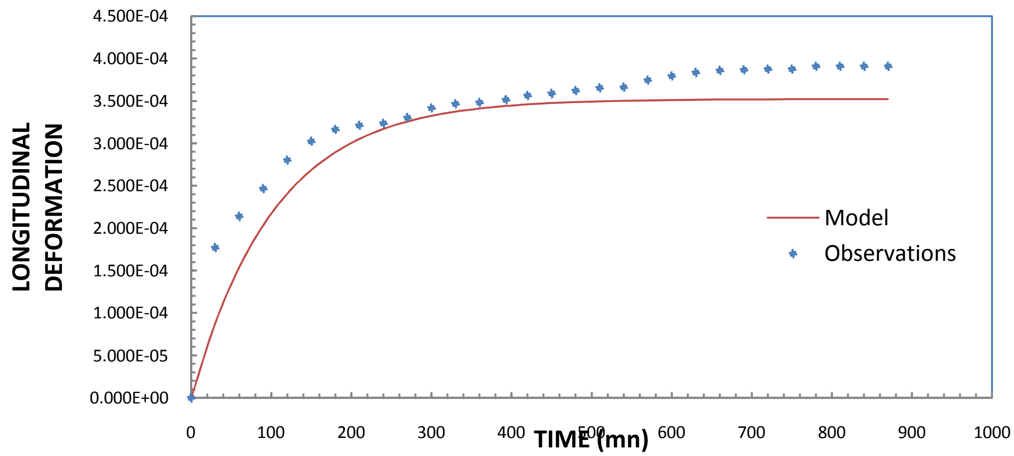


**Figure-6a**
**Evolution of the longitudinal strain of *Tectona grandis Lf* in time**

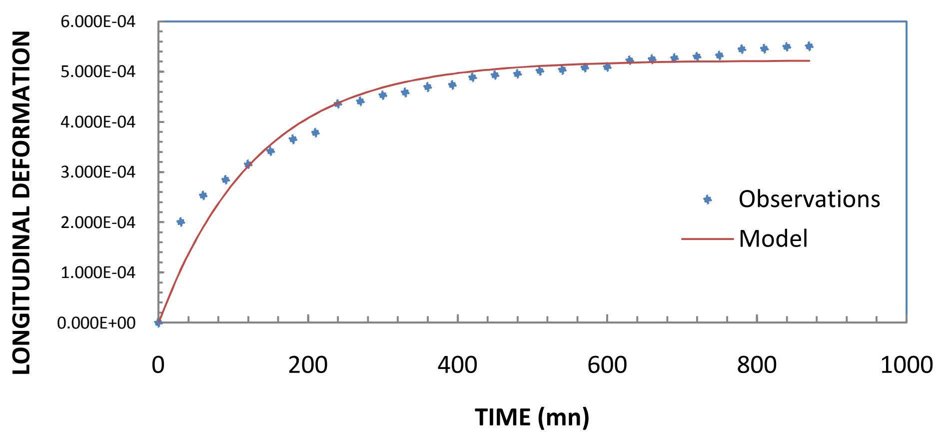


**Figure-6b**
**Evolution of the longitudinal strain of *Diospyros mespiliformis* in time**

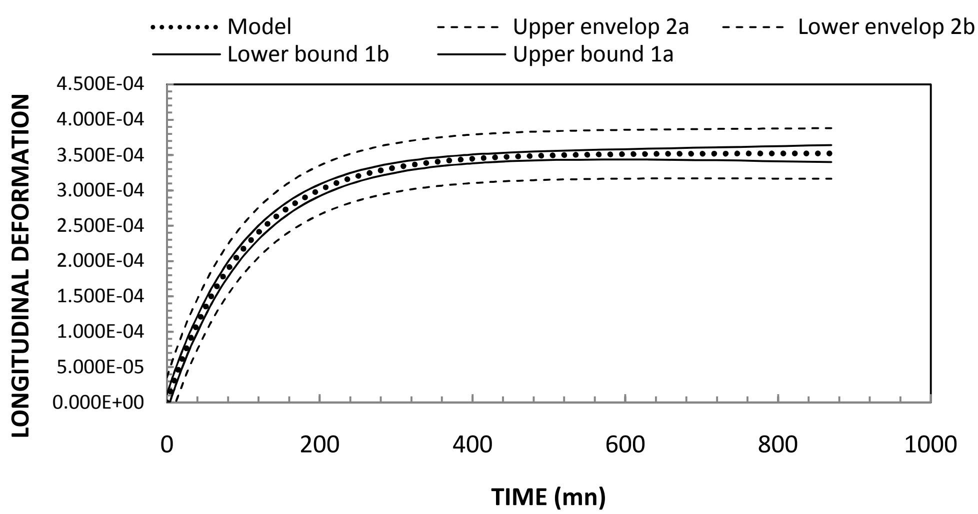


**Figure-7a**
**Limits of the field of prediction of the linear viscoelastic model of *Tectona grandis Lf***

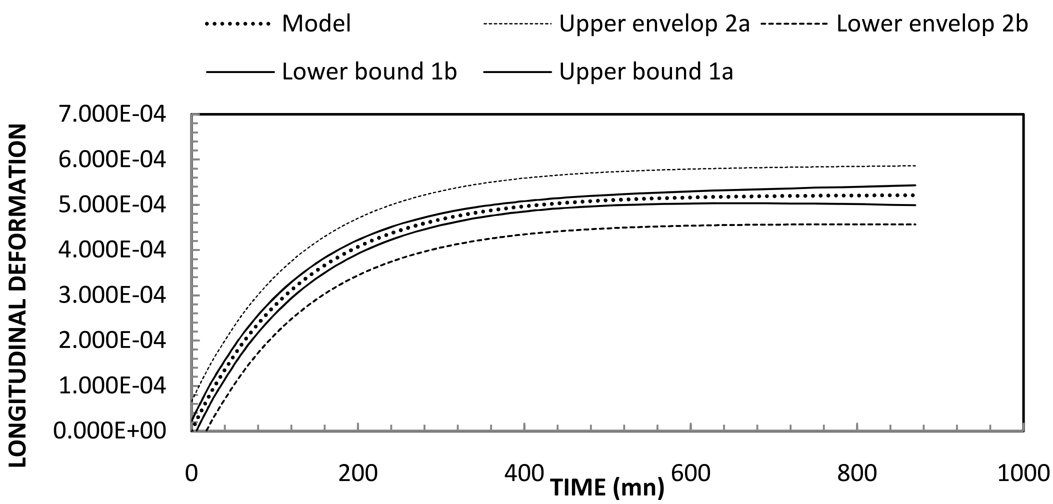


**Figure-7b**
**Limits of the field of prediction of the linear viscoelastic model of *Diospyros mespiliformis***

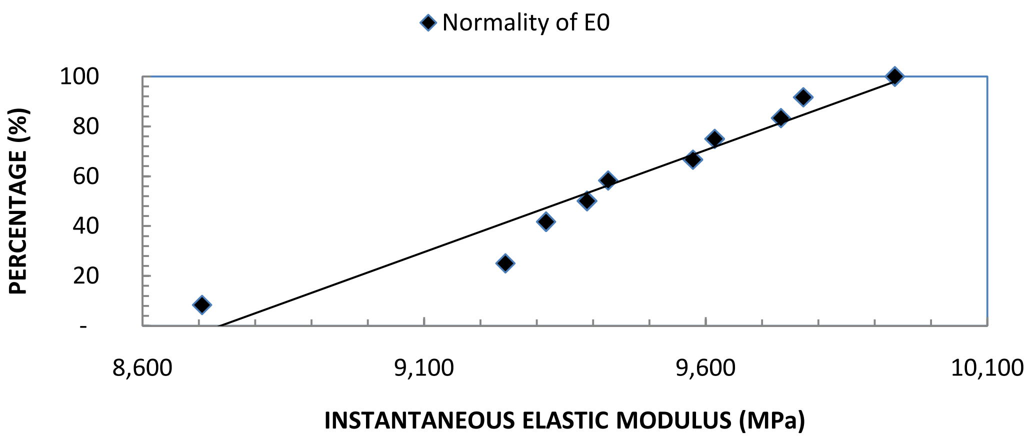


**Figure-8a**
**Normal distribution of the instant elastic modulus of *Tectona grandis L.f***

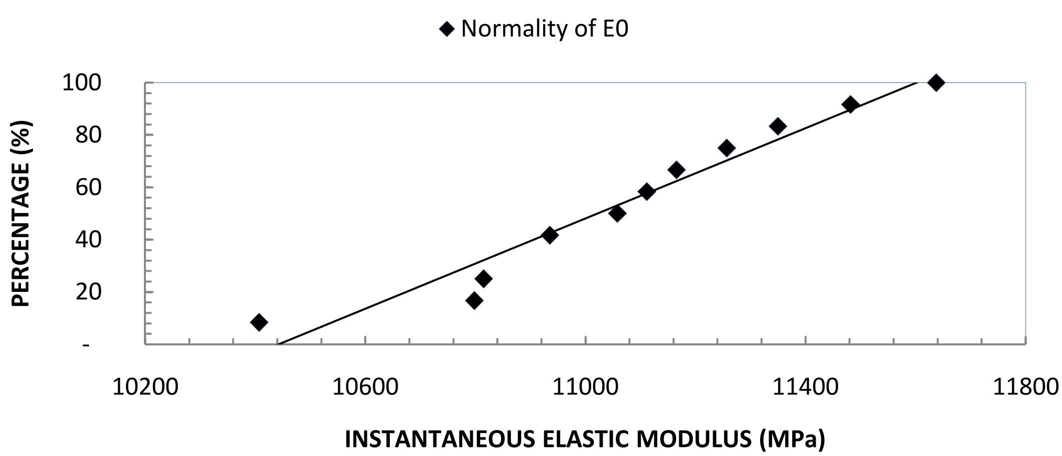


**Figure-8b**
**Normal distribution of the instant elastic modulus of *Diospyros mespiliformis***

**Table-1**
**Main creep test results**

| N° | Data | *Tectona grandis L.f* | | *Diospyros mespiliformis* | |
|---|---|---|---|---|---|
| | | **Modeling** | **Validation** | **Modeling** | **Validation** |
| 1 | Moisture content (%) | 12 | 12 | 12 | 12 |
| 2 | Initial strain $\varepsilon_0$ (%) | 0.2085 | 0.2069 | 0.2323 | 0.2385 |
| 3 | Final strain $\varepsilon_f$ (%) | 0.2453 | 0.2436 | 0.2873 | 0.2919 |
| 4 | Creep coefficient K | 1.18 | 1.18 | 1.24 | 1.22 |
| 5 | Creep $\Delta\varepsilon$ (%) | 0.0368 | 0.0367 | 0.0550 | 0.0534 |
| 6 | Stress (MPa) | 19.6 | 19.6 | 26 | 26 |
| 7 | Instantaneous elastic modulus $E_0$ (MPa) | 9 440 | | 11 080 | |

**Table-2**
**Synthesis of the optimal values of the rheological parameters of the model**

| N° | Data | | *Tectona grandis L.f* | *Diospyros mespiliformis* |
|---|---|---|---|---|
| 1 | Dynamic elastic modulus E | MPa | 55 640 | 49 820 |
| 2 | Dynamic viscosity constant η | MPa.mn | 5 800 000 | 6 580 000 |
| | | MPa.s | 348 000 000 | 394 800 000 |

**Table-3**
**Statistical indicators**

| Indicators | | | *Tectona grandis L.f* | *Diospyros mespiliformis* |
|---|---|---|---|---|
| **Notation** | **Expression** | **Designation** | **Calculated values** | |
| $S_{xx}$ | $\sum_{i=1}^{n}(x_i-\bar{x})^2$ | Sum of squares of x | 2 022 489 | 2 022 489 |
| $SS_E$ | $\sum_{i=1}^{n}(y_i-\hat{y}_i)^2$ | Error sum of squares | $4.18\ 10^{-9}$ | $1.377\ 10^{-8}$ |
| $SS_R$ | $\sum_{i=1}^{n}(\hat{y}_i-\bar{y})^2$ | Regression sum of squares | $2.175\ 10^{-7}$ | $5.269\ 10^{-7}$ |
| $SS_T$ | $\sum_{i=1}^{n}(y_i-\bar{y})^2$ | Total corrected sum of squares | $2.18\ 10^{-7}$ | $4.691\ 10^{-7}$ |
| $\hat{\sigma}^2$ | $\frac{SS_E}{n-2}$ | Unbiaised estimator | $1.49\ 10^{-10}$ | $4.419\ 10^{-10}$ |
| se($\hat{\beta}_1$) | $\sqrt{\frac{\hat{\sigma}^2}{S_{xx}}}$ | Estimated standard error of the slope | $8.594\ 10^{-9}$ | $1.56\ 10^{-8}$ |
| se($\hat{\beta}_0$) | $\sqrt{\hat{\sigma}^2(\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}})}$ | Estimated standard error of the intercept | $4.354\ 10^{-6}$ | $7.902\ 10^{-6}$ |
| $\hat{\beta}_0$ | E | Estimator of E | 55 640 | 49 820 |
| $\hat{\beta}_1$ | η | Estimator of η | 5 800 000 | 6 580 000 |

The number n of observations is equal to 30.

**Table-4**
**Parameter $\hat{\beta}_1$ test**

| Statistical test | *Tectona grandis L.f* | *Diospyros mespiliformis* |
|---|---|---|
| **Hypotheses** | $H_0 : \hat{\beta}_1=0$ , $H_1 : \hat{\beta}_1 \neq 0$ | |
| $t_0 = \frac{\hat{\beta}_1}{se(\hat{\beta}_1)}$ | $6.749\ 10^{+14}$ | $4.219\ 10^{+14}$ |
| **For α=1% and using a t-Distribution table, we have :** $t_{\alpha/2,(n-2)}$ | 2.763 | |
| **Conclusion** | $t_0 > t_{\alpha/2,(n-2)}$ therefore we reject $H_0$. | |
| | Hence $\hat{\beta}_1$=5 800 000 | Hence $\hat{\beta}_1$=6 580 000 |

**Table-5**
**Parameter $\hat{\beta}_0$ test**

| **Statistical test** | ***Tectona grandis L.f*** | ***Diospyros mespiliformis*** |
|---|---|---|
| **Hypotheses** | $H_0 : \hat{\beta}_0 = 0$ <br> $H_1 : \hat{\beta}_0 \neq 0$ | |
| $t_0 = \frac{\hat{\beta}_0}{se(\hat{\beta}_0)}$ | $1.278\ 10^{+10}$ | $6.305\ 10^{+9}$ |
| **For α=1% and using t-Distribution table, we have :** $t_{\alpha/2,(n-2)}$ | 2.763 | |
| **Conclusion** | $t_0 > t_{\alpha/2,(n-2)}$ therefore we reject $H_0$. | |
| | Hence $\hat{\beta}_0$=55 640 | Hence $\hat{\beta}_0$=49 820 |

**Table-6**
**Rheological parameters of behavioral curves**

| **Graph** | ***Tectona grandis L.f*** | | ***Diospyros mespiliformis*** | |
|---|---|---|---|---|
| | **E*** | **η*** | **E*** | **η*** |
| 1a | $5.448\ 10^{4}$ | $5.800\ 10^{6}$ | $4.857\ 10^{4}$ | $6.580\ 10^{6}$ |
| 1b | $5.685\ 10^{4}$ | $5.800\ 10^{6}$ | $5.114\ 10^{4}$ | $6.580\ 10^{6}$ |
| 2a | $5.094\ 10^{4}$ | $5.800\ 10^{6}$ | $4.495\ 10^{4}$ | $6.580\ 10^{6}$ |
| 2b | $6.149\ 10^{4}$ | $5.800\ 10^{6}$ | $5.627\ 10^{4}$ | $6.580\ 10^{6}$ |

**Table-7**
**Test of statistical adequacy**

| Statistical test | | *Tectona grandis L.f* | *Diospyros mespiliformis* |
|---|---|---|---|
| **Coefficient of determination $R^2$** | Formula | $R^2 = 1 - \frac{SS_E}{SS_T}$ | |
| | Value | 98.08% | 97.06% |
| **Interpretation** | | The model helps account for 98% of the explained values. | The model helps account for 97% of the explained values |

**Table-8**
**Statistical indicator of the mean analysis**

| **Indicator** | | | ***Tectona grandis Lf*** | ***Diospyros mespiliformis*** |
|---|---|---|---|---|
| **Notation** | **Expression** | **Designation** | **Calculated values** | |
| S | $\sum_{i=1}^{n} E_{Oi}$ | Sum | 113 279.2 | 132 955.2 |
| $\overline{E}_0$ | $\frac{S}{n}$ | Sample mean | 9 439.93 | 11 079.61 |
| $S_{xx}$ | $\sum_{i=1}^{n} (E_{Oi} - \overline{E}_0)^2$ | Sum of the squares of sample deviation | 1 142 440 | 1 229 263 |
| $s^2$ | $\frac{S_{xx}}{n-1}$ | Sample variance | 103 858.22 | 111 751.2 |
| $s$ | $\sqrt{s^2}$ | Sample standard deviation | 322.27 | 334.29 |

The number of observations n is equal to 12.

**Table-9**
**Mean test**

| **Statistical test** | ***Tectona grandis Lf*** | ***Diospyros mespiliformis*** |
|---|---|---|
| **Hypotheses** | $H_0: \overline{E}_0 = \mu$ <br> $H_1: \overline{E}_0 > \mu$, μ=9 440 | $H_0: \overline{E}_0 = \mu$ <br> $H_1: \overline{E}_0 > \mu$, μ=11 080 |
| $t_0 = \frac{\overline{E}_0 - \mu}{s/\sqrt{n}}$ | $-6.12\ 10^{-5}$ | $-3.36\ 10^{-4}$ |
| **For α=1% and using t-Distribution table, we have :** $t_{\alpha,(n-1)}$ | | |
| **Conclusion** | $t_0 < t_{\alpha,(n-1)}$ Therefore we accept $H_0$. | |
| | Hence $E_0$=9 440 | Hence $E_0$=11 080 |

**Table-10**
**Confidence interval of the mean**

| Statistical value | *Tectona grandis Lf* | *Diospyros mespiliformis* |
|---|---|---|
| **For α=1% and using t-Distribution table, we have :** $t_{\alpha/2,(n-1)}$ | 3.106 | |
| $\Delta E = t_{\alpha/2,(n-1)} \cdot \frac{s}{\sqrt{n}}$ | 289 | 300 |
| $E_0$ | 9 440 | 11 080 |
| **Lower bound** | 9 151 | 10 780 |
| **Upper bound** | 9 729 | 11 379 |

**Table-11**
**Thirteenth observation prediction**

| Statistical value | *Tectona grandis Lf* | *Diospyros mespiliformis* |
|---|---|---|
| **For α=1% and using t-Distribution table, we have :** $t_{\alpha/2,(n-1)}$ | 3.106 | |
| $\Delta E = t_{\alpha/2,(n-1)} \cdot s\sqrt{1+\frac{1}{n}}$ | 1 040 | 1 081 |
| $E_0$ | 9 440 | 11 080 |
| **Lower bound** | 8 400 | 9 999 |
| **Upper bound** | 10 480 | 12 161 |

## Conclusion

On the base of the creep tests carried out on two wood species (Diospyros mespiliformis and Tectona grandis L.f), the key parameters of the linear viscoelastic behavior have been determined. The work allows to determine the characteristics of the spring and damper of the Kelvin-Voigt model associated with the own creep of each species. These tests were used to determine experimentally creep coefficients and instantaneous elastic modulus. This identification completes the development of the linear viscoelastic model of these two species which will help to predict their delayed deformation.

## References

1. Trouy-Triboulot M.C. and Triboulot P., Matériau bois, structure et caractéristiques, Technique de l'Ingénieur, traité de construction, C925, 26 **(2009)**
2. Foudjet A., Contribution à l'étude rhéologique du matériau bois. Thèse ès Science en Génie Civil, Université Claude Bernard de Lyon **(1986)**
3. Bourahla N., Résistance des matériaux de base, GECOTEC, Alger **(2003)**
4. Bousaid E., Contribution à la modélisation des effets différés du bois et du béton sous conditions climatiques variables, Application aux structures mixtes bois-béton, Thèse en Structures, INSA de Lyon **(2003)**
5. Chevalier Y., Comportement élastique et viscoélastique des composites. Technique de l'Ingénieur, traité de Plastiques et composites, A 7 750, 32 **(2009)**
6. Husson J.M., Loi de comportement viscoélastique avec effet mémoire. Application à la mécanosorption dans le bois, Thèse en Génie Civil. Université de Limoges **(2009)**
7. Montgomery D.C., Runger G.C., Applied Statistics and Probability for Engineers, Third edition, John Wiley and Sons, Inc. Arizona State University **(2003)**
8. Nocedal J. and Wright S.J., Numerical Optimization, Springer, New York **(1999)**
9. Salençon J., Viscoélasticité pour le calcul des structures. Les Editions de l'Ecole Polytechnique, Les Presses des Ponts et Chaussées, 156 **(2009)**